

12

Gases

- Why does a plastic bottle left in a car overnight look crushed on a chilly morning?
- How hard is air pushing on your body?
- How long can the Sun shine?

BE SURE YOU KNOW HOW TO:

- Draw force diagrams (Section 3.1).
- Use Newton's second and third laws to analyze interactions of objects (Section 3.7 and 3.8).
- Use the impulse-momentum principle (Section 6.3).

When you inflate the tires of your bicycle in a warm basement in winter, they tend to look a bit flat when you take the bike outside. The same thing happens to a basketball—you need to pump it up before playing outside on a cold day. An empty plastic bottle left in a car looks crushed on a chilly morning. What do all those phenomena have in common, and how do we explain them?

IN CHAPTER 11, we learned that sound propagates due to the compression and decompression of air. But what exactly is being compressed? To answer this question and the ones above, we need to investigate what makes up a gas and how certain properties of gases can change.

12.1 Structure of matter

When we look at objects that surround us, we do not see their internal structure—water in a cup looks homogeneous, and the cup itself looks like one piece of material. However, they must be made up of *something*. What are the building blocks of matter? To begin to answer this question, consider a simple observational experiment.

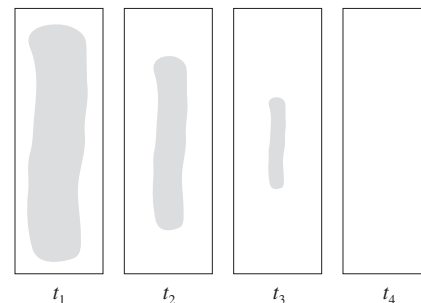
Imagine that you dip a cotton ball in rubbing alcohol and wipe it across a piece of paper (**Figure 12.1**). The wet alcohol strip disappears gradually, with the edges of the strip disappearing first. You observe the same behavior when you wipe water or acetone on the paper, except that the water strip disappears more slowly and the acetone strip disappears more quickly. This phenomenon is the same one we observe with wet clothes and puddles as they dry.

Since the alcohol disappeared gradually, it is reasonable to suggest that it is made of “pieces” too small to be seen. If the alcohol were composed of one piece, it would be gone all at once. However, the model of small pieces does not explain *how* the disappearance occurs. Let’s try to construct some possible *mechanisms* that explain how the alcohol disappears (these mechanisms would also be applicable to the drying of water or acetone). Three of the many possible mechanisms are described below:

- Mechanism 1.** The little pieces of liquid move to the *inside* of the paper and are still there, even though the paper looks dry.
- Mechanism 2.** The air surrounding the paper somehow pulls the liquid pieces out of the paper.
- Mechanism 3.** The pieces of liquid are moving—they bump into each other and slowly bump each other out of the paper one by one.

All of these mechanisms seem viable. Testing Experiment **Table 12.1** will help us decide if we can rule out any of them. To make predictions based on these three mechanisms, we will assume that the pieces comprising the alcohol have mass.

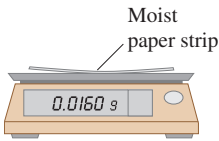
FIGURE 12.1 A disappearing moist strip on a piece of paper.



TESTING EXPERIMENT TABLE 12.1

Testing various mechanisms for the drying of wet objects

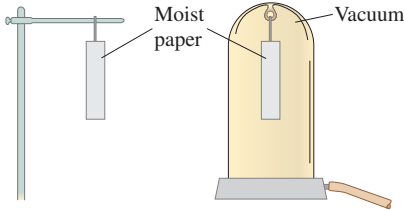


Testing experiment 1	Prediction	Outcome
<p>Weigh a dry strip of paper on a sensitive scale. Record its mass. Then moisten the paper with alcohol and record the mass of the wet paper. What happens to the mass of the strip when it dries?</p> 	<p>Mechanism 1. If the alcohol pieces go “inside” the paper, then the mass of the paper after it dries should be greater than it was before it was made wet and exactly the same as it was just after it was made wet.</p> <p>Mechanism 2. If the air is somehow responsible for removing the alcohol pieces, then after the paper dries, it should have the same mass as it had before it was made wet.</p> <p>Mechanism 3. If the moving alcohol pieces bump each other out of the paper, then after the paper dries it should have the same mass as it had before it was made wet.</p>	<p>After the paper dries, its mass is the same as before it was made wet.</p>

Conclusion 1

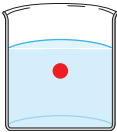
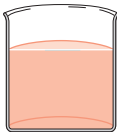
The alcohol pieces left the paper. Mechanism 1 is disproved by this experiment, but Mechanisms 2 and 3 are consistent with it. This leaves us with only two possible mechanisms.

(CONTINUED)

Testing experiment 2	Prediction	Outcome
<p>Moisten two strips of paper. Place one inside a sealed glass jar attached to a vacuum pump. Place the other strip outside the jar. Pump the air from the jar.</p> 	<p>Mechanism 2. If the air is somehow responsible for removing the alcohol pieces out of the strip, then the strip inside the vacuum jar with little air should dry more slowly than the strip outside the jar.</p> <p>Mechanism 3. If the moving alcohol pieces bump each other out of the paper, then the strip inside the vacuum jar should not dry more slowly than the strip outside the jar.</p>	<p>The paper inside the evacuated jar dries faster.</p>

Conclusion 2

Mechanism 2 is disproved by this experiment. Mechanism 3 is not disproved by it.¹ Now we have only one mechanism that has not been disproved.

Testing experiment 3	Prediction	Outcome
<p>Add a droplet of colored alcohol to a glass of clear alcohol. What happens to the colored droplet?</p> 	<p>Mechanism 3. If the small pieces of clear alcohol are moving and bumping each other, then they should bump the colored pieces and cause them to spread.</p>	<p>The color slowly spreads throughout the clear alcohol.</p> 

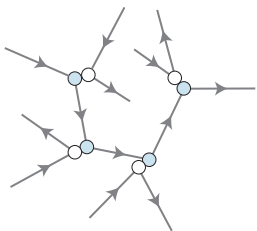
Conclusion 3

Mechanism 3 is supported by this experiment.

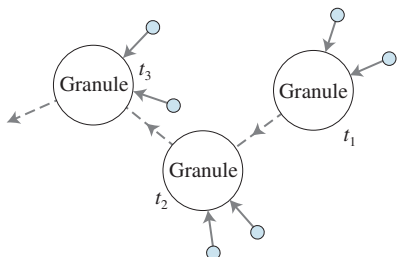
¹Note that while Mechanism 3 was not disproved in Experiment 2, the mechanism does not explain why the strip in the vacuum jar actually dried faster.

FIGURE 12.2 Random motion of water particles and pollen granules.

- (a) The shaded water particle moves randomly due to collisions with other water particles.



- (b) Water molecules collide with the pollen granule and cause it to move in random directions.



Based on these experiments, it is reasonable to assume that alcohol and other liquids are composed of smaller objects, called **particles**, which move randomly in all directions. These particles need empty space between them so that particles of other materials can move between them, as happened in Experiment 3 in Table 12.1. This model of the internal structure of alcohol can be used to explain many other phenomena that we