Two-phase flow with phase change: Flow Boiling and condensation

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Outline

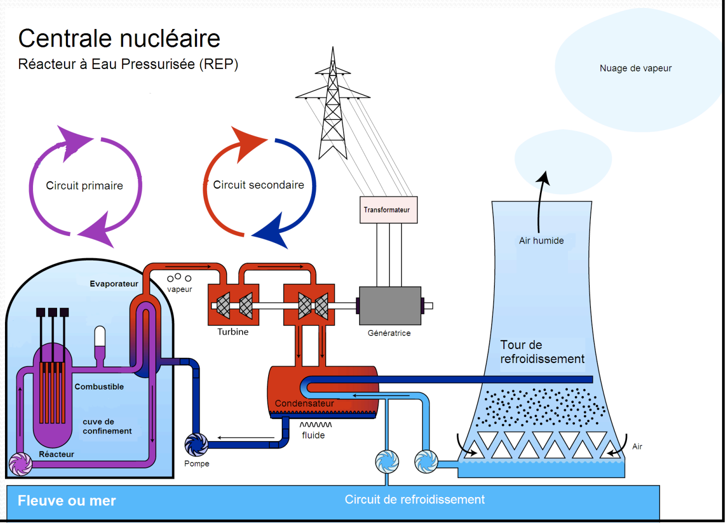
- Industrial applications of two-phase flow with phase change
- Derivation of averaged balance equations for two-phase flows
- Closure laws for wall friction
- Heat Transfer Coefficient in flow boiling
- Convective condensation

Industrial applications: Nuclear Power Plants

PWR : water at 155 bars and 320°C

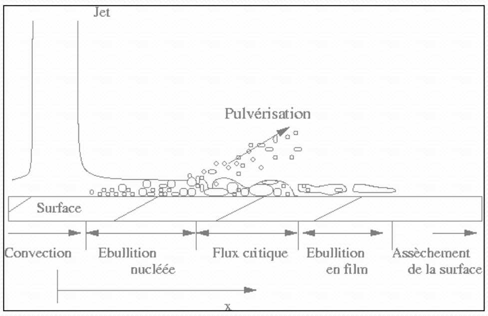
Accidents (LOCA, RIA)

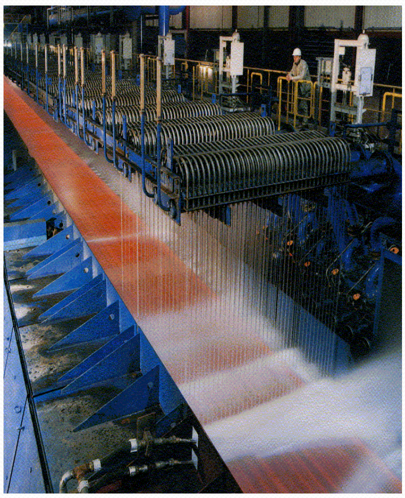
Prediction of Boiling crisis



Industrial applications: Steel industry

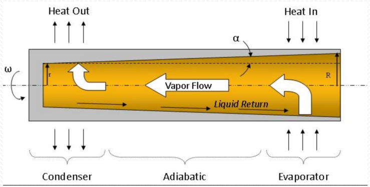
- Cooling down of the rolled steel plates by water jet impingement.



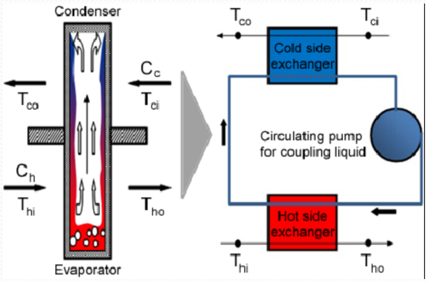


Industrial applications:

- Cooling electronic devices by two-phase flow loops



Heat pipe

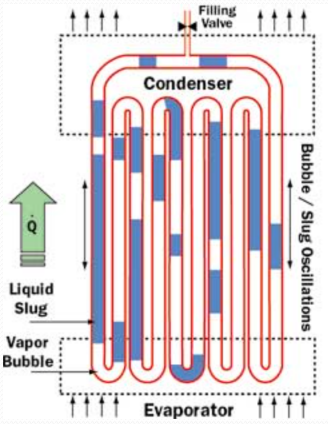


Thermo siphon

Loop heat pipe

Industrial applications:

- Cooling electronic devices by two-phase flow loops

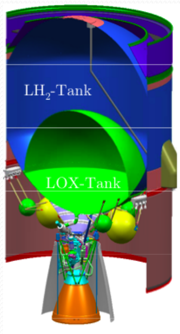


Pulsating Heat Pipe

https://www.youtube.com/watch?v=rG-fneOv1Z8

Industrial applications: space industry

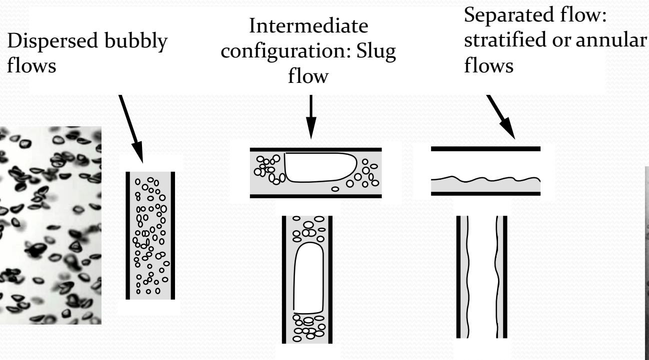
- Propulsion of launchers: fluid management



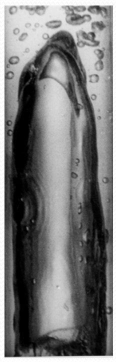
ESC-B/Vinci Engine

3rd stage of Ariane V launcher with cryogenic reservoirs with LOx and LH2
Wall heated by solar radiations
➡No thermal convection in microgravity
➡ Boiling incipience at the wall of the reservoirs.

Problematic of two-phase liquid-vapour flows



What is the flow pattern? Stability?
What is the phase distribution?
Which are the transfers between phases?



General Methodology

- Multiscale analysis and modelling:

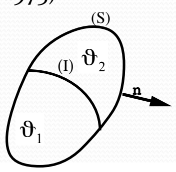
- Local instant equations
- Time averaged equations
- Time and space averaged equations

Lost of information:
Need for closure laws

- Local analysis (bubble motion, stability of a liquid film) -> phenomenogical models, closure laws for averaged equations, calculation of the mean values of velocity, pressure, enthalpy....

Local instant equations

(Ishii, 1975)



•Balance for parameter ϕ_k in phase k

$$\frac{\partial \phi_k}{\partial t} + \nabla \cdot (\phi_k \boldsymbol{u}_k) = \Pi_k - \nabla \cdot (\boldsymbol{\Gamma}_k)$$

•Interfacial balance

$$\nabla \cdot \boldsymbol{\Gamma}_i - 2\kappa \boldsymbol{\Gamma}_i \cdot \boldsymbol{n}_{ik} + \sum_{k=1,2} \left[\phi_k (\boldsymbol{u}_k - \boldsymbol{u}_i) + \boldsymbol{\Gamma}_k \right] \cdot \boldsymbol{n}_{ik} = 0$$

	ϕ_k	Π_k	Γ_k	Γ_i
Mass	ρ_k	0	0	
Momentum	$\rho_k \boldsymbol{U}_k$	$\rho_k \boldsymbol{g}$	$-\Sigma_k = p_k I - \tau_k$	$-\sigma I$
Energy	$\rho_k \left(e_k + \frac{U_k^2}{2} \right)$	$\rho_k r + \rho_k \boldsymbol{g}.\boldsymbol{U}_k$	$-\boldsymbol{U}_k \cdot \Sigma_k + q_k$	
Chemical Specy	$\rho_k C_k$		$\boldsymbol{J}_k = -\rho_k D \nabla C_k$	

Averaged phase equations

- Definition of averaged values

- Statistical average
  Steady flow
- Time average

$$\overline{\phi}(\boldsymbol{r},t) = \lim_{N \longrightarrow \infty} \left(\frac{1}{N} \sum_{i=1}^{N} \phi_i(\boldsymbol{r},t) \right)$$

$$\overline{\phi}(\boldsymbol{r},t) = \lim_{T \to \infty} \left(\frac{1}{T} \int_T \phi dt \right)$$

$\Rightarrow$ Reynolds Relations

$$\overline{\lambda\phi + \varphi} = \lambda\overline{\phi} + \overline{\varphi}$$

$$\overline{\overline{\phi}\varphi} = \overline{\phi}\,\overline{\varphi}$$

$$\overline{\frac{\partial \phi}{\partial t}} = \frac{\partial \overline{\phi}}{\partial t} \quad ; \quad \overline{\Delta\phi} = \Delta\overline{\phi}$$

- Fonction of phase presence χ_k

$$\chi_k(\boldsymbol{x},t) = 1 \quad si \quad \boldsymbol{x} \in k$$

$$\chi_k(\boldsymbol{x},t) = 0 \quad si \quad \boldsymbol{x} \notin k$$

- Presence of phase k $\alpha_k = \overline{\chi}_k$

- Interfacial area concentration $\overline{\delta_I} = \alpha_I$

Averaged phase equations

Instantaneous value $\phi = \overline{\phi} + \phi'$

Phase averaged $\overline{\overline{\phi}}_k = \dfrac{\overline{\chi_k \phi_k}}{\alpha_k}$ $\quad \overline{\chi_k \phi'_k} = 0 \quad$ $\overline{\overline{\phi}}^I = \dfrac{\overline{\delta_I \phi}}{\alpha_I}$ $\quad \overline{\delta_I} = \alpha_I$

Statistical average: $\overline{\phi} = \alpha_l \overline{\overline{\phi}}_l + \alpha_g \overline{\overline{\phi}}_g + \alpha_i \overline{\overline{\phi}}_i^{\,i}$

$$\frac{\partial \alpha_k \overline{\overline{\phi}}_k}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\phi_k \boldsymbol{u}_k}} + \alpha_k \overline{\overline{\boldsymbol{\Gamma}_k}} \right) - \alpha_k \overline{\overline{\Pi}}_k + \alpha_i \overline{\overline{\left[\phi_k (\boldsymbol{u}_k - \boldsymbol{u}_i) + \boldsymbol{\Gamma}_k \right] \cdot \boldsymbol{n}_{ik}}}^{\,i} = 0$$

$$\alpha_i \left[\overline{\overline{\nabla_i \cdot (\boldsymbol{\Gamma}_i)}}^{\,i} - \overline{\overline{2\kappa \boldsymbol{\Gamma}_i \boldsymbol{n}_{ik}}}^{\,i} \right] + \alpha_i \sum_{k=l,g} \overline{\overline{\left[\phi_k (\boldsymbol{u}_k - \boldsymbol{u}_i) + \boldsymbol{\Gamma}_k \right] \cdot \boldsymbol{n}_{ik}}}^{\,i} = 0$$

Averaged phase equations

- Mass conservations

$$\frac{\partial \alpha_k \overline{\overline{\rho_k}}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k \boldsymbol{u_k}}} \right) = -\alpha_i \overline{\overline{\left[\rho_k \left(\boldsymbol{u_k} - \boldsymbol{u}_i \right) \right] \cdot \boldsymbol{n}_{ik}}}^i = -\alpha_i \overline{\overline{\dot{m}_k}}^i \qquad \sum_{k=l.g} \overline{\overline{\dot{m}_k}}^i = 0$$

- Momentum balance

$$\frac{\partial \alpha_k \overline{\overline{\rho_k \boldsymbol{u_k}}}}{\partial t} + \nabla \cdot \left(\alpha_k \overline{\overline{\rho_k \boldsymbol{u_k} \otimes \boldsymbol{u_k}}} \right) - \alpha_k \rho_k \boldsymbol{g} + \nabla \left(\alpha_k \overline{\overline{p_k}} \right) - \nabla \cdot \left(\alpha_k \overline{\overline{\boldsymbol{\tau_k}}} \right)$$
$$= -\alpha_i \overline{\overline{\left[\rho_k \boldsymbol{u_k} \left(\boldsymbol{u_k} - \boldsymbol{u}_i \right) \cdot \boldsymbol{n}_{ik} + p_k \boldsymbol{n}_{ik} - \boldsymbol{\tau_k} \cdot \boldsymbol{n}_{ik} \right]}}^i = -\alpha_i \overline{\overline{\dot{m}_k \boldsymbol{u_{ki}}}}^i + \alpha_i \boldsymbol{I_k}$$

$$\alpha_i \left[\overline{\overline{\nabla_i \sigma}}^i - \overline{\overline{2\kappa\sigma \boldsymbol{n}_{ik}}}^i \right] = \alpha_i \left[\sum_{k=l,g} \left(\boldsymbol{I_k} - \overline{\overline{\dot{m}_k \boldsymbol{u_{ki}}}}^i \right) \right]$$

- Total enthalpie balance

$$\frac{\partial \alpha_k \rho_k \overline{\overline{h_{tk}}}}{\partial t} + \nabla \cdot \left(\alpha_k \rho_k \overline{\overline{h_{tk} \boldsymbol{u}_k}} \right) = \nabla \cdot \left(\alpha_k \overline{\overline{\boldsymbol{u}_k \boldsymbol{\tau}_k}} \right) - \nabla \cdot \left(\alpha_k \overline{\overline{q_k}} \right) + \alpha_k \left(\rho_k \overline{\overline{r}} + \rho_k \overline{\overline{\boldsymbol{g}.\boldsymbol{u}_k}} \right) + \frac{\partial \alpha_k \overline{\overline{p_k}}}{\partial t}$$
$$- \alpha_i \overline{\overline{\left[\dot{m}_k h_{tk} + \left(p_k \boldsymbol{u}_i - \boldsymbol{u}_k . \boldsymbol{\tau}_k + \boldsymbol{q_k} \right) \cdot \boldsymbol{n}_{ik} \right]}}^i$$

$$\sum_{k=l,g} \alpha_i \overline{\overline{\left[\dot{m}_k \left(h_k + \frac{1}{2} \frac{\dot{m}_k^2}{\rho_k^2} - \frac{\boldsymbol{n}_{ik} \cdot \boldsymbol{\tau_k} \cdot \boldsymbol{n}_{ik}}{\rho_k} \right) + \boldsymbol{q_k} \cdot \boldsymbol{n}_{ik} \right]}}^i = 0$$

Averaged phase equations

- Interfacial momentum balance

$$\nabla_i \cdot \sigma + 2\kappa\sigma \boldsymbol{n}_{il} + \sum_{k=l,g} \left[\dot{m}_k \boldsymbol{U}_k + p_k \boldsymbol{n}_{ik} - \tau_k \cdot \boldsymbol{n}_{ik} \right] = 0$$

In the direction normal to the interface

$$2\kappa\sigma + \left[\dot{m}_l (U_{\ln} - U_{gn}) + p_l - p_g - (\tau_{\ln n} - \tau_{gnn}) \right] = 0$$

Without flow and phase change

↓

Laplace law: $p_l - p_g + 2\kappa\sigma = 0$

Along the interface

$$\nabla_i \cdot \sigma + \left[\dot{m}_l (\boldsymbol{U}_{lt} - \boldsymbol{U}_{gt}) - \tau_{lt} + \tau_{gt} \right] = 0$$

with $\boldsymbol{U}_{t1} = \boldsymbol{U}_{t2}$

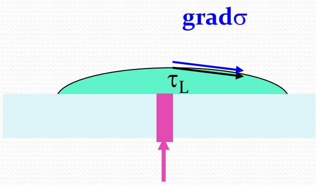
↓

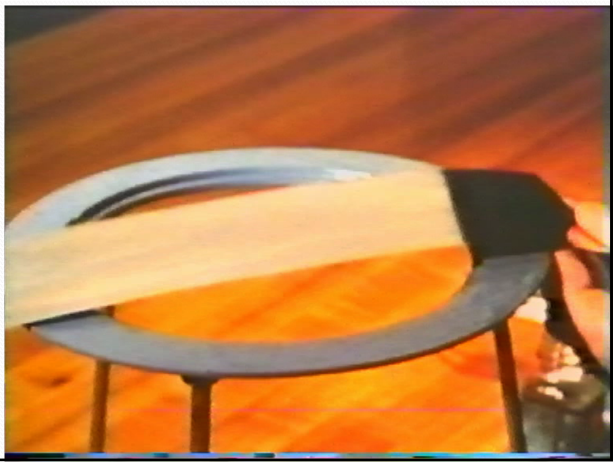
$$\tau_{1t} - \tau_{2t} = \nabla_I \sigma$$

Marangoni convection

$$\boldsymbol{\tau}_L - \boldsymbol{\tau}_G = \boldsymbol{t}.(\boldsymbol{\Sigma}_L - \boldsymbol{\Sigma}_G)\boldsymbol{n} = \boxed{grad_s \sigma} \rightarrow$$

Gradient de tension de surface dû à un gradient de température





Averaged Phase Equations

•Total enthalpy balance of phase k

$$h_{tk} = e_k + \frac{1}{2}U_k^2 + \frac{p_k}{\rho_k}$$

$$\frac{\partial}{\partial t}\rho_k h_{tk} + \nabla\cdot\left[\rho_k h_{tk}\boldsymbol{u_k}\right] = \rho_k\left(\boldsymbol{g_k}\cdot\boldsymbol{u_k} + r\right) + \nabla\cdot\left(\boldsymbol{\tau}_k\cdot\boldsymbol{u_k}\right) - \nabla\cdot\boldsymbol{q_k} + \frac{\partial p_k}{\partial t}$$

•Interfacial Balance

$$\sum_{k=l,g}\left[\dot{m}_k\left(h_k + \frac{1}{2}\frac{\dot{m}_k^2}{\rho_k^2} - \frac{\boldsymbol{n_{ik}}\cdot\boldsymbol{\tau_k}\cdot\boldsymbol{n_{ik}}}{\rho_k}\right) + \boldsymbol{q_k}\cdot\boldsymbol{n_{ik}}\right] = 0$$

$$\Rightarrow \quad \dot{m}_l\left(h_g - h_l\right) = \dot{m}_l h_{gl} = \boldsymbol{q}_g\cdot\boldsymbol{n}_{ig} + \boldsymbol{q}_l\cdot\boldsymbol{n}_{il} = \left(\boldsymbol{q}_l - \boldsymbol{q}_g\right)\cdot\boldsymbol{n}_{il}$$

$\dot{m}_l h_{lv} \approx \boldsymbol{q}_l.\boldsymbol{n}_{il} > 0$ vaporization

$\dot{m}_l h_{lv} \approx \boldsymbol{q}_l.\boldsymbol{n}_{il} < 0$ condensation

Resolution of equations

3 mass conservation equations
3x3 momentum balance equations
3 enthalpy balance
1 topological equation $\alpha_l + \alpha_g = 1$

16 equations

21 unknowns :

$$\alpha_l, \alpha_g, \alpha_i, \overline{\overline{\dot{m}_l}}, \overline{\overline{\dot{m}_g}}$$
$$\overline{\overline{\boldsymbol{u}_l}}, \overline{\overline{\boldsymbol{u}_g}}, \overline{\overline{p_l}}, \overline{\overline{p_g}}, \overline{\boldsymbol{I}_l}, \overline{\boldsymbol{I}_g}$$
$$\overline{\overline{h_l}}, \overline{\overline{h_g}}$$

Need for closure laws

2 mass conservation equations
2x3 momentum balance equations
3 enthalpy balance
1 topological equation $\alpha_l + \alpha_g = 1$

12 Equations

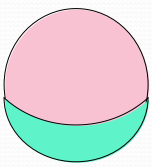
14 unknowns
$$\begin{cases} \alpha_l, \alpha_g, \alpha_i, \overline{\overline{\dot{m}_l}}, \\ \overline{\overline{\boldsymbol{u}_l}}, \overline{\overline{\boldsymbol{u}_g}}, \overline{\overline{p_l}}, \overline{\overline{p_g}}, \\ \overline{\overline{h_l}}, \overline{\overline{h_g}} \end{cases}$$

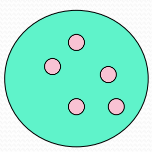
Modeling of $\overline{\overline{\boldsymbol{I}_l}}, \overline{\overline{\boldsymbol{I}_g}}$

1 closure law $\overline{\overline{p_l}} = \overline{\overline{p_g}}$
+1 transport equation for α_i

(Kocamustafaogullari & Ishii M., 1983, Morel et al., 1999)

Equations integrated over the tube section





A : tube section $\quad R_k = \dfrac{A_k}{A} = \dfrac{1}{A}\int_A \alpha_k dA$

A_g : section occupied by the gas phase $\quad R_g = \dfrac{A_g}{A}$

A_l : section occupied by the liquid phase $\quad R_l = \dfrac{A_l}{A} = 1 - R_g$

R_g (-): mean void fraction

J_l, j_g (m/s) : superficial velocities $\quad j_l = \dfrac{Q_l}{A}$

U_l, U_g (m/s) : mean velocities $\quad U_g = \dfrac{Q_g}{A_g} = \dfrac{j_g}{R_g} \quad U_l = \dfrac{Q_l}{A_l}$

x (-) : quality $\quad x = \dfrac{\dot{m}_v}{\dot{m}}$

$\dot{m}$ (kg/s): mass flow rate

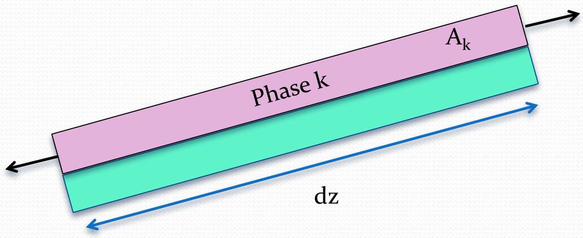
G (kg/m²/s) : mass flux

$$j_g = \frac{Gx}{\rho_g} \qquad j_l = \frac{G(1-x)}{\rho_l}$$

$$U_g = \frac{Gx}{\rho_g R_g} \qquad U_l = \frac{G(1-x)}{\rho_l R_l}$$

$$x = \frac{1}{1 + \frac{\rho_l}{\rho_g}\frac{U_l}{U_g}\frac{(1-R_g)}{R_g}} \qquad R_g = \frac{x\frac{\rho_l}{\rho_g}\frac{U_l}{U_g}}{1 - x\left(1 - \frac{\rho_l}{\rho_g}\frac{U_l}{U_g}\right)}$$

Mass conservation in the tube section



$$\frac{\partial \rho_k A_k dz}{\partial t} = \rho_k A_k U_k\big|_z - \rho_k A_k U_k\big|_{z+dz} - \dot{M}_k A dz$$

$$\frac{1}{A}\frac{\partial \rho_k A_k}{\partial t} + \frac{1}{A}\frac{\partial \rho_k A_k U_k}{\partial z} = \frac{\partial \rho_k R_k}{\partial t} + \frac{\partial \rho_k R_k U_k}{\partial z} = -\dot{M}_k$$

Mass conservation equations

$$\frac{\partial R_k \rho_k}{\partial t} + \frac{\partial}{\partial z}(R_k \rho_k U_k) = -\dot{M}_k \quad \textit{with} \quad \dot{M}_k = -\frac{1}{A}\int_A \alpha_I \overline{\left[\rho_k(\boldsymbol{U}_k - \boldsymbol{U}_I)\right]\cdot \boldsymbol{n}_{ik}}^{\,i} dA$$

$\dot{M}_k$: mass flow rate per unit volume from the phase k through the interface

U_l , U_g : mean liquid and gas velocities in the tube section

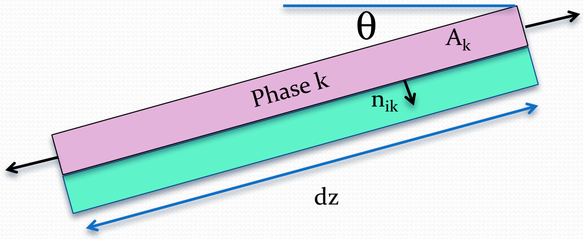
$R_g + R_l = 1$

vapor $$\frac{\partial \rho_g R_g}{\partial t} + \frac{\partial \rho_g R_g U_g}{\partial z} = -\dot{M}_g = \dot{M}_l$$

liquid $$\frac{\partial \rho_l (1-R_g)}{\partial t} + \frac{\partial \rho_l (1-R_g) U_l}{\partial z} = -\dot{M}_l$$

Mixture $$\frac{\partial\left[\rho_l (1-R_g) + \rho_g R_g\right]}{\partial t} + \frac{\partial\left[\rho_l (1-R_g) U_l + \rho_g R_g U_g\right]}{\partial z} = 0$$

Momentum balance in the tube section



$$P_i < \alpha_i n_{ik}.n_z > Adz$$
$$= P_i < \nabla \alpha_k . n_z > Adz$$
$$= P_i \nabla R_k . n_z Adz$$
$$= P_i \frac{dR_k}{dz} Adz$$

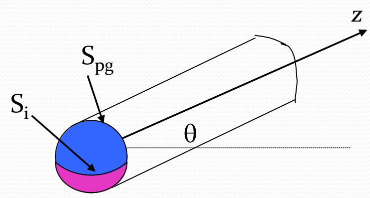
$$\frac{\partial \rho_k U_k A_k dz}{\partial t} = \rho_k A_k U_k^2\Big|_z - \rho_k A_k U_k^2\Big|_{z+dz} + P_k A_k\Big|_z - P_k A_k\Big|_{z+dz} + P_i dz \frac{dA_k}{dz}$$

$$-\rho_k g A_k dz \sin\theta - \tau_{pk} S_{pk} dz + \tau_{ik} S_{ik} dz - \dot{M}_k u_i Adz$$

$$\frac{\partial \rho_k U_k R_k}{\partial t} + \frac{\partial \rho_k R_k U_k^2}{\partial z} = -\frac{\partial P_k R_k}{\partial z} + P_i \frac{dR_k}{dz} - \rho_k g R_k \sin\theta - \tau_{pk} \frac{S_{pk}}{A} + \tau_{ik} \frac{S_{ik}}{A} - \dot{M}_k u_i$$

Momentum balance equations

Model with one pressure $p_l = p_g = p$



wall shear stress

Interfacial shear stress

vapor $$\frac{\partial \rho_g R_g U_g}{\partial t} + \frac{1}{A}\frac{\partial \rho_g R_g U_g^2 A}{\partial z} = -R_g \frac{\partial p}{\partial z} + \frac{\tau_{pg} S_{pg}}{A} + \frac{\tau_{ig} S_i}{A} - \rho_g R_g g \sin\theta + \dot{M}_l U_i$$

liquid $$\frac{\partial \rho_l (1-R_g) U_l}{\partial t} + \frac{1}{A}\frac{\partial \rho_l (1-R_g) U_l^2 A}{\partial z} = -(1-R_g)\frac{\partial p}{\partial z} + \frac{\tau_{pl} S_{pl}}{A} + \frac{\tau_{il} S_i}{A} - \rho_l (1-R_g) g \sin\theta - \dot{M}_l U_i$$

mixture $$\frac{\partial\left[\rho_l (1-R_g) U_l + \rho_g R_g U_g\right]}{\partial t} + \frac{1}{A}\frac{\partial\left[\rho_l (1-R_g) U_l^2 A + \rho_g R_g U_g^2 A\right]}{\partial z}$$

$$\tau_{ig} = -\tau_{il} = \tau_i$$

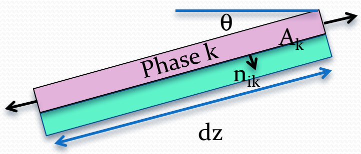
$$= -\frac{\partial p}{\partial z} + \frac{(\tau_{pl} + \tau_{pg}) S_p}{A} - \left[\rho_l (1-R_g) + \rho_g R_g\right] g \sin\theta$$

Energy conservation in the tube section

$$\frac{\partial \rho_k\left(e_k+\frac{U_k^2}{2}\right)A_k dz}{\partial t}+\frac{\partial \rho_k\left(e_k+\frac{U_k^2}{2}\right)U_k A_k dz}{\partial z}=q_{pk}S_{pk}dz+q_{ik}S_{ik}dz+r_kA_kdz$$

$$-\frac{\partial P_k A_k U_k}{\partial z}-P_i dzA\frac{dR_k}{dt}-\rho_k g U_k A_k dz\sin\theta+\tau_{ik}U_iS_{ik}dz-\dot{M}_k H_{ik} Adz$$

$$P_i<u_i n_{ik}\alpha_i>Adz=P_i<\frac{d\alpha_k}{dt}>Adz=P_i\frac{dR_k}{dt}Adz$$



Total Enthalpy $\quad H_{tk}=e_k+\frac{U_k^2}{2}+\frac{P_k}{\rho_k}-gz\sin\theta\approx e_k+\frac{P_k}{\rho_k}\simeq H_k$

$$\frac{\partial\rho_kR_kH_{tk}}{\partial t}+\frac{\partial\rho_kR_kH_{tk}U_k}{\partial z}=q_{pk}\frac{S_{pk}}{A}+q_{ik}\frac{S_{ik}}{A}+r_kR_k-R_k\frac{dP}{dt}+\frac{\tau_{ik}U_iS_{ik}}{A}-\dot{M}_kH_{ik}$$

Enthalpy balance equations

Parameters

Total enthalpy (J/kg) $\quad H_{tk}=H_k+\frac{U_k^2}{2}-gz\sin\theta\approx H_k$

Source per unit volume r_K (W/kg) $\quad$ Heat flux q (W/m²) $\quad$ negligible

vapor $\quad \frac{\partial\rho_gR_gH_{tg}}{\partial t}+\frac{1}{A}\frac{\partial\rho_gR_gH_{tg}U_gA}{\partial z}=R_gr_g+\frac{q_{pg}S_{pg}}{A}+\frac{q_{ig}S_i}{A}+\dot{M}_lH_{ig}+R_g\frac{\partial p}{\partial t}+\xi\frac{\tau_iS_iU_i}{A}$

liquid

$$\frac{\partial\rho_l(1-R_g)H_{tl}}{\partial t}+\frac{1}{A}\frac{\partial\rho_l(1-R_g)H_{tl}U_lA}{\partial z}=(1-R_g)r_l+\frac{q_{pl}S_{pl}}{A}+\frac{q_{il}S_i}{A}-\dot{M}_lH_{il}+(1-R_g)\frac{\partial p}{\partial t}-\xi\frac{\tau_iS_iU_i}{A}$$

mixture

$$\frac{\partial\left[\rho_gR_gH_{tg}+\rho_l(1-R_g)H_{tl}\right]}{\partial t}+\frac{1}{A}\frac{\partial\left[\rho_gR_gH_{tg}U_{gl}A+\rho_l(1-R_g)H_{tl}U_lA\right]}{\partial z}$$
$$=(1-R_g)r_l+R_gr_g+\frac{q_pS_p}{A}+\frac{\partial p}{\partial t}$$

$$\boxed{\dot{M}_l(H_{ig}-H_{il})+\frac{S_i}{A}(q_{ig}+q_{il})=0}$$

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Solving the system of 6 equations

$$\left\{\begin{array}{l} \dfrac{\partial \rho_g R_g}{\partial t} + \dfrac{\partial \rho_g R_g U_g}{\partial z} = \dot{M}_l \\ \dfrac{\partial \rho_l (1-R_g)}{\partial t} + \dfrac{\partial \rho_l (1-R_g) U_l}{\partial z} = -\dot{M}_l \end{array}\right.$$

$$U_g = \frac{Gx}{\rho_g R_g} \quad et \quad U_l = \frac{G(1-x)}{\rho_l R_l}$$

$$\left\{\begin{array}{l} \dfrac{\partial \rho_g R_g U_g}{\partial t} + \dfrac{1}{A}\dfrac{\partial \rho_g R_g U_g^2 A}{\partial z} = -R_g \dfrac{\partial p}{\partial z} + \dfrac{\tau_{pg} S_{pg}}{A} + \dfrac{\tau_{ig} S_i}{A} - \rho_g R_g g \sin\theta + \dot{M}_l U_i \\ \dfrac{\partial \rho_l (1-R_g) U_l}{\partial t} + \dfrac{1}{A}\dfrac{\partial \rho_l (1-R_g) U_l^2 A}{\partial z} = -(1-R_g)\dfrac{\partial p}{\partial z} + \dfrac{\tau_{pl} S_{pl}}{A} + \dfrac{\tau_{il} S_i}{A} - \rho_l (1-R_g) g \sin\theta - \dot{M}_l U_i \end{array}\right.$$

$$\tau_{ig} = -\tau_{il} = \tau_i$$

$$\left\{\begin{array}{l} \dfrac{\partial \rho_g R_g H_g}{\partial t} + \dfrac{1}{A}\dfrac{\partial \rho_g R_g H_g U_g A}{\partial z} = R_g r_g + \dfrac{q_{pg} S_{pg}}{A} + \dfrac{q_{ig} S_i}{A} + \dot{M}_l H_{ig} \\ \dfrac{\partial \rho_l (1-R_g) H_l}{\partial t} + \dfrac{1}{A}\dfrac{\partial \rho_l (1-R_g) H_l U_l A}{\partial z} = (1-R_g) r_l + \dfrac{q_{pl} S_{pl}}{A} + \dfrac{q_{il} S_i}{A} - \dot{M}_l h_{il} \end{array}\right.$$

$$\dot{M}_l H_{lg} + \frac{S_i}{A}(q_{ig} + q_{il}) = 0$$

6 main unknowns $R_g, U_g, U_l, p, H_l, H_g$ or G, x, R_g, p, H_l, H_g

Unknowns to be modelled $\dot{M}_l, \tau_{pl}, \tau_{pg}, \tau_{ig}, U_i, q_{pg}, q_{pl}, q_{il}, S_{pg} / S, S_i$

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Equations for the mixture

Remark: the vapour phase is generally at saturation temperature T_{sat}

For the 2 phases in thermodynamical equilibrium $H_l(T_{sat}), H_g(T_{sat})$ are known

Enthalpy balance gives access to quality x

$$\frac{1}{A}\frac{\partial\left[\rho_g R_g H_g U_g + \rho_l (1-R_g) H_l U_l\right]}{\partial z} = \frac{q_p S_p}{A}$$

$$\frac{\partial\left[GxH_{g,sat} + G(1-x)H_{l,sat}\right]}{\partial z} \approx G(H_{g,sat} - H_{l,sat})\frac{dx}{dz} \quad \Rightarrow \quad G h_{lg} \boxed{\frac{dx}{dz}} = \frac{q_p S_p}{A} = \frac{q_p 4}{D}$$

Equations of mass conservation and enthalpy balance are linked

Simplification : no need for modelling the interfacial terms

System of 6 equations ➡ System of 4 equations

1 Mass conservation equation $\quad G\dfrac{dx}{dz} = \dot{M}_l$

2 Momentum balances

1 Enthalpy balance for the mixture

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Equations for the mixture

If the velocities of the 2 phases are linked

2 equations of momentum balance are replaced by:

1 equation for the momentum balance of the mixture:

$$\frac{1}{A}\frac{\partial\left(\rho_l(1-R_g)U_l^2+\rho_g R_g U_g^2\right)A}{\partial z}=\frac{d}{dz}\left[\frac{Gx^2}{\rho_g R_g}+\frac{G(1-x)^2}{\rho_l(1-R_g)}\right]$$

$$=-\frac{\partial p}{\partial z}+\frac{\tau_p S_p}{A}-\left(\rho_l(1-R_g)+\rho_g R_g\right)g\sin\theta$$

+ 1 relation f (U_g , U_l , R_g)=0

Homogeneous model $U_g = U_l$ ➔ system of 3 équations

Simplification: no modelling of the interfacial area concentration and interfacial shear stress needed.

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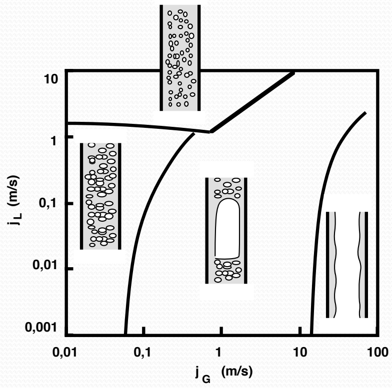
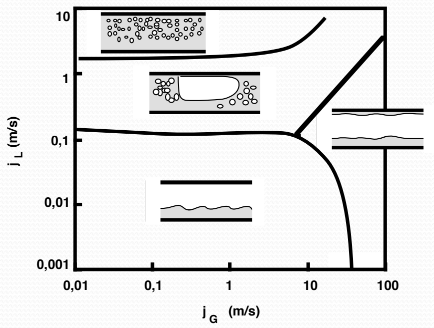
Closure laws

- Void fraction
- Interfacial perimeter S_i, wetted perimeters S_{pg}, S_{pl} depend of the flow topology
- Wall shear stress τ_p and interfacial shear stress τ_i
- Wall heat flux q_p and interfacial heat flux q_i, specific modelling in boiling and condensation.

Flow patterns in adiabatic two-phase flows



Air-water flow in vertical tube of 5 cm dia., Taitel *et al.*, (1980)

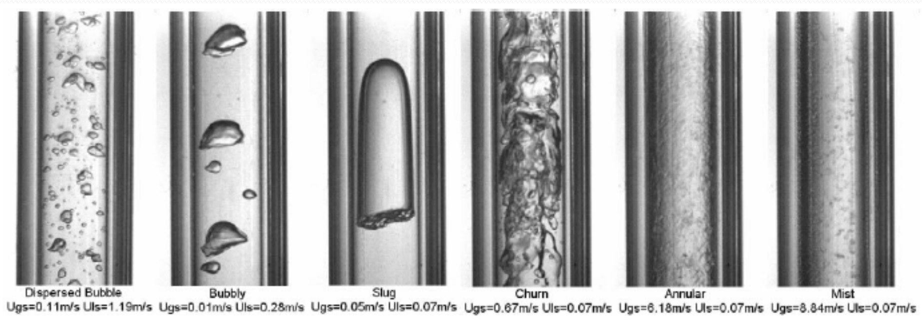


Air-water flow in horizontal tube of 5.1 cm dia., Mandhane, (1974)

Two-phase flow with phase change: same flow patterns + 1 configuration vapor + liquid droplet.

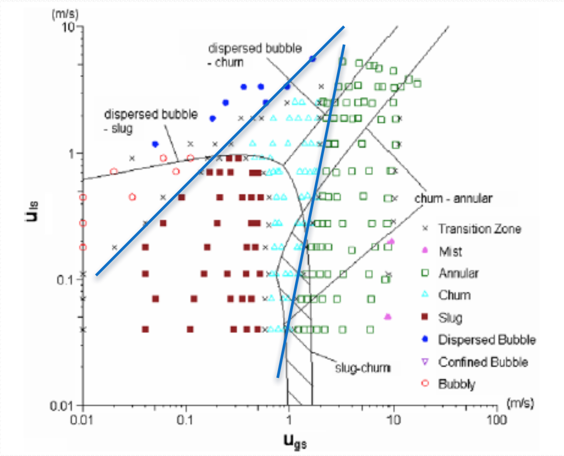
Flow patterns in convective boiling at low heat flux

Chen & Karayannis, 2006-→ refrigerant R134a in 4.26mm dia. tube



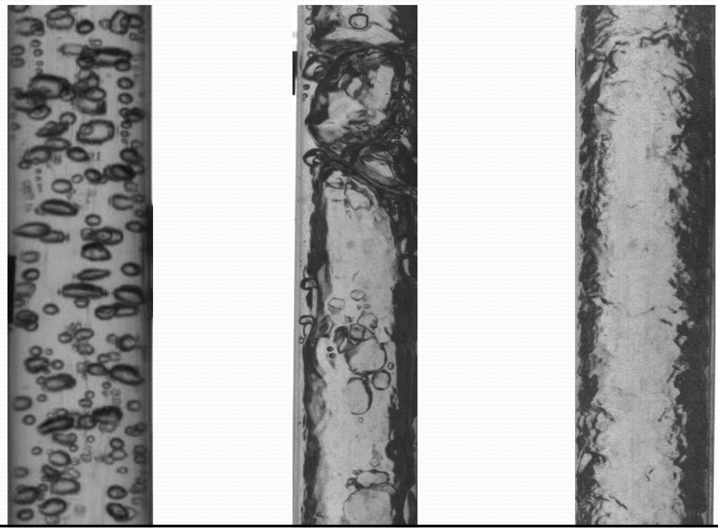
Flow patterns in convective boiling at low heat flux

Chen & Karayannis, 2006

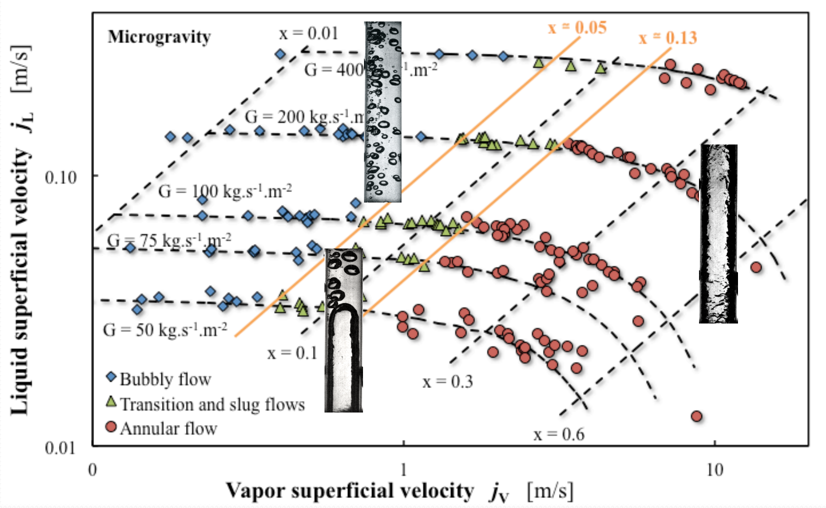


Flow patterns in convective boiling

Flow of boiling HFE7000 in a vertical tube of 6 mm diameter



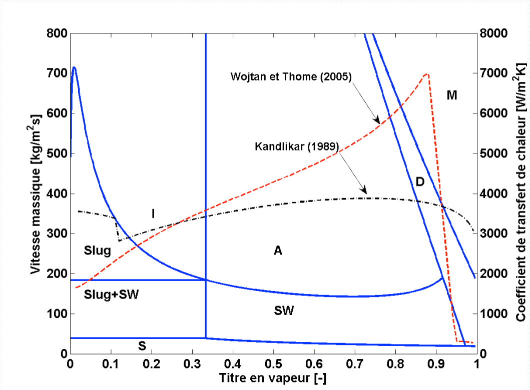
Flow patterns in convective boiling at low heat flux



HFE 7000 –6mm diameter tube

Flow patterns in convective boiling

Wojtan et al. (2005) for horizontal flows

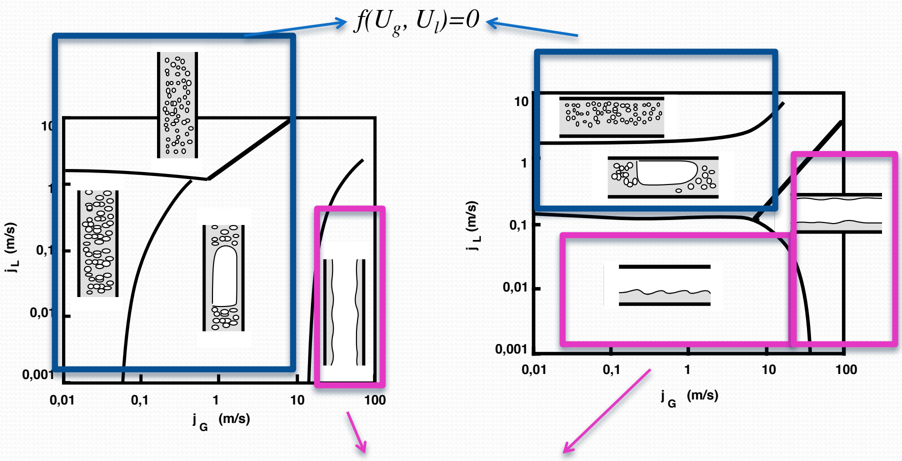


Flow pattern map and heat transfer coefficient for R134a , D=10 mm q_p= 10 kW/m², T_{sat} = 10°C et G=300 kg/m²s .

I : Intermittent, A : Annular flow, SW : Stratified wavy, S : stratified, Slug flow, D : dry out, M : mist flow

Two-phase flow with phase change: same flow patterns + 1 configuration additionnal configuration vapor + liquid droplet (Mist Flow)

Phase velocities



Two fluid model: dynamics of the 2 phases controlled by the interfacial shear stress.

Closure laws for the void fraction

Homogenous model : *Hypothesis:* $U_l = U_g = U_M$ $\quad R_g = \dfrac{x}{x + (1-x)\dfrac{\rho_g}{\rho_l}}$

➡ Dispersed flow with small bubble drift velocity $/U_l$

Drift flux models

Zuber and Findlay (1965)

$$U_g = C_0 U_m + U_\infty = C_0\left(j_g + j_l\right) + U_\infty$$

Dispersed Bubbles $\quad U_\infty = 1{,}53\left[\dfrac{g\left(\rho_l - \rho_g\right)\sigma}{\rho_l^2}\right]^{1/4}$

$C_0 = 1.1$

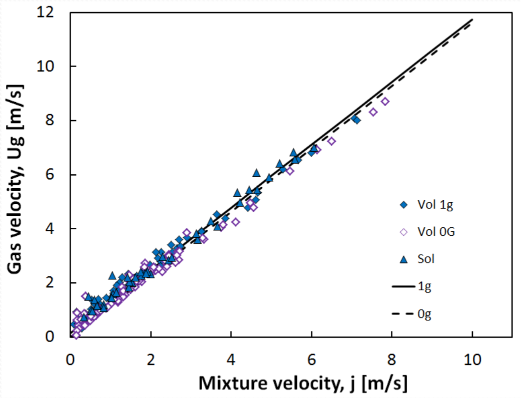
Taylor bubbles $\quad U_\infty = C_\infty\sqrt{gD}$

$C_0 = 1.2$ $\quad C_\infty = 0.35\,(vertical)$

$C_\infty = 0.5\,(horizontal)$

Mean gas velocity

E. Trejo, IMFT, (2018)



$$U_G = \frac{j_v}{\alpha} = C_0(j_l + j_v) + U_\infty$$

Drift-flux model for bubbly and slug flow

1g→ $U_\infty = 0.35\sqrt{gD}$

0g→ $U_\infty = 0$

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Closure laws for the void fraction

$$U_g = C_0 U_m + U_\infty = C_0\left(j_g + j_l\right) + U_\infty$$

Churn flow: Ishii (1977)

$$C_0 = 1.2 - 0.2\sqrt{\frac{\rho_v}{\rho_l}}, \quad U_\infty = \sqrt{2}\left(\frac{\sigma g\left(\rho_l - \rho_v\right)}{\rho_l^2}\right)^{0.25}$$

Annular flows: Zuber et al. (1967) $C_0 = 1.0, \quad U_\infty = 23\sqrt{\frac{\mu_l j_l}{\rho_v D}}\left(\frac{\rho_l - \rho_v}{\rho_l}\right)$

Cioncolinio and Thome (2012) $R_g = \dfrac{hx^n}{1 + (h-1)\,x^n}$

$$h = a + (1-a)\left(\frac{\rho_v}{\rho_l}\right)^{a_1} \qquad a = -2.129 \quad a_1 = -0.2186$$

$$n = b + (1-b)\left(\frac{\rho_v}{\rho_l}\right)^{b_1} \qquad b = 0.3487 \quad b_1 = 0.515$$

Awad and Muzychka (2010)

$$R_g = \frac{0.5}{1 + 0.28X^{0.71}} + \frac{0.5}{1 + X^{16/19}}$$

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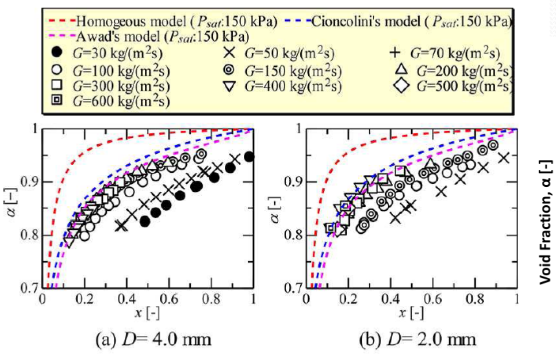
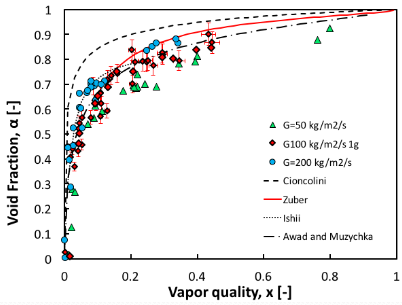
Closure laws for the void fraction

Comparison with experimental data





Gomyo & Asano (2016) FC 72

Trejo, 2018, HFE7000

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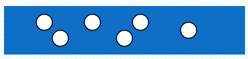
Closure law for Interfacial area concentration and interfacial shear stress

Dispersed flows



$$\alpha_i = \frac{S_i}{A} = \frac{3R_g}{R}$$

R bubble/drop radius given by

$$We_c = \frac{\rho_c (U_l - U_g)^2\, 2R}{\sigma}$$

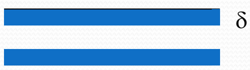
= 3 bubbles

= 10 droplets

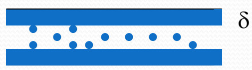
$$\tau_{ic} = -\frac{1}{4}\frac{C_D \rho_c |U_g - U_l|(U_g - U_l)}{2}$$

Annular Flow

$$R_g = \left(1 - \frac{2\delta}{D}\right)^2 \quad \text{et} \quad \frac{S_i}{A} = \frac{4}{D}\sqrt{R_g}$$



$$\alpha_i = \frac{S_i}{A} = \frac{4}{D}\sqrt{1-(1-R_g)(1-E)} + \frac{6R_g}{d_{32}}(1-R_g)E$$



Droplet entrainment rate

$$f_i = 0{,}005\left(1 + 300\frac{\delta}{D}\right)$$

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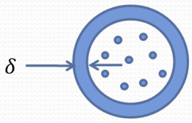
Annular flow with droplet entrainment

Cioncolini and Thome model (2011)

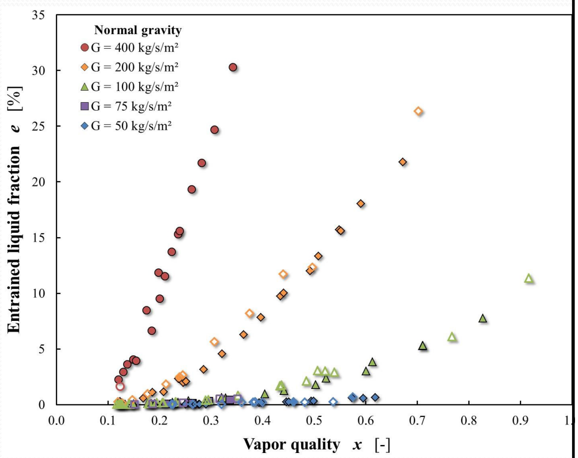


$$E=\left(1+279{,}6\,We_C^{-0{,}8395}\right)^{-2{,}209}$$

with $10 < We_C < 10^5$

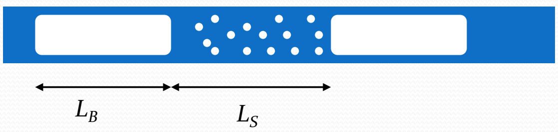
$$We_C=\frac{\rho_C j_v^2 D}{\sigma}$$

with $\rho_C=\dfrac{E(1-x)+x}{\dfrac{E(1-x)}{\rho_l}+\dfrac{x}{\rho_v}}$



Interfacial area concentration and interfacial shear stress

Slug flows



$$R_g=\frac{L_B}{L_B+L_S}+R_{gS}\frac{L_S}{L_B+L_S}\qquad R_{gS}=R_{gBS}\exp\left[-10\frac{R_g-R_{gBS}}{R_{gSA}-R_{gBS}}\right]$$

$R_{gBS}=0.25$
$R_{gSA}=0.8$

Interfacial area concentration:

$$\alpha_i=\frac{S_i}{A}=\frac{4}{D}\frac{L_B}{L_B+L_S}+\frac{6R_{gS}}{d_{32}}\frac{L_S}{L_B+L_S}$$

Interfacial shear stress:

$$\tau_{il}=-\frac{1}{4}\frac{C_D\rho_l\left|U_g-U_l\right|\left(U_g-U_l\right)}{2}$$

$$C_D=9{,}8\left(\frac{L_S}{L_B+L_S}\right)^3$$

Closure laws

- Void fraction
- Interfacial perimeter S_i, wetted perimeters S_{pg}, S_{pl} depend of the flow topology
- Wall shear stress τ_p and interfacial shear stress τ_i
- Wall heat flux q_p and interfacial heat flux q_i, specific modelling in boiling and condensation.

Closure law for the wall shear stress: homogeneous models

Hypothesis: $U_l = U_g = U_M$ → Dispersed flow with small bubble drift velocity $/U_l$

$$\frac{\partial\left(\rho_l(1-R_g)U_l+\rho_g R_g U_g\right)}{\partial t}+\frac{\partial\left(\rho_l(1-R_g)U_l^2+\rho_g R_g U_g^2\right)}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\tau_p S_p}{A}-\left(\rho_l(1-R_g)+\rho_g R_g\right)g\sin\theta$$

$$\frac{\partial \rho_M U_M}{\partial t}+\frac{\partial}{\partial z}\left[\rho_M U_M^2\right]=\frac{\partial G}{\partial t}+\frac{\partial}{\partial z}\left[\frac{G^2}{\rho_M}\right]=-\frac{dP}{dz}+\frac{\tau_p S_p}{A}-\rho_M g\sin\theta$$

$$\left(\frac{dp}{dz}\right)_{fr}=\frac{\tau_p S_p}{A}=-\frac{S_p}{A}\frac{1}{2}f_{pm}\frac{G^2}{\rho_M}=-\frac{S_p}{A}\frac{1}{2}f_{pM}\rho_M U_M^2 \quad \text{with} \quad \rho_M=R_g\rho_g+(1-R_g)\rho_l$$

f_{pM} wall friction factor

$$\begin{cases} f_{pM}=\dfrac{16}{Re_M} & \text{si } Re_M<2000 \\ f_{pM}=0{,}079\,Re_M^{-0.25} & \text{si } Re_M>2000 \end{cases}$$

with $Re_M=\dfrac{GD}{\mu_M}$, $\mu_M=R_g\mu_g+(1-R_g)\mu_l$

Closure law for the wall shear stress: homogeneous models

Authors	Definitions
[McAdams et al. (1942)]	$\mu_{TP} = \left(\frac{x}{\mu_V} + \frac{1-x}{\mu_L}\right)^{-1}$
[Cicchitti et al. (1960)]	$\mu_{TP} = x \cdot \mu_V + (1-x) \cdot \mu_L$
[Dukler et al. (1964)]	$\mu_{TP} = \rho_{TP} \cdot \left(x \cdot \frac{\mu_V}{\rho_V} + (1-x) \cdot \frac{\mu_L}{\rho_L}\right)$
[Beattie and Whalley (1982)]	$\mu_{TP} = \theta \cdot \mu_V + (1-\theta) \cdot (1 + 2.5 \cdot \theta) \cdot \mu_L$
	$\theta = \left[1 + \left(\frac{\rho_V}{\rho_L}\right) \cdot \left(\frac{1-x}{x}\right)\right]^{-1}$
[Lin et al. (1991)]	$\mu_{TP} = \frac{\mu_L \cdot \mu_V}{\mu_V + x^{1.4} \cdot (\mu_L - \mu_V)}$
[Fourar and Bories (1995)]	$\mu_{TP} = \rho_{TP} \cdot \left(\sqrt{x \cdot \nu_V} + \sqrt{(1-x) \cdot \nu_L}\right)^2$
[Davidson et al. (1943)]	$\mu_{TP} = \mu_L \cdot \left[1 + x \cdot \left(\frac{\rho_L}{\rho_V} - 1\right)\right]$
[García et al. (2003)]	$\mu_{TP} = \frac{\mu_L \cdot \rho_V}{x \cdot \rho_L + (1-x) \cdot \rho_V}$
[Awad and Muzychka (2008)] No 1	$\mu_{TP} = \mu_L \cdot \frac{2 \cdot \mu_L + \mu_V - 2 \cdot (\mu_L - \mu_V) \cdot x}{2 \cdot \mu_L + \mu_V + (\mu_L - \mu_V) \cdot x}$
[Awad and Muzychka (2008)] No 2	$\mu_{TP} = \mu_V \cdot \frac{2 \cdot \mu_V + \mu_L - 2 \cdot (\mu_V - \mu_L) \cdot (1-x)}{2 \cdot \mu_V + \mu_L + (\mu_V - \mu_L) \cdot (1-x)}$

Closure law for the wall shear stress: separated flows models like Lockhart and Martinelli model

Frequently used in flow boiling to predict the wall shear stress

$$\frac{\partial(R_l\rho_l U_l + R_g\rho_g U_g)}{\partial t} + \frac{\partial}{\partial z}\left(R_l\rho_l U_l^2 + R_g\rho_g U_g^2\right) = -\frac{\partial P}{\partial z} - \left(R_l\rho_l + R_g\rho_g\right)g\sin\theta + \frac{S_p\tau_p}{A}$$

Modelling of the frictionnal pressure gradient using Martinelli multipliers

$$\left(\frac{dP}{dz}\right)_{fr} = \frac{\tau_p S_p}{A} = \phi_l^2\left(\frac{dP}{dz}\right)_l = \phi_g^2\left(\frac{dP}{dz}\right)_g$$

$$\phi_l^2 = \left(1 + \frac{C}{X} + \frac{1}{X^2}\right) \qquad \phi_g^2 = \left(1 + CX + X^2\right)$$

$$\left(\frac{dP}{dz}\right)_l = -\frac{S_p}{A} f_{pl} \frac{\rho_l j_l^2}{2} \qquad \left(\frac{dP}{dz}\right)_g = -\frac{S_p}{A} f_{pg} \frac{\rho_g j_g^2}{2}$$

$$X = \left[\left(\frac{dP}{dx}\right)_l \Big/ \left(\frac{dP}{dx}\right)_g\right]^{1/2} = \frac{j_l}{j_g}\sqrt{\frac{\rho_l}{\rho_g}\frac{f_{pl}}{f_{pg}}}$$

$$f_{pl} = K\left(\frac{j_l D_H}{\nu_l}\right)^{-n} \qquad f_{pg} = K\left(\frac{j_g D_H}{\nu_g}\right)^{-n} \qquad D_H = \frac{4A}{S_p}$$

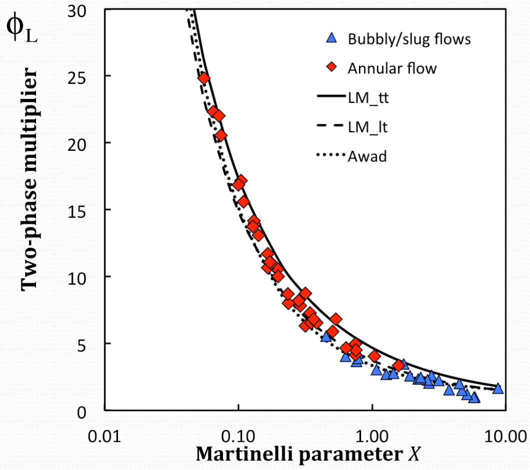
K=16, n=1 in laminar flow
K=0.079, n=1/4 in turbulent flow

Liquide	Gaz	C
Turbulent	Turbulent	20
Laminaire	Turbulent	12
Turbulent	Laminaire	10
Laminaire	Laminaire	5

$R_g = \left(1 + X^{0.8}\right)^{-0.378}$ proposed by L&M, but not always relevant

Closure law for the wall shear stress: Lockhart and Martinelli model

Comparison with experimental data HFE7000- 6mm

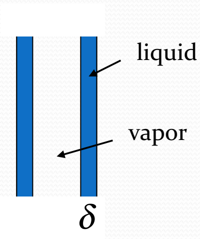


$$\phi_l^2 = \left(1 + \frac{C}{X} + \frac{1}{X^2}\right) \qquad \phi_g^2 = \left(1 + CX + X^2\right)$$

$$X = \left[\left(\frac{dP}{dx}\right)_l \Big/ \left(\frac{dP}{dx}\right)_g\right]^{1/2} = \frac{j_l}{j_g}\sqrt{\frac{\rho_l}{\rho_g}\frac{f_{pl}}{f_{pg}}}$$

Closure law for the wall and interfacial shear stresses: two-fluid model

2 momentum balance equations: example for a vertical upflow



$$\frac{\partial \rho_g R_g U_g^2}{\partial z} = \frac{d}{dz}\frac{G^2 x^2}{\rho_g R_g} = -R_g \frac{\partial P}{\partial z} + \frac{\tau_{ig} S_i}{A} + \dot{M}_l U_i - \rho_g R_g g$$

$$\frac{d}{dz}\frac{G^2(1-x)^2}{\rho_l R_l} = -R_l \frac{dP}{dz} + \frac{\tau_{il} S_i}{A} + \frac{\tau_p S_p}{A} - \dot{M}_l U_i - \rho_l (1 - R_g) g$$

$$U_i \approx U_l \qquad \frac{S_i}{A} = \frac{4}{D}\sqrt{R_g} \qquad \dot{M}_l = G\frac{dx}{dz}$$

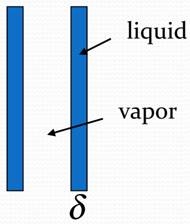
In saturated boiling x is calculated by the enthalpy balance

2 unknowns P et R_g

Elimination of the pressure gradient between the 2 equations

$$R_l \frac{d}{dz}\frac{G^2 x^2}{\rho_g R_g} - R_g \frac{d}{dz}\frac{G^2(1-x)^2}{\rho_l R_l} = \frac{\tau_{ig} 4}{D}\sqrt{R_g} - R_g \frac{\tau_p 4}{D} + (\rho_l - \rho_g) R_g R_l g$$

Closure law for the wall and interfacial shear stresses: annular flow model without entrainment



Calculation of R_g

$$\frac{dR_g}{dz}G^2\left(\frac{R_l x^2}{\rho_g R_g^2}+\frac{R_g(1-x)^2}{\rho_l R_l^2}\right)=$$
$$-\frac{\tau_{ig}4}{D}\sqrt{R_g}+R_g\frac{\tau_p 4}{D}-(\rho_l-\rho_g)R_g R_l g$$
$$+G^2\frac{dx}{dz}\left(\frac{2xR_l}{\rho_g R_g}+\frac{(1-x)(2R_g-1)}{\rho_l R_l}\right)$$

Modelling of τ_i (Wallis, 1969) : $\tau_i=-\frac{1}{2}f_i\rho_g\left|U_g-U_l\right|\left(U_g-U_l\right)$

well adapted to centimetric tubes $\quad f_i=0{,}005\left(1+300\frac{\delta}{D}\right)=0{,}005\left(1+150(1-\sqrt{R_g})\right)$

$\tau_p=-\frac{1}{2}f_{pl}\rho_l U_l^2 \quad ; \quad f_{pl}=C\,\mathrm{Re}_l^{-n}$ with $\mathrm{Re}_l=\frac{U_l D}{\nu_l}$

$$\frac{dp}{dz}=-\frac{d}{dz}\frac{G^2x^2}{\rho_g R_g}-\frac{d}{dz}\frac{G^2(1-x)^2}{\rho_l R_l}+\frac{\tau_p 4}{D}-(\rho_g R_g+\rho_l R_l)g$$

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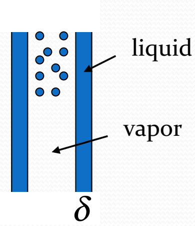
Annular flow with droplet entrainment



R_{lF}= liquid hold up in the liquid film
R_{le}= liquid hold up in the entrained droplets
R_g= void fraction $\qquad R_{lF}+R_{le}+R_g=1$

E entrainment rate
R_D deposition flux (kg/m²/s)
R_A entrainment flux (kg/m²/s)

Mass conservation equations

Gas $\quad \frac{d}{dz}\rho_g R_g U_g=\dot{M}_l$

Film $\quad \frac{d}{dz}\rho_l R_{lF}U_{lF}=\frac{d}{dz}G(1-x)(1-E)=-\dot{M}_l+(R_D-R_A)\frac{S_i}{A}$

Droplets $\quad \frac{d}{dz}\rho_l R_{le}U_{le}=\frac{d}{dz}G(1-x)E=(R_A-R_D)\frac{S_i}{A}$

Momentum balance equations

Gas $\quad \frac{\partial\rho_g R_g U_g^2}{\partial z}=\frac{d}{dz}\frac{G^2x^2}{\rho_g R_g}=-R_g\frac{\partial p}{\partial z}+\frac{\tau_{ig}S_i}{A}+\dot{M}_l U_i-\rho_g R_g g-F_D$

Film $\quad \frac{\partial\rho_l R_{lF}U_{lF}^2}{\partial z}=\frac{d}{dz}\frac{G^2\left[(1-x)(1-E)\right]^2}{\rho_l R_{lF}}=-R_{lF}\frac{\partial p}{\partial z}+\frac{\tau_{il}S_i}{A}-\dot{M}_l U_i-\rho_l R_{lF}g+(R_D U_{eF}-R_A U_{Fe})\frac{S_i}{A}$

Droplets $\quad \frac{\partial\rho_l R_{le}U_{le}^2}{\partial z}=\frac{d}{dz}\frac{G^2\left[(1-x)E\right]^2}{\rho_l R_{le}}=-R_{le}\frac{\partial p}{\partial z}-\rho_l R_{le}g+(R_A U_{Fe}-R_D U_{eF})\frac{S_i}{A}+F_D$

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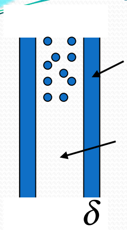
Annular flow with droplet entrainment



At equilibrium R_D=R_A deposition rate = entrainment rate

Momentum balance equations

$$\tau'_i \frac{S_i}{A}$$

Gas+ Droplets
$$\frac{\partial \rho_g R_g U_g^2}{\partial z} + \frac{\partial \rho_l R_{le} U_{le}^2}{\partial z} = -\left(R_g + R_{le}\right)\frac{\partial p}{\partial z} - (\rho_g R_g + \rho_l R_{le})g + \dot{M}_l U_i + \boxed{\frac{\tau_{ig} S_i}{A} + (R_A U_{Fe} - R_D U_{eF})\frac{S_i}{A}}$$

Film
$$\frac{\partial \rho_l R_{lF} U_{lF}^2}{\partial z} = \frac{d}{dz}\frac{G^2\left[(1-x)(1-E)\right]^2}{\rho_l R_{lF}} = -R_{lF}\frac{\partial p}{\partial z} - \dot{M}_l U_i - \rho_l R_{lF} g + \boxed{\frac{\tau_{il} S_i}{A} + (R_D U_{eF} - R_A U_{Fe})\frac{S_i}{A}}$$

Homogeneous mixture of gas and droplets $\longrightarrow$ $U_{le} = U_g \Rightarrow R_{le} = R_g \frac{\rho_g}{\rho_l}\frac{1-x}{x}E$

$$R_{lF} = 1 - R_g\left(1 + \frac{\rho_g}{\rho_l}\frac{1-x}{x}E\right)$$

Closure laws

- Void fraction
- Interfacial perimeter S_i, wetted perimeters S_{pg}, S_{pl} depend of the flow topology
- Wall shear stress τ_p and interfacial shear stress τ_i
- Wall heat flux q_p and interfacial heat flux q_i, specific modelling in boiling and condensation.

Convective Boiling

- Characteristic dimensionless numbers
- Convective boiling regimes
- Boiling incipience
- Wall heat flux in convective boiling
- Boiling crisis: DNB and dry-out
- Film Boiling

Characteristic dimensionless numbers

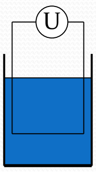
- Physical properties: $\rho_l, \rho_g, \nu_l, \nu_g, \lambda_l, \lambda_g$ σ C_{pl}, C_{pv}, h_{lv},
- Control parameters: D, G, g, T_{sat}-T_{le}, T_p-T_{sat} *or* q_p
- 15 parameters – 4 dimensions (M L t T)= 11 independant dimensionless numbers

$$Re_l = \frac{V_k D}{\nu_l} = \frac{GD}{\mu_l}, \quad Pe_l = \frac{V_l D}{a_l}, \quad \mathrm{Fr}_l = \frac{V_l^2}{gD}, \quad \mathrm{Ja}_l = \frac{C_{pl}\theta_l}{h_{lv}} = \frac{C_{pl}\left(T_{sat} - T_{le}\right)}{h_{lv}}, \quad \mathrm{Ec}_l = \frac{V_l^2}{C_{pl}\left(T_{sat} - T_{le}\right)} \text{ou} \frac{V_l^2}{h_{lv}},$$

$$\mathrm{We}_G = \frac{\rho_g V_g^2 D}{\sigma}, \frac{\rho_g}{\rho_l}, \frac{\nu_g}{\nu_l}, \frac{\lambda_g}{\lambda_l}, \frac{C_{pg}}{C_{pl}}, \quad \frac{\left(T_{sat} - T_{le}\right)}{T_p - T_{sat}} \quad \text{or} \quad \mathrm{Bo} = \frac{q_p}{Gh_{lv}}$$

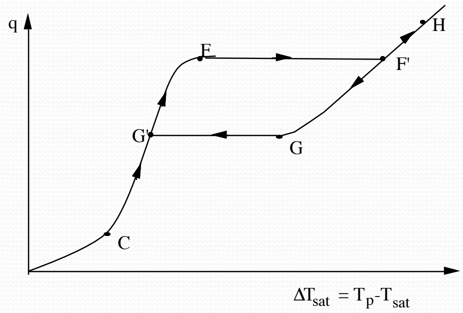
- Consequence: q_p or T_p-T_{sat} can be expressed versus the dimensionless numbers
- Simplification: Ec_l<<1,

Nukiyama Experiment (1932)



Wire heated by Joule effect: imposed heat flux $q = \dfrac{UI}{\pi d l}$

Determination of T_p from the measurement of the wire resistance U/I



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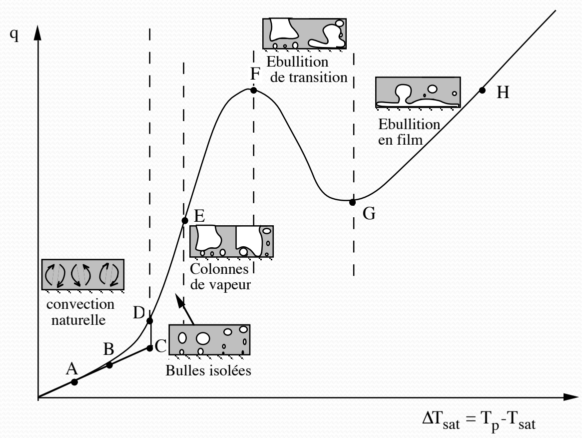
Drew and Müller experiments (1934)

Wall heated at constant temperature



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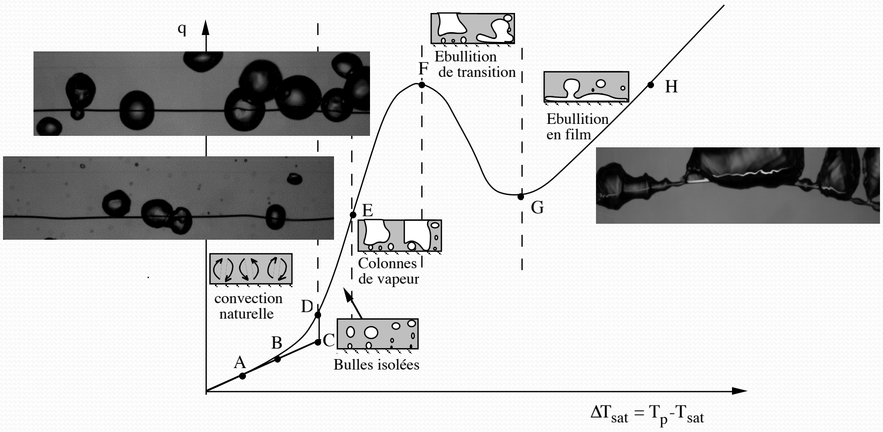
Drew and Müller experiments (1934)

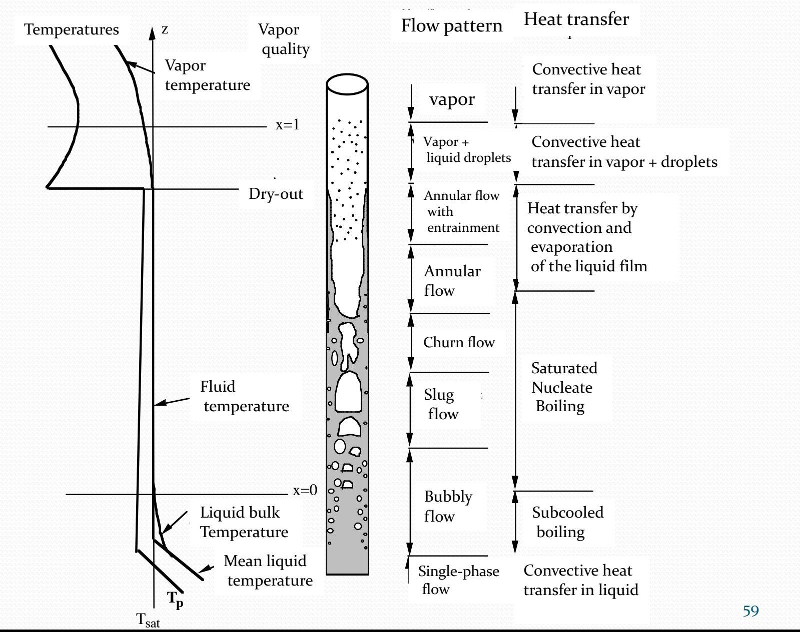
Wall heated at constant temperature



Boiling crisis

- Imposed heat flux
- Degradation of the heat transfer
- Rapid increase of the wall temperature
- « Burn out »

Convective boiling regimes



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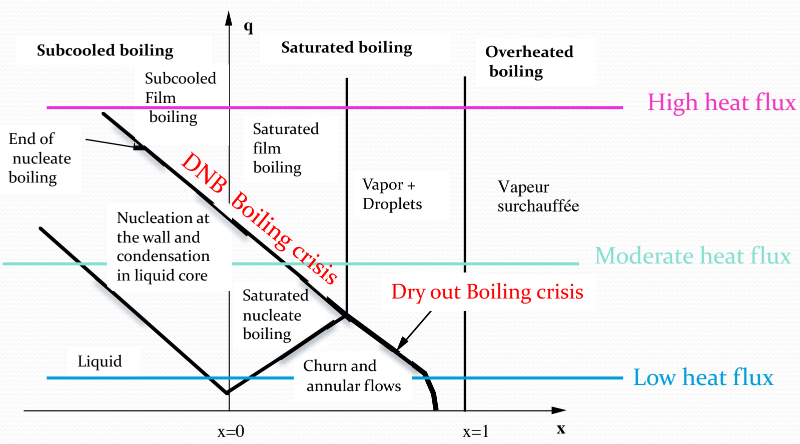
Convective boiling regime for imposed wall heat flux



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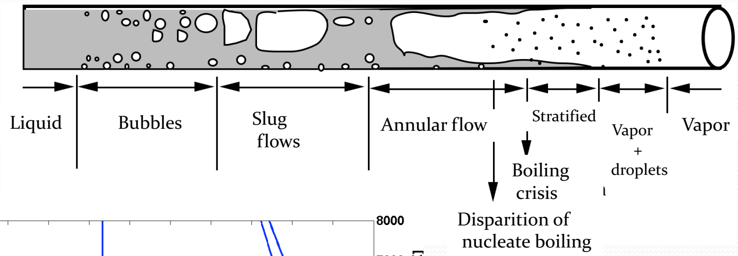
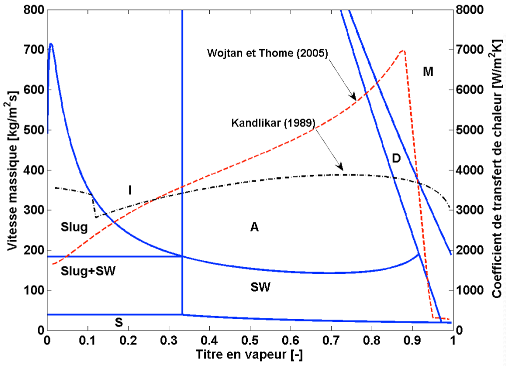
Convective boiling regime in horizontal tube





Constant mass flux G

Increase quality along the tube x

Wojtan *et al.* (2005)

Heat Transfer Coefficient

Single-phase liquid flow

$$q_p = h_l\,(T_p - T_l\,(z))$$

$$Nu = \frac{h_l D}{k_l} = f(\mathrm{Re},\mathrm{Pr})$$

$$Nu = \frac{h_l D}{\lambda_l} = 0{,}023\left(\frac{GD}{\mu_l}\right)^{0,8} \mathrm{Pr}^{1/3}$$

Circular tube (Dittus-Boelter, 1930)

$$GC_{pl}\frac{dT_l(z)}{dz} = \frac{q_p S_p}{A}$$

G mass flux
h_l HTC

Constant heat flux

$$T_l(z) - T_{le} = \frac{q_p S_p}{AGC_{pl}}(z - z_e)$$

$$T_p(z) - T_l(z) = \frac{q_p}{h_l}$$

Constant wall temperature

$$q_p = \frac{GC_{pl}A}{S_p}\frac{dT_l(z)}{dz} = h_l\left[T_p - T_l(z)\right]$$

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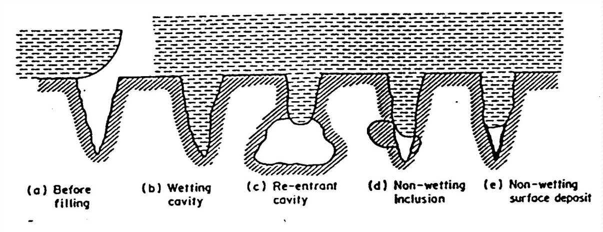
Boiling incipience



Entrapment of vapor (gas) embryos in cavities of the wall

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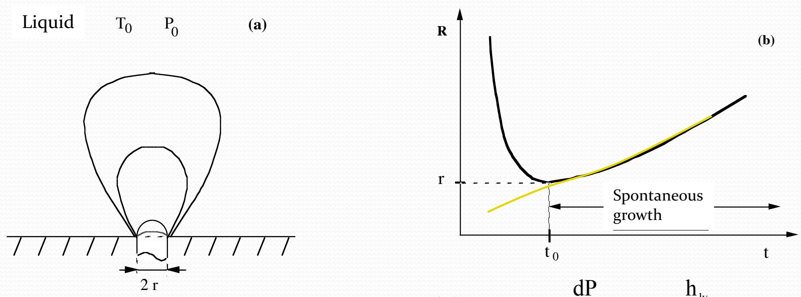
Activation of vapor embryos

Non wetting liquid R>r



Liquid at $T>T_{sat}(P_0)$

$$R = \frac{2\sigma(T)}{P_{sat}(T) - P_0}$$

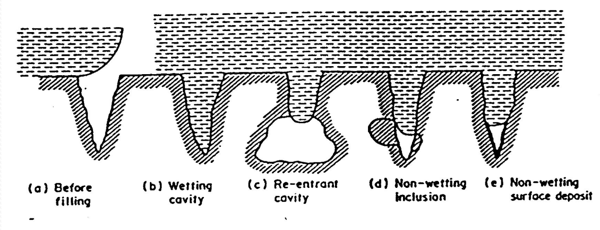
$$\frac{dP}{dT} = \frac{h_{lv}}{T_{sat}(T_0)(v_v - v_l)}$$

Activation of a cavity of radius r for T_0 : $T_0 - T_{sat}(P_0) > \frac{2\sigma(T_0)T_{sat}(P_0)v_v}{r\,h_{lv}}$

Wetting liquid R < r

→ spontaneous growth if: $T_0 - T_{sat}(P_0) > \frac{2\sigma(T_0)T_{sat}(P_0)v_v}{R_{initial}h_{lv}}$

Boiling Incipience



Entrapment of vapor (gas) embryos in cavities of the wall

Forst and Dzakowic criterion (1967)

Correction due to fluid properties

Wall temperature has to be high enough to activate boiling

$$T_p - T_{sat} > \left[\frac{8\sigma T_{sat} q_p}{\lambda \rho_v h_{lv}}\right]^{1/2} \frac{1}{Pr_l}$$

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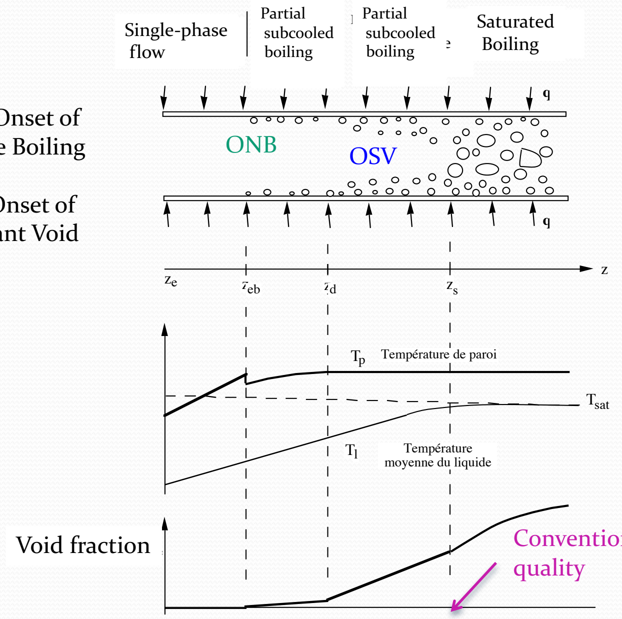
Wall heat flux in subcooled boiling

ONB = Onset of Nucleate Boiling

OSV = Onset of Significant Void



Conventional origine of quality

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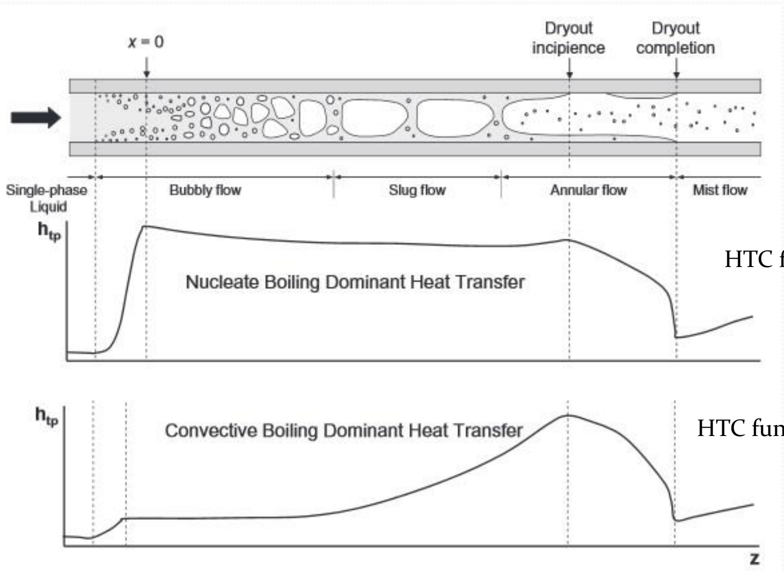
Heat transfer modes in convective boiling



HTC function of q

HTC function of x, G

Rohsenow model

Heat transfer by convection in the bubble wakes: analogy with single-phase convection

$$Nu_b = \frac{hL_b}{k_l} = A\,Re_b^m\,Pr^n$$

$$h = \frac{q}{T_p - T_{sat}} \qquad Re_b = \frac{\rho_v U_b L_b}{\mu_l} \qquad Pr = \frac{\nu_l}{\alpha_l} = \frac{\mu_l C_{pl}}{k_l}$$

$$U_b = \frac{q}{\rho_v h_{lv}} \qquad L_b = C_b \theta \left[\frac{2\sigma}{g(\rho_l - \rho_v)}\right]^{1/2}$$

Characteristic length scale linked to the bubble detachment diameter

$$q = \mu_l h_{lv} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{C_p(T_p - T_{sat}(P_0))}{C_{sf}\,Pr^s\,h_{lv}}\right]^{1/r}$$

≈3

$C_{sf} = 2^{1/2} C_b\,\theta / A$, $r = 1-m$, $s = 1-n$

s=1,7 (or 1 for water)

Specific constant dependant of the couple solid/fluid

Heat transfer in subcooled boiling

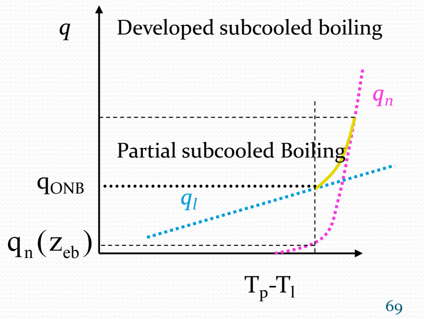
Rohsenow model (1973), validated with experiments of Hino et Ueda (1985)

$$q_p = q_l + q_n \qquad \text{avec} \qquad q_l = h_l\,(T_p - T_l\,(z))$$

Contribution due to bubble nucleation

Contribution due to single phase convection

$$q_n = \mu_l h_{lg}\left[\frac{g(\rho_l - \rho_g)}{\sigma}\right]^{1/2} Pr^{-5}\left[\frac{C_{pl}(T_p - T_{sat})}{C_{sf} h_{lg}}\right]^3$$



Superposition models

$$h = \left(h_l^p + h_n^p\right)^{\frac{1}{p}}$$

p=2 for Kutateladze (1961)
p=3 for Steiner et Taborek (1992)

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Heat transfer in subcooled Boiling: toward mechanistic models

In subcooled boiling, vapor is at saturation temperature and liquid is subcooled.

Enthalpy balance equation for the mixture

$$\frac{q_p S_p}{A} = \frac{\partial\left[Gxh_{g,sat} + G(1-x)\left(C_{pl}(T_l - T_{sat}) + h_{l,sat}\right)\right]}{\partial z}$$

$$= \boxed{G(h_{lg} + C_{pl}(T_{sat} - T_l))\frac{dx}{dz}} + \boxed{G(1-x)C_{pl}\frac{dT_l}{dz}}$$

Part of the heat flux for phase change

Part of the heat flux for liquid heating

Global model are not able to partition the heat flux between phase-change and liquid heating

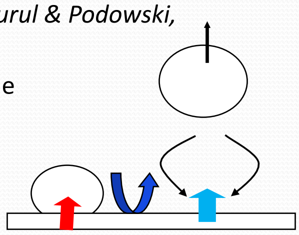
Models based on heat flux partitionning

Wall heat transfer in convective boiling is often splitted into different contributions in the advanced mechanistic models (*Kurul & Podowski, 1990; Basu et al., 2005, Yeoh et al., 2008*):

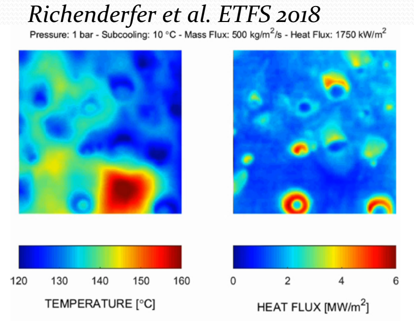
evaporation of the liquid micro-layer below the bubble

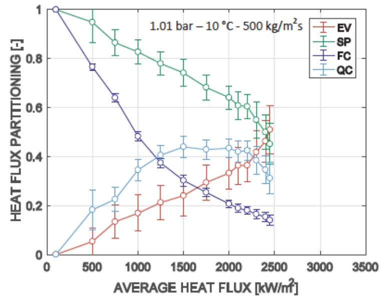
unsteady conduction in the liquid after bubble departure (quenching)

forced convection between the nucleation sites



Richenderfer et al. ETFS 2018

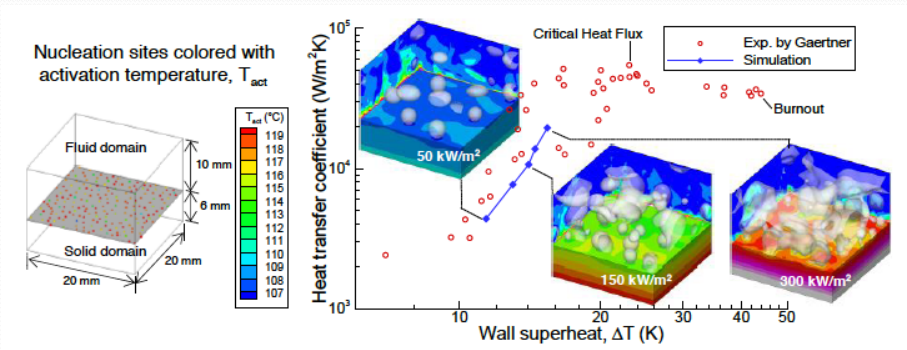




Models based on heat flux partitionning

Numerical studies:

- on DNS of isolated bubble vaporization
  - *Kunkelmann & Stephan, Int. J. Fluid Refrig. (2010),*
  - *Son, Dhir, Ramanujapu, J. Heat Transfer (1999)*
  - *Sato & Niceno J. Comput. (2015)*
  - *Huber et al. IJHMT (2017)*
- with multiple bubble nucleation (*Sato & Niceno, 2016*)



Models based on heat flux partitionning:

Contribution of different heat transfer modes: Judd et Wang (1976), Del Valle et Kenning (1985), Dhir (1991)

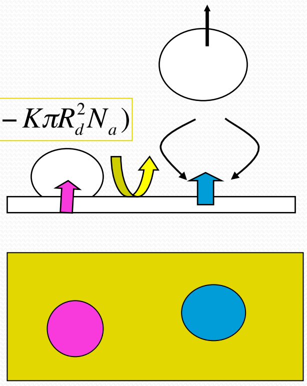
$$q_p = q_e + q_{CI} + q_{CONV}$$

$$q_e = \rho_g h_{lg} \frac{4}{3}\pi R_d^3 N_a f$$

Vaporisation of liquid microlayer

$$q_{CONV} = h_l (T_p - T_l)(1 - K\pi R_d^2 N_a)$$

Single-phase convection between the nucleation sites



Unsteady conduction during rewetting of the wall

$$q_{CI} = K\pi R_d^2 N_a q_b = 2\sqrt{\pi \rho_l C_{pl} \lambda_l} K R_d^2 \sqrt{f} N_a \left(T_p - T_l\right)$$

Parameters to model: R_d, N_a, f

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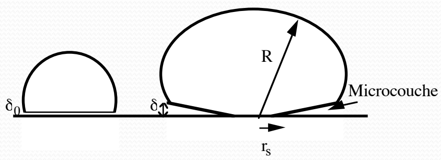
Bubble growth rate

Models based on liquid microlayer evaporation: Cooper and Lloyd (1969) and Van Stralen *et al.* (1975)

$$R = C_1 t^n$$

$$\delta_0(r) = C_2 \sqrt{\nu_l t_c}$$

$$t_c = (r/C_1)^{(1/n)}$$



$\delta < \delta_0$

$$\rho_l h_{lv} \frac{d\delta}{dt} = -k_l \frac{T_p - T_{sat}}{\delta} \quad \text{soit} \quad \delta_0^2 - \delta^2 = 2k_l \frac{T_p - T_{sat}}{\rho_l h_{lv}}\left(t - t_c\right)$$

$$Ja = \frac{\rho_l C p_l (T_p - T_{sat})}{\rho_v h_{lv}}$$

Vaporized liquid mass

$$\rho_l \left\{ \int_0^{r_s} \delta_0 2\pi r dr + \int_{r_s}^{R} (\delta_0 - \delta) 2\pi r dr \right\} = \rho_v \frac{2}{3}\pi R^3 \Rightarrow \begin{cases} R = C_1 \sqrt{t} = \dfrac{2{,}5}{Pr^{1/2}} Ja \sqrt{\alpha_l t} \\ \text{pour} \quad k_p >> k_l \end{cases}$$

General relations

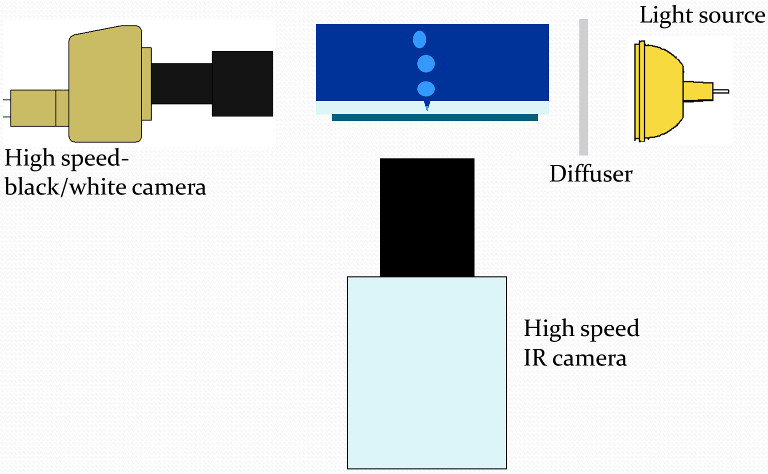
$$R(t) = f(Pr, \frac{k_l}{k_p}, \frac{\alpha_l}{\alpha_p}) Ja \sqrt{\alpha_l t}$$

High coupling between the liquid micro-layer evaporation and conduction in the wall

If $Fo = \alpha_p t_c / e_p^2 \ll 1$ -> $T_p \approx cte$

Technische Universität Darmstadt

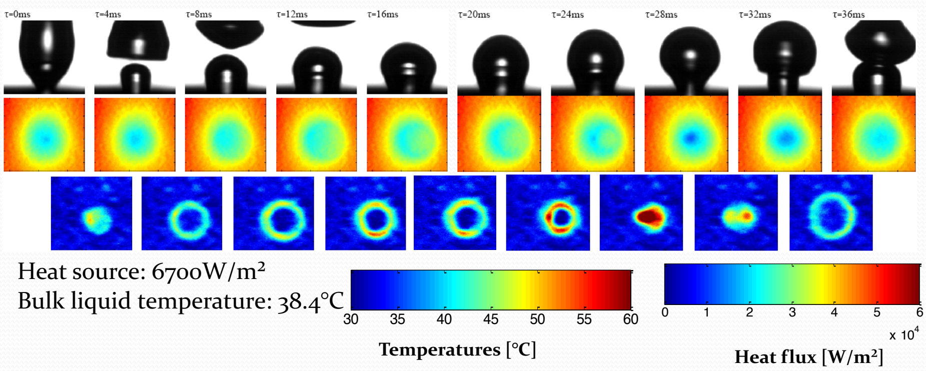
Measurement method



Results: Local temperature and heat flux

Energy balance for each pixel element and two successive temperature images

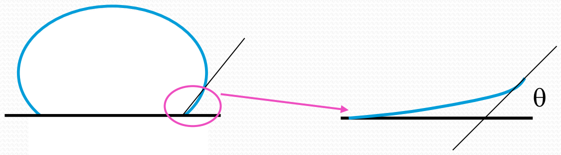
$$\dot{q}_{fluid} = \dot{q}_{el} + \lambda_{hf} \cdot \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \delta_{hf} \cdot \rho_{hf} \cdot c_{hf} \cdot \frac{\partial T}{\partial \tau}$$



Heat source: 6700W/m²
Bulk liquid temperature: 38.4°C

Toward Direct Numerical simulation of Boiling

Multi-scale problem: Dhir, (2002), Stephan, (2002)

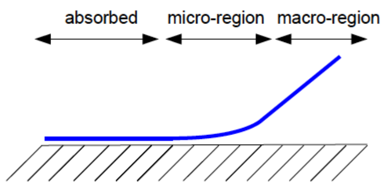


Macroscopic scale
Hydrodynamic and transfer around the bubble

Microscopic scale
Evaporation of liquid microlayer - coupled with the resolution of the heat transfer by conduction in the wall (Mathieu et al., 2002)

Matching of the 2 solutions

Toward Direct Numerical simulation of Boiling



Macroscopique scale– code DIVA
Hydrodynamic and heat and mass transfers around the bubble

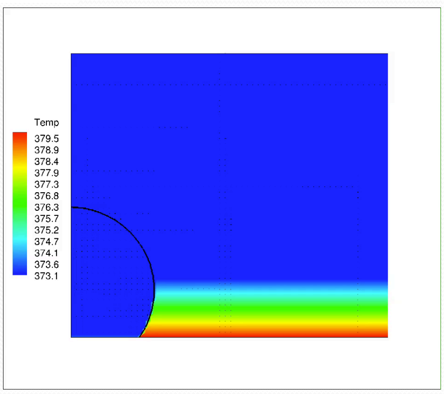
Microscopique scale– model of Stephan and Busse (2002)
Evaporation at the contact line
Huber et al, IJHMT, 2017

$$\frac{R_{dep}}{R_{Fritz}} = 1 + \alpha Ja^{n}$$

$$Ja = \frac{\rho_l C_{pl}(T_p - T_{sat})}{\rho_v h_{lv}}$$

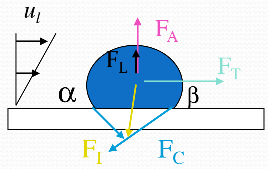
α=0.00219 and n=1.43

$$R_F = 0.01040\theta\sqrt{\frac{\sigma}{g(\rho_L-\rho_V)}}$$



Bubble detachment diameters and frequency

Shear flow on a horizontal wall



$$F_A = \rho_l V g \mathbf{e_z}$$

$$F_C(\alpha,\beta) = F_{Cx}\mathbf{e_x} + F_{Cz}\mathbf{e_z}$$

$$F_{Tx} = \frac{1}{2}\rho_L C_D \pi R^2 U^2 \qquad F_{Lz} = \frac{1}{2}\rho_L C_L \pi R^2 U^2$$

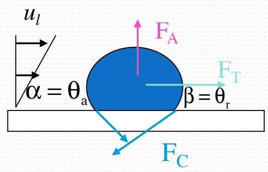
During the bubble growth F_I is weak.

Detachment occurs when $F_{Tx}+F_{Cx}>0$ sliding along the wall

$F_{Az}+F_{Cz}+F_{Lz}>0$ lift-off from the wall

Bubble detachment diameters and frequency

Shear flow on a horizontal wall



Model of Winterton (1972)

Detachment parallel to the wall

Capillary force:

$$F_{Cx} = -\frac{\pi}{2}\sigma r_s(\cos\theta_r - \cos\theta_a) = -\frac{\pi}{2}\sigma R\sin\theta(\cos\theta_r - \cos\theta_a) = -\frac{\pi}{2}\sigma R F(\theta)$$

Drag force: $$F_{Tx} = \frac{1}{2}\rho_L C_D \pi R^2 U^2$$

Detachment occurs when: $$\frac{1}{2}C_D\rho_l U^2 R^2\pi > \frac{\pi}{2}\sigma R F(\theta)$$

$$C_D = 18.7\,Re_B^{-0.68}$$
$$Re_B = U2R/\nu$$

$$\frac{1}{2}C_D\rho_l U^2 R^2\pi > \frac{\pi}{2}\sigma R\sin\theta$$

Bubble detachment diameters

Numerous correlations based on a critical Bond number: $Bo = \dfrac{g(\rho_l - \rho_v) d_d^2}{\sigma}$

$Ja = \dfrac{\rho_l Cp_l (T_p - T_{sat})}{\rho_v h_{lv}}$

Authors	Correlation
Firtz[3]	$D_d = 0.0146\theta \left(\frac{2\sigma}{g(\rho_l - \rho_v)}\right)^{1/2}$ $\theta = 35°$ for mixtures and 45° for water
Ruckenstein[11]	$D_d = \left[\frac{3\pi^2 \rho_l \alpha_l^2 g^{0.5} (\rho_l - \rho_v)^{0.5}}{\sigma^{3/2}}\right] Ja^{4/3} \left[\frac{2\sigma}{g(\rho_l - \rho_v)}\right]^{1/2}$
Cole[12]	$D_d = 0.04\, Ja \left[\frac{2\sigma}{g(\rho_l - \rho_v)}\right]^{1/2}$
Cole and Rohsenow[13]	$D_d = C\, Ja^{5/4} \left[\frac{2\sigma g_c}{g(\rho_l - \rho_v)}\right]^{1/2}$ $C = 1.5 \times 10^{-4}$ for water and 4.65×10^{-4} for others
Van Stralen and Zijl[14]	$D_d = 2.63 \left(\frac{Ja^2 \alpha_l^2}{g}\right)^{1/3} \left[1 + \left(\frac{2\pi}{3Ja}\right)^{0.5}\right]^{1/4}$
Kim and Kim20	$D_d = 0.1649 \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{1/2} Ja^{0.7}$
Fazel and Shafaee[21]	$D_d = 40 \left[\mu_v \left(\frac{q}{h_{lv}\rho_v}\right) / \sigma \cos\theta\right]^{1/3} \left[\frac{\sigma}{g(\rho_l - \rho_v)}\right]^{1/2}$
Hamzenkhani et al.[22]	$D_d = \sqrt{\left(\frac{\sigma}{\Delta\rho g}\right)\left(\frac{\mu_v V_b}{\sigma \cos\theta}\right)^{0.25}\left(\frac{\rho_l C_{pl} \Delta T}{\rho_v h_{lv}}\right)^{0.775}\left[\frac{g\rho_l \Delta\rho}{\mu_l^2}\left(\frac{\sigma}{g\Delta\rho}\right)^{1.5}\right]^{0.05}}$

V_b=bubble velocity

Bubble detachment diameters and frequency

Frequency of detachment: $f = \dfrac{1}{t_w + t_g}$

waiting time (t_w), growth time (t_g)

Correlations $f^n d_d = cste$

- $n = 2$ Inertial growth
- $n = 1/2$ Diffusive growth

Example: boiling water at atmospheric pressure $f^2 d_d = \dfrac{4}{3}\dfrac{g(\rho_l - \rho_v)}{C\rho_l}$ $C \approx 1$

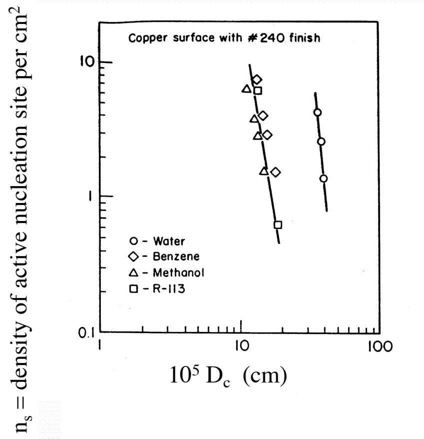
Model of Mikic et Rohsenow $\sqrt{f} d_d = 0.83 Ja \sqrt{\pi \alpha_l}$

Stephan[32]	$f D_d = \frac{1}{\pi}\left[\frac{g}{2}\left(D_d + \frac{4\sigma}{\rho_l g D_d}\right)\right]^{\frac{1}{2}}$
Sakashita and Ono[33]	$f = 0.6 \left[\frac{g(\rho_l - \rho_v)}{\rho_l}\right]^{\frac{2}{3}} \left\{\nu_l \left[\frac{g(\rho_l - \rho_v)\rho_l^2 \nu_l^4}{\sigma^3}\right]^{-0.25}\right\}^{-\frac{1}{3}}$
Hamzekhani et al.[34]	$f = 0.015 \left(\frac{\Delta\rho^{0.25} g^{0.75}}{\sigma^{0.25}}\right)\left(\frac{q}{\Delta\rho^{0.25} g^{0.75} \sigma^{0.75}}\right)^{0.44}\left(\frac{\Delta\rho^{0.5} g^{0.5} D_d}{\sigma^{0.5}}\right)^{0.88}$

Density of active nucleation sites

Density of active nucleation site (for T_w) : $n_s \sim (T_w - T_{sat})^m$ with m=4 ou 5

$n_s \sim q^m$



Gaertner & Westwater $n_s \sim q^{2.1}$

Mikic et Rohsenow

$$n_s \approx [D_{c,max}/D_c]^m$$

$$D_c = 4\sigma T_{sat}/\rho_v h_{lv}(T_w - T_{sat}), m = 6.5$$

Kocamustahaogullari et Ishii

$$n_s^+ = f(\rho^+) R_c^{+(-4.4)}$$

$$n_s^+ = n_s D_d^2,$$

$$f(\rho^+) = 2.157 \times 10^{-7} \rho^{+(-3.2)} (1 + 0.0049\rho^+)^{4.13}$$

$$\rho^+ \equiv (\rho_l - \rho_v)/\rho_v, \quad R_c^+ = 2R_c/D_d$$

Heat Transfer Coefficient in saturated boiling

Strong evolution of the flow pattens along the tube: bubbly flow, slug flow and annular flow.

Quality calculated from enthalpy balance: $x(z) = \dfrac{4q_p}{DGh_{lg}}(z - z_s)$

Different models and correlation for the prediction of the heat transfers

Chen correlation (1966) $h = Sh_n + Fh_l$

$$h_l = 0{,}023 \frac{\lambda_l}{D} \left(\frac{G(1-x)D}{\mu_l} \right)^{0.8} Pr^{1/3}$$

$$h_n = 0{,}00122 \left[\frac{k_l^{0,79} C_{pl}^{0,45} \rho_l^{0,49}}{\sigma^{0,5} \mu_l^{0,29} h_{lg}^{0,24} \rho_g^{0,24}} \right] (T_p - T_{sat})^{0.24} (p_{sat}(T_p) - p_l)^{0.75}$$

$$X = \frac{1-x}{x} \sqrt{\frac{\rho_g}{\rho_l} \frac{f_{pl}}{f_{pg}}}$$

$$S = 1 \Big/ \left[1 + 2{,}53 \cdot 10^{-6} \left(\frac{DG(1-x)}{\mu_l} F(X_{tt}) \right)^{1,25} \right]^{1,17}$$

$$F(X_{tt}) = 2{,}35 \left[0{,}213 + \frac{1}{X_{tt}} \right]^{0,736} \quad \text{if} \quad X_{tt}^{-1} > 0{,}1$$

$$F(X_{tt}) = 1 \quad \text{if} \quad X_{tt}^{-1} \le 0{,}1$$

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Heat Transfer Coefficient in saturated boiling

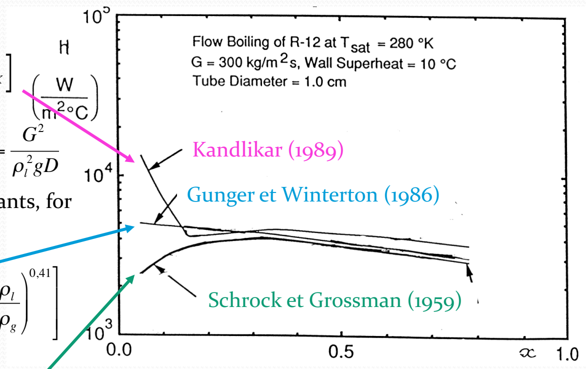
Other correlations

$$h = h_l\left[C_1 C_0^{C_2}(25Fr)^{C_5} + C_3 Bo^{C_4} F_K\right]$$

$$C_0 = \left(\frac{1-x}{x}\right)^{0,8}\sqrt{\frac{\rho_g}{\rho_l}} \qquad Bo = \frac{q}{G h_{lg}} \qquad Fr = \frac{G^2}{\rho_l^2 g D}$$

(different fluids : water, N2, refrigerants, for vertical and horizontal tubes)

$$h = h_l\left[1 + 3000\left(\frac{q}{G h_{lg}}\right)^{0,86} + \left(\frac{x}{1-x}\right)^{3/4}\left(\frac{\rho_l}{\rho_g}\right)^{0,41}\right]$$



$$h = 7{,}39\cdot 10^3\, h_l\left[\frac{q}{G h_{lg}} + 0{,}00015\left(\frac{1}{X_{tt}}\right)^{0,66}\right] \quad \text{et} \quad h_l = 0{,}023\frac{\lambda_l}{D}\left(\frac{G(1-x)D}{\mu_l}\right)^{0,8} Pr^{1/3}$$

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Fitting of experimental resultats (Kandlikar, 1989)

$$H = H_l\left[C_1 C_0^{C_2}(25Fr)^{C_5} + C_3 Bo^{C_4} F_K\right]$$

with $C_0 = \left(\frac{1-x}{x}\right)^{0,8}\sqrt{\frac{\rho_g}{\rho_l}}$ $\qquad Bo = \frac{q}{G h_{lg}}$ $\qquad Fr = \frac{G^2}{\rho_l^2 g D}$

	$C_0 < 0.65$ Région convective	$C_0 > 0.65$ Région de l'ébullition nucléée
C_1	1,1360	0,6683
C_2	-0,9	-0,2
C_3	667,2	1058
C_4	0,7	0,7
C_5	0,3	0,3

Fluide	F_K
Eau	1,00
R-11	1,30
R-12	1,50
R-13B1	1,31
R-22	2,20
R-113	1,30
R-114	1,24
R-152a	1,10
Azote	4,70

C_5 =0 for vertical tubes and horizontal tubes when Fr>0,04.

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Fitting of experimental resultats (Kim & Mudawar 2013)

New correlation

$$h_{tp} = (h_{nb}^2 + h_{nb}^2)^{0.5}$$

$$h_{nb} = \left[2345\left(Bo\frac{P_H}{P_F}\right)^{0.7} P_R^{0.38}(1-x)^{-0.51}\right]\left(0.023Re_f^{0.8}Pr_f^{0.4}\frac{k_f}{D_h}\right)$$

$$h_{cb} = \left[5.2\left(Bo\frac{P_H}{P_F}\right)^{0.08} We_{fo}^{-0.54} + 3.5\left(\frac{1}{X_{tt}}\right)^{0.94}\left(\frac{\rho_v}{\rho_f}0.25\right)\right]\left(0.023Re_f^{0.8}Pr_f^{0.4}\frac{k_f}{D_h}\right)$$

where $Bo = \frac{q"_H}{Gh_{fg}}$, $P_R = \frac{P}{P_{crit}}$, $Re = \frac{G(1-x)D_h}{\mu_f}$, $We_{fo} = \frac{G^2 D_h}{\rho_f \sigma}$,

$X_{tt} = \left(\frac{\mu_f}{\mu_v}\right)^{0.1}\left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_v}{\rho_f}\right)^{0.5}$,

$q"_H$: effective heat flux average over heated perimeter of channel,
P_H : heated perimeter of channel, P_F : wetted perimeter of channel.

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Model of evaporation of a liquid film in annular flow

Cioncolini et Thome (2011)

Hypotheses : Turbulent liquid film and heat transfer by evaporation through the liquid film. No nucleation at the wall.

$$H = 0.0776\frac{\lambda_l}{\delta}\left(\frac{\delta u_*}{\nu_l}\right)^{0.9} \mathrm{Pr}^{0.52}$$ δ film thickness

with $10 < \delta^+ < 800$ $\quad 0.86 < \mathrm{Pr} < 6.1$

$$\rho_c = \rho_g R_g +$$

$$\rho_l u_*^2 = \tau_p = \frac{1}{2} f \rho_c V_c^2 \quad \text{and} \quad f = 0.172 We_c^{-0.372}$$

$$U_{le} = U_g \Rightarrow R_{le} = R_g \frac{\rho_g}{\rho_l}\frac{1-x}{x}E$$

ρ_c, $V_c \approx j_v$ density et velocity of the vapour core

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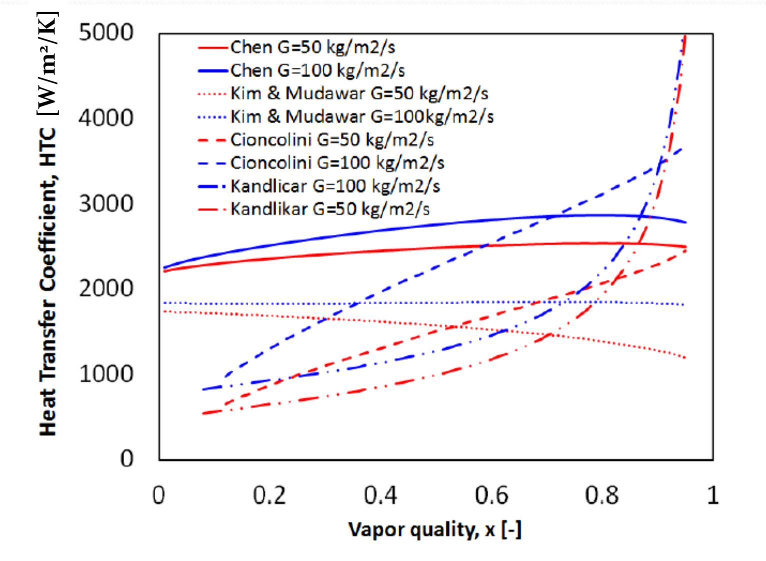
Comparison of different correlations



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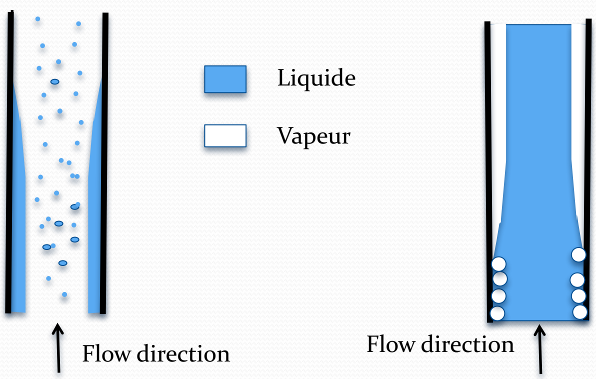
Critical heat flux in convective boiling

At low heat flux: progressive increase progressive of quality → annular flow with evaporation of the liquid film, decrease of the film thickness → dryout of the liquid film

At high heat flux, boiling crisis occurs for low qualities x → strong vapour production at the wall → inversed annular flow



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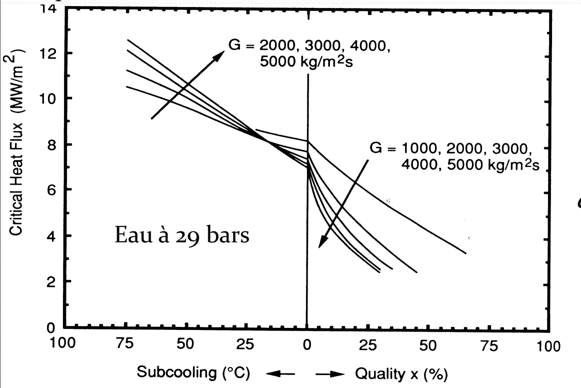

Science Academy of Russia

Katto et Ohno (1984)
water, ammoniac, benzen, alcohols, hydrogen, nitrogen, refrigerants R12, R21, R22, R113,

$$q_{crit} = q_0\left(1 + K\,\frac{h_l(T_{sat}) - h_l(T_{le})}{h_{lg}}\right)$$

q_o and K depend of:

$$\gamma = \frac{\rho_v}{\rho_l} \quad We = \frac{G^2 l}{\rho_l \sigma} \quad l/D \quad G \quad h_{lv}$$

Bowring, (1972) for water

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Correlation of Katto et Ohno (1984)

$$q_{crit} = q_0\left(1 + K\,\frac{h_l(T_{sat}) - h_l(T_{le})}{h_{lg}}\right)$$

$$\gamma = \frac{\rho_v}{\rho_l} \qquad We = \frac{G^2 l}{\rho_l \sigma}$$

$L = h_{lg}$

$C = 0.25$ pour $l/D < 50$

$C = 0.25 + 0.0009\left[\left(\frac{l}{D}\right) - 50\right]$ if $50 < l/D < 150$

$C = 0.34$ pour $l/D > 150$

$$q_{01} = CGLWe^{-0.043}\left(D/l\right) \qquad q_{02} = 0.1GL\gamma^{0.133}We^{-1/3}\left(1 + 0.0031(l/D)\right)^{-1}$$

$$q_{03} = 0.098GL\gamma^{0.133}We^{-0.433}\left(\frac{(l/D)^{0.27}}{1 + 0.0031(l/D)}\right)$$

$$q_{04} = 0.0384GL\gamma^{0.6}We^{-0173}\left(\frac{1}{1 + 0.280We^{-0.233}(l/D)}\right)$$

$$q_{05} = 0.234GL\gamma^{0.513}We^{-0.433}\left(\frac{(l/D)^{0.27}}{1 + 0.0031(l/D)}\right)$$

$$K_1 = \frac{1.043}{4CWe^{-0.043}} \qquad K_2 = \frac{5}{6}\,\frac{0.0124 + D/l}{\gamma^{0.133}We^{-1/3}} \qquad K_3 = 1.12\,\frac{1.52We^{-0.233} + D/l}{\gamma^{0.6}We^{-0.173}}$$

$\gamma < 0.15$

if	$q_{01} < q_{02}$	then	$q_0 = q_{01}$		
if	$q_{01} > q_{02}$	then	$q_0 = q_{02}$	for	$q_{02} < q_{03}$
			$q_0 = q_{03}$	for	$q_{03} \leq q_{02}$
if	$K_1 > K_2$	then	$K = K_1$		
if	$K_1 \leq K_2$	then	$K = K_2$		

$\gamma > 0.15$

if	$q_{01} < q_{05}$	then	$q_0 = q_{01}$		
if	$q_{01} > q_{05}$	then	$q_0 = q_{05}$	for	$q_{04} < q_{05}$
			$q_0 = q_{04}$	for	$q_{05} \leq q_{04}$
if	$K_1 > K_2$	then	$K = K_1$		
if	$K_1 \leq K_2$	then	$K = K_2$	for	$K_2 < K_3$
			$K = K_3$	for	$K_2 \geq K_3$

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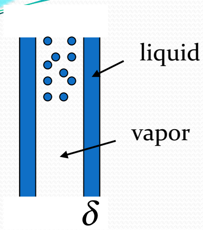
Dryout of the wall



Whalley *et al.* (1974), Govan *et al.*,(1988).

R_{lF}= liquid hold up in the liquid film
R_{le}= liquid hold up in the entrained droplets
R_g= void fraction $\quad R_{lF}+R_{le}+R_g=1$

Mass conservation equations

$$\begin{cases} \text{Gas} & \dfrac{d}{dz}\rho_g R_g U_g = \dot{M}_l \\ \text{Film} & \dfrac{d}{dz}\rho_l R_{lF} U_{lF} = \dfrac{d}{dz} G(1-x)(1-E) = -\dot{M}_l + (R_D - R_A)\dfrac{S_i}{A} \\ \text{Droplets} & \dfrac{d}{dz}\rho_l R_{le} U_{le} = \dfrac{d}{dz} G(1-x)E = (R_A - R_D)\dfrac{S_i}{A} \end{cases}$$

E entrainment rate
R_D deposition flux (kg/m²/s)
R_A entrainment flux (kg/m²/s)

Momentum balance equations for the liquid film and for the vapour core with entrained droplet.

Enthalpy balance equation $\quad \dfrac{dx}{dz} = \dfrac{4q_p}{DGh_{lv}}$

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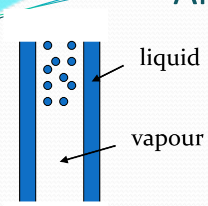
Annular flow with entrainment



Balance between entrainement and redosition of the droplets $R_D=R_A$

Momentum balances equations

$$\tau'_i \frac{S_i}{A}$$

Gas+ Droplets
$$\frac{\partial \rho_g R_g U_g^2}{\partial z} + \frac{\partial \rho_l R_{le} U_{le}^2}{\partial z} = -\left(R_g + R_{le}\right)\frac{\partial p}{\partial z} - (\rho_g R_g + \rho_l R_{le})g + \dot{M}_l U_i + \boxed{\frac{\tau_{ig} S_i}{A} + (R_A U_{Fe} - R_D U_{eF})\frac{S_i}{A}}$$

Film
$$\frac{\partial \rho_l R_{lF} U_{lF}^2}{\partial z} = \frac{d}{dz}\frac{G^2\left[(1-x)(1-E)\right]^2}{\rho_l R_{lF}} = -R_{lF}\frac{\partial p}{\partial z} - \dot{M}_l U_i - \rho_l R_{lF} g + \boxed{\frac{\tau_{il} S_i}{A} + (R_D U_{eF} - R_A U_{Fe})\frac{S_i}{A}}$$

Homogeneoux mixture gas + droplets $\quad U_{le} = U_g \Rightarrow R_{le} = R_g \dfrac{\rho_g}{\rho_l}\dfrac{1-x}{x}E$

$$R_{lF} = 1 - R_g\left(1 + \frac{\rho_g}{\rho_l}\frac{1-x}{x}E\right)$$

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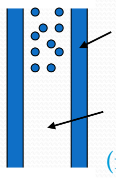
Annular flow with entrainment



$$R_{lF} = 1 - R_g\left(1 + \frac{\rho_g}{\rho_l}\frac{1-x}{x}E\right)$$

Momentum balance equations

(1) $$\frac{d}{dz}\left[\frac{G^2 x}{\rho_g R_g}\left(x + (1-x)E\right)\right] = -R_g\left(1 + \frac{\rho_g}{\rho_l}\frac{1-x}{x}E\right)\frac{\partial p}{\partial z} - \rho_g R_g\left(1 + \frac{1-x}{x}E\right)g + \dot{M}_l U_i + \frac{\tau'_i 4}{D}\sqrt{R_g}$$

(2) $$\frac{d}{dz}\frac{G^2\left[(1-x)(1-E)\right]^2}{\rho_l R_{lF}} = -R_{lF}\frac{\partial p}{\partial z}_i - \rho_l R_{lF} g - \frac{\tau'_i 4}{D}\sqrt{R_g}$$

Enthalpy balance equation

(3) $\frac{dx}{dz} = \frac{4q}{\dot{m}h_{lv}}$ if T_p is imposed $q = \frac{\lambda\left(T_p - T_{sat}\right)}{\delta}$ or $q = h\left(T_p - T_{sat}\right)$

Iterative resolution

Calculation of x using (3)

Elimination of p between (1) and (2) and calculation of R_g

Calculation of $\delta = \frac{D}{2}\left[1 - \sqrt{1 - R_{lF}}\right]$

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Dryout of the wall

Annular flow model with droplet entrainment $\frac{dx}{dz} = \frac{4q_p}{DGh_{lv}}$

Calculation of the heat flux: thin film, negligible convective terms

$$\rho_l Cp\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial y}\left[\left(\lambda_l + \lambda_t\right)\frac{\partial T}{\partial y}\right] \approx 0 \quad \Rightarrow \quad \left(\lambda_l + \lambda_t\right)\frac{\partial T}{\partial y} = q$$

Laminar liquid film $q_p = \lambda_l \frac{T_p - T_{sat}}{\delta}$

Turbulent film $\left(a_l + a_t\right)\frac{\partial T}{\partial y} = \frac{q_p}{\rho_l Cp} \quad \Rightarrow \quad a_l \frac{T_{sat} - T_p}{q/\rho_l Cp} = \int_0^\delta \frac{dy}{1 + \frac{a_t}{a_l}} = \int_0^\delta \frac{dy}{1 + \frac{\nu_t}{\nu_l}\frac{Pr_l}{Pr_t}}$

Resolution by using a given turbulent eddy profile $Pr_t \approx 1$

Dukler (1959)

$$\frac{\nu_t}{\nu_l} = 0{,}01y^+\left[1 - \exp(-0{,}01y^+)\right]$$

with $y^+ = \frac{yu_*}{\nu} < 20$

Other expressions

$y^+ < 5 \quad \nu_t = 0$

$5 < y^+ < 30 \quad \frac{\nu_t}{\nu_l} = \frac{y^+}{5} - 1$

$y^+ > 30 \quad \frac{\nu_t}{\nu_l} = \frac{y^+}{2{,}5} - 1$

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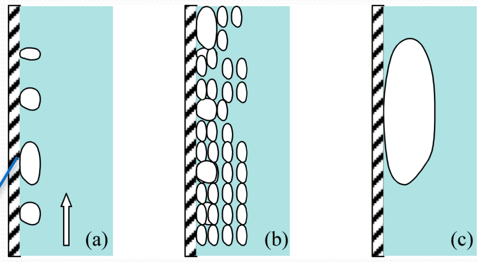
Critical Heat Flux: Departure from Nucleated Boiling (DNB type)

No predictive model. Different scenarii proposed.



In weakly subcooled boiling

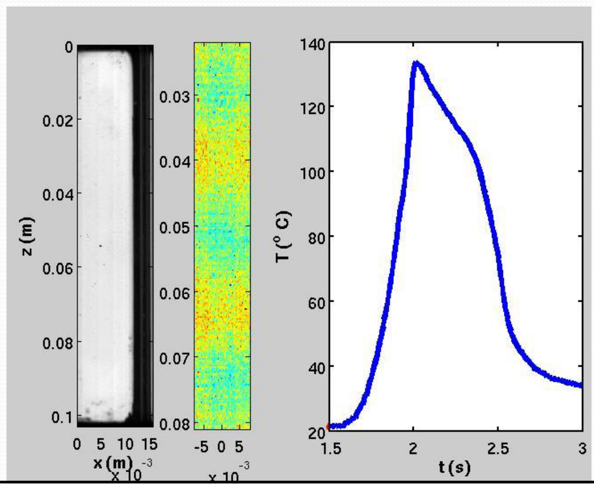
Local phenomenum: formation of dry spot below the bubble by total evaporation of the liquid microlayer: Theofanous *et al.*, (2002)

accumulation of bubbles - Tong et Hewitt (1972) ($R_g \approx 0.8$)

Balance between evaporation and recondensation of large vapour bubbles: Lee et Mudamar (1988) et Celata *et al.* (1999)

Film Boiling

Vapour film at the wall→ high increase in the wall temperature



Film Boiling

Inversed annular flow

→ Heat transfer by conduction across the vapour film

$$q_p = \lambda_l \frac{T_p - T_{sat}}{\delta}$$

→ Enthalpy Balance

$$G\left(h_{lv} + C_{pl}\left(T_{sat} - T_l\right)\right)\frac{dx}{dz} = \frac{4q_p}{D}$$

→ Momentum balance equation

$$\frac{d}{dz}\frac{G^2x^2}{\rho_g R_g} = -R_g\frac{\partial P}{\partial z} + \frac{\tau_{ig}S_i}{A} + \frac{\tau_p S_p}{A} + \dot{M}_l U_i - \rho_g R_g g$$

$$\frac{d}{dz}\frac{G^2(1-x)^2}{\rho_l R_l} = -R_l\frac{dP}{dz} + \frac{\tau_{il}S_i}{A} - \dot{M}_l U_i - \rho_l(1-R_g)g$$

Post CHF regimes

Transition boiling: Tong et Young (1974)

$$q_{et} = q_f + q_n \exp\left[-0{,}0394\frac{x^{2/3}}{dx/dz}\left(\frac{T_p - T_{sat}}{55{,}6}\right)^{1+0{,}0029(T_p - T_{sat})}\right]$$

Film boiling around cylinder: Bromley (1960)

$$h = 0{,}62\left[\frac{\lambda_g^3\rho_g(\rho_l - \rho_g)h_{lg}}{\mu_g(T_p - T_{sat})\lambda_H}\right]^{1/4} \qquad \lambda_H = 2\pi\left(\frac{\sigma}{g(\rho_l - \rho_g)}\right)^{1/2}$$

Vapour flow with entrained droplets: Dougall et Rohsenow (1963)

$$Nu_g = \frac{h_g D}{k_g} = 0{,}023\left[\left(\frac{GD}{\mu_g}\right)\left(x + \frac{\rho_g}{\rho_l}(1-x)\right)\right]^{0.8} Pr_{g,T_{sat}}^{0.4}$$

Homogeneous model

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Conclusion

- Strong evolution of the flow patterns in flow boiling
- Boiling incipience: numerous models (effect of wall wettability, cavity size..)
- HTC in convective Boiling: numerous correlations, promosing mechanistic models, which require local closure laws.
- CHF with dryout (reasonnable predictions), CHF DNB type (open problem)

Condensation of pure vapour

Dropwise condensation
High heat flux

Filmwise condensation frequently observed with wetting liquids

Filmwise condensation

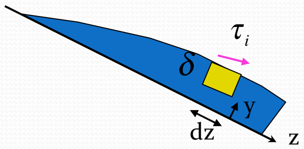
Local heat transfer coefficient: $$h(z) = \frac{q}{T_i - T_p} = \frac{q}{T_{sat} - T_p}$$

Global heat transfer coefficient: $$\bar{h}(z) = \frac{1}{z}\int_0^z h(z)dz$$

Predominant thermal resistance through the liquid film.

Filmwise condensation of pure vapour

Non inertial model of Rohsenow: laminar flow



Momentum balance equation along z axis

$$\left(\rho_L g \sin\theta - \frac{dP}{dz}\right) + \mu \frac{d^2u}{dy^2} = 0$$

Equality of pressure gradients
In liquid and vapour phases

$$\frac{dP}{dz} = \rho_v g \sin\theta + \left(\frac{dP}{dz}\right)_m = \rho_v^* g \sin\theta$$

Pressure gradient in the vapour phase

Integration between y and δ

$$\left(\rho_L g \sin\theta - \frac{dP}{dz}\right)(\delta - y) + \tau_i - \mu\left(\frac{\partial u}{\partial y}\right) = 0$$

$$u(y) = \frac{\left(\rho_L - \rho_v^*\right) g \sin\theta}{\mu}\left(\delta y - \frac{y^2}{2}\right) + \frac{\tau_i y}{\mu}$$

Mass flow rate per unit of width *b*

$$\frac{\dot{M}}{b} = \rho_L \int_0^\delta u\,dy = \frac{\rho_L\left(\rho_L - \rho_v^*\right) g \sin\theta}{\mu}\frac{\delta^3}{3} + \frac{\rho_L \tau_i}{\mu}\frac{\delta^2}{2}$$

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Thermal balance at the liquid-vapour interface

Heat flux: condensation of vapour+ cooling at the mean film temperature T_m

$$u(y) = \frac{\left(\rho_L - \rho_v^*\right) g \sin\theta}{\mu}\left(\delta y - \frac{y^2}{2}\right) + \frac{\tau_i y}{\mu}$$

$$T = \frac{T_{sat} - T_p}{\delta} y + T_p$$

$$\bar{u} = \frac{1}{\delta}\int_0^\delta u\,dy = \frac{\rho_L - \rho_v}{\mu} g \frac{\delta^2}{3} + \frac{\tau_i \delta}{2\mu}$$

$$T_m = \frac{\int_0^\delta uT\,dy}{\bar{u}\delta} = \frac{5}{8}T_{sat} + \frac{3}{8}T_p$$

$$q = \frac{\lambda\left(T_{sat} - T_p\right)}{\delta} = \frac{1}{b}\frac{d\dot{M}}{dz}\left(h_{lv} + C_p(T_{sat} - T_m)\right) = \frac{1}{b}\frac{d\dot{M}}{dz}\left(h_{lv} + \frac{3}{8}C_p(T_{sat} - T_p)\right) = \frac{1}{b}\frac{d\dot{M}}{dz}h_{Lv}^*$$

$$\frac{d\dot{M}}{dz} = \frac{d\dot{M}}{d\delta}\frac{d\delta}{dz} = \frac{b\lambda\left(T_{sat} - T_p\right)}{\delta h_{Lv}^*} = b\left[\frac{\rho_L\left(\rho_L - \rho_v^*\right) g \sin\theta}{\mu}\delta^2 + \frac{\rho_L \tau_i}{\mu}\delta\right]\frac{d\delta}{dz}$$

$$\Rightarrow \quad \dot{M}(z) = b\left[\frac{\rho_L\left(\rho_L - \rho_v^*\right) g \sin\theta}{\mu}\frac{\delta^3}{3} + \frac{\rho_L \tau_i}{\mu}\frac{\delta^2}{2}\right]$$

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$$\dot{M}(z) = b\left[\frac{\rho_L(\rho_L - \rho_v^*)g\sin\theta}{\mu}\frac{\delta^3}{3} + \frac{\rho_L\tau_i}{\mu}\frac{\delta^2}{2}\right]$$

$$\frac{d\dot{M}}{dz} = \frac{d\dot{M}}{d\delta}\frac{d\delta}{dz} = b\left[\frac{\rho_L(\rho_L - \rho_v^*)g\sin\theta\delta^2}{\mu} + \frac{\rho_L\tau_i\delta}{\mu}\right]\frac{d\delta}{dz} = \frac{b\lambda(T_{sat} - T_p)}{\delta h_{Lv}^*}$$

$$\Rightarrow \quad \rho_L(\rho_L - \rho_v^*)g\sin\theta h_{Lv}^*\frac{\delta^4}{4} + \rho_L\tau_i h_{Lv}^*\frac{\delta^3}{3} = \lambda\mu(T_{sat} - T_p)z$$

$$\delta^4 + \frac{\tau_i}{(\rho_L - \rho_v^*)g\sin\theta}\frac{4}{3}\delta^3 = \frac{4\lambda\mu(T_{sat} - T_p)}{\rho_L(\rho_L - \rho_v^*)g\sin\theta h_{Lv}^*}z = \frac{\mu^2}{\rho_L(\rho_L - \rho_v^*)g\sin\theta}\frac{4\lambda(T_{sat} - T_p)}{\mu h_{Lv}^*}z$$

L_f reference length $\qquad L_f = \left[\frac{\nu^2}{g\sin\theta}\right]^{1/3} \qquad \delta^* = \delta\left[\frac{\rho_L(\rho_L - \rho_v^*)g\sin\theta}{\mu^2}\right]^{1/3} = \frac{\delta}{L_f}$

$$L_f^4\delta^{*4} + \underbrace{\boxed{\frac{\tau_i}{(\rho_L - \rho_v^*)g\sin\theta L_f}}}_{\tau_i^*}\frac{4}{3}\delta^{*3}L_f^4 = \underbrace{\boxed{\frac{4C_p(T_{sat} - T_p)}{\Pr h_{Lv}^*}\frac{z}{L_f}}}_{z^*}L_f^4 \quad \Rightarrow \quad \delta^{*4} + \frac{4}{3}\delta^{*3}\tau_i^* = z^*$$

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Nusselt number characteristic of the heat transfer

$$Nu = \frac{\bar{h}L_f}{\lambda}$$

Mean heat transfer coefficient: $\quad \bar{h}(z) = \frac{1}{z}\int_0^z h(z)dz = \frac{1}{z}\int_0^z \frac{\lambda}{\delta}dz = \frac{1}{z^*}\int_0^{z^*}\frac{\lambda}{L_f\delta^*}dz^*$

$$(4\delta^{*3} + 4\delta^{*2}\tau_i^*)d\delta^* = dz^* \qquad \frac{1}{z^*}\int_0^{z^*}\frac{4\lambda}{L_f}(\delta^{*2} + \delta^*\tau_i^*)d\delta^* = \frac{4\lambda}{L_f z^*}\left(\frac{\delta^{*3}}{3} + \frac{\delta^{*2}}{2}\tau_i^*\right)$$

$$\Rightarrow \quad Nu = \frac{\bar{h}L_f}{\lambda} = 4\left(\frac{\delta^{*3}}{3z^*} + \frac{\delta^{*2}}{2z^*}\tau_i^*\right)$$

Reynolds number $\qquad Re_L = \frac{\rho_L\bar{u}D_h}{\mu} \qquad D_h = \frac{4b\delta}{b} = 4\delta$

$$Re_L = \frac{4}{3}\frac{\rho_L(\rho_L - \rho_v^*)g\sin\theta}{\mu^2}\delta^3 + \frac{4\tau_i\rho_L}{2\mu^2}\delta^2 = \frac{4}{3}\delta^{*3} + 4\tau_i^*\delta^{*2}$$

Nusselt model: $\quad \tau_i = 0$

$$\delta^{*4} = z^* \qquad Re_L(z) = \frac{4}{3}\delta^{*3} \qquad Nu = \frac{4\delta^{*3}}{3z^*} = \frac{4}{3\delta^*} \quad \Rightarrow \quad Nu = \left(\frac{4}{3}\right)^{4/3}Re_L^{-1/3} = 1{,}47\,Re_L^{-1/3}$$

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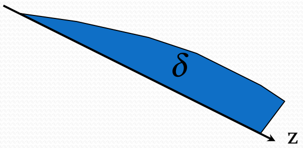
Filmwise condensation of pure vapour

Non inertial model of Rohsenow



$$\delta^{*4} + \frac{4}{3}\delta^{*3}\tau_i^* = z^* \quad avec \quad L_f = \left[\frac{\nu_l^2}{g\sin\theta}\right]^{1/3}$$

$$\delta^* = \frac{\delta}{L_f} = \delta\left[\frac{\rho_l(\rho_l-\rho_v^*)g\sin\theta}{\mu_l^2}\right]^{1/3}$$

$$z^* = \frac{z}{L_f}\frac{4\lambda(T_p - T_{sat})}{\mu_l h_{lv}^*} \qquad \tau_i^* = \frac{\tau_i}{(\rho_l-\rho_v^*)g\sin\theta L_f}$$

$$Nu = \frac{\bar{h}}{k_\ell}\left[\frac{\mu_\ell^2}{\rho_\ell^2 g}\right]^{1/3}$$

$$\overline{Nu} = \frac{\bar{h}L_f}{\lambda} = 4\left(\frac{\delta^{*3}}{3z^*} + \frac{\delta^{*2}}{2z^*}\tau_i^*\right)$$

$$Re_l = \left(\frac{4}{3}\delta^{*3} + 2\delta^{*2}\tau_i^*\right)$$

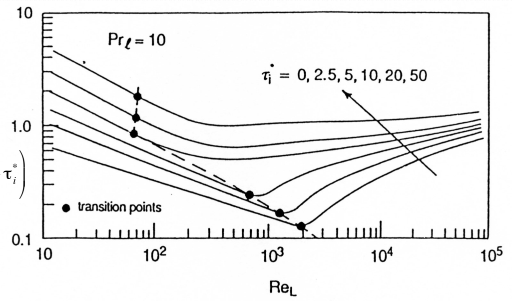


Figure Variation of the mean film condensation heat transfer coefficient with Reynolds numbe and τ_i^* as predicted by the analytical model of Rohsenow et al. [9.56]. *(Adapted from [9.56] wit permission, copyright © 1956, American Society of Mechanical Engineers.)*

For a given value of z, τ_i^* is calculated. Then z^* is calculated and δ^*

Graphe ➡ Nu, Re_l

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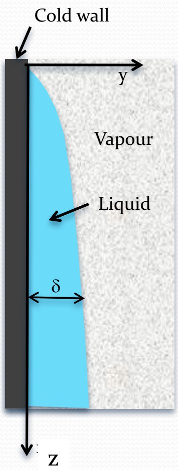
Filmwise condensation of pure vapour with inertia effects (Sparrow et Gregg, 1959)



$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0$$

$$\boxed{\begin{aligned} u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} &= \nu_L\frac{\partial^2 u}{\partial y^2} + \frac{\rho_L - \rho_V}{\rho_L}g \\ u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial y} &= a_L\frac{\partial^2 T}{\partial y^2}\end{aligned}}$$

Inertia-> Boundary layer resolution

$$\psi(z,\eta) = 4a_L C_1 z^{3/4} f(\eta) \quad avec \quad \eta = C_1 y z^{-1/4} \quad et \quad C_1 = \left[\frac{gC_{pL}(\rho_L - \rho_V)}{4\nu_L\lambda_L}\right]^{1/4}$$

$$h(\eta) = \frac{T_{sat} - T}{T_{sat} - T_p} \qquad u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial z}$$

Boundary conditions: at $y=0$, $u=v=0$, $T=T_p$

at y= δ , $T=T_{sat}$ and $\frac{\partial u}{\partial y} = 0$

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Filmwise condensation of pure vapour with inertia effects

$$1+f'''+\frac{1}{\Pr}\left[3ff''-2f'^2\right]=0 \qquad \text{with} \qquad f'(0)=0 \quad h(0)=1$$

$$3f'h'+h''=0 \qquad f(0)=0 \quad h(\eta_\delta)=0 \qquad f''(\eta_\delta)=0$$

Energy balance at the interface

$$\int_0^z\left[\lambda_L\left(\frac{\partial T}{\partial y}\right)_{y=\delta}\right]dx=\frac{\dot M}{b}h_{LV}=\int_0^\delta \rho_L u\, h_{LV}\, dy$$

Implicite equation for the calculation of δ versus z

$$-\frac{3f(\eta_\delta)}{h'(\eta_\delta)}=Ja=\frac{C_{pL}\left(T_{sat}-T_p\right)}{h_{LV}} \quad \text{with} \quad \eta_\delta=C_1\delta z^{-1/4}$$

Convective heat transfer coefficient *h* and *Nu*

$$h=\frac{q}{T_{sat}-T_p}=\frac{\lambda_L}{T_{sat}-T_p}\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\lambda_L h'(0)C_1x^{-1/4}=\lambda_L\left(0.68+Ja^{-1}\right)^{1/4}C_1z^{-1/4}$$

$$Nu_x=\left[\frac{g(\rho_L-\rho_V)z^3h_{LV}(1+0.68Ja)}{4\nu_L\lambda_L\left(T_{sat}-T_p\right)}\right]^{1/4}$$

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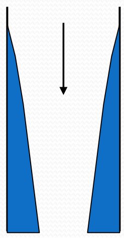
Condensation in a vertical tube in downward flow



$$\frac{\partial \rho_g R_g U_g^2}{\partial z}=\frac{d}{dz}\frac{G^2x^2}{\rho_g R_g}=-R_g\frac{\partial p}{\partial z}+\frac{\tau_{ig}S_i}{A}+\dot M_i U_i+\rho_g R_g g$$

$$\frac{\partial p}{\partial z}=-\frac{1}{R_g}\frac{d}{dz}\frac{G^2x^2}{\rho_g R_g}+\frac{\tau_{ig}S_i}{R_g A}+\rho_g g=\rho_g^* g$$

$$R_g=\left(1-\frac{2\delta}{D}\right)^2\approx 1-\frac{4\delta}{D} \qquad S_i=\pi(D-2\delta)$$

Iterative resolution:

For a given value *z*, x is known

Guess value for δ,

modeling of τ_i , calculation of $\rho_g^*,\tau_i^*,\delta^*,z^*$

Verification of $\delta^{*4}+\frac{4}{3}\delta^{*3}\tau_i^*=z^*$

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Some correlations for the Nusselt number

With $\tau_i = 0$

Laminar Flow $Re<30$ $\qquad Nu = 1{,}47\,\mathrm{Re}_z^{-1/3}$

Laminar wavy flow $30<Re_z<1800$ $\qquad Nu = \dfrac{\mathrm{Re}_z}{1{,}08\,\mathrm{Re}_z^{1{,}22} - 5{,}2}$

Inertial regime (Sparrow et Gregg, 1959)

$$Nu = \left(0{,}68Ja+1\right)^{1/4}\left(\frac{g\rho_L(\rho_L-\rho_v)h_{Lv}^{*}z^3}{4\mu\lambda\left(T_{sat}-T_p\right)}\right)^{1/4} \qquad Ja = \frac{C_p(T_{sat}-T_p)}{h_{Lv}}$$

Wavy turbulent liquid film

Correlation of Kirkbridge $\qquad Nu = 0{,}0077\,\mathrm{Re}^{0{,}4}$

Colburn (1933) $Pr<0.05$ $\qquad Nu = 0{,}056\,\mathrm{Re}^{0{,}2}\,\mathrm{Pr}^{1/3}$

Grober (1961) $1<Pr<5$ $\qquad Nu = 0{,}0131\,\mathrm{Re}^{1/3}$

Extension of Colburn correaltion with $\tau_i \neq 0$

Dukler model →extension of Rohsenow model with a eddy viscosity model

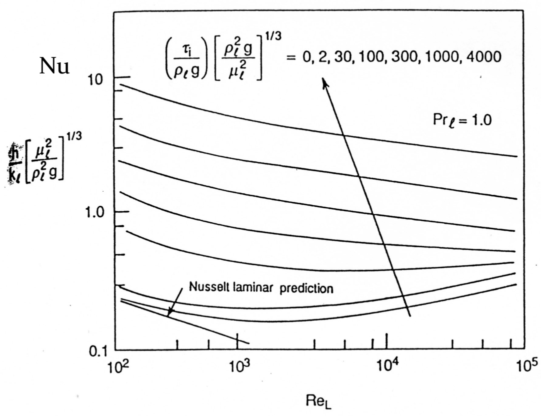

Figure Variation of the local film condensation heat transfer coefficient with Reynolds number and τ_i as predicted by the analytical model of Dukler [9.42]. *(Adapted from* [9.42] *with permission, copyright © 1960, American Institute of Chemical Engineers.)*

$$Nu = 0{,}065\,\mathrm{Pr}^{1/2}\sqrt{\tau_i^{*}}$$

$$\tau_i^{*} = \frac{\tau_i}{\rho_L\left(g\nu_L\right)^{2/3}}$$

Application : calcul of the heat transfer coefficient in condensation on a flat plate without vapour flow, with and without inertia effects.

Calculate the numerical value of the HTC at the end of a plate of 10 cm long at a temperature of 80°C, with condensation of water vapour at 100°C. Given values:

$$\rho_L = 958\ kg/m^3,\ \rho_V = 0.597\ kg/m^3, \nu_L = 2.9\ 10^{-7} m^2/s,$$
$$C_{pL} = 4185\ J/kg/K, \lambda_L = 0.679\ W/m/K,\quad h_{LV} = 2257\ kJ/kg.$$

Compare the expressions of the Nusselt numbers in both cases.

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John G. Collier, John R. Thome, Convective Boiling and Condensation, Third edition, Oxford University Press, 1994.

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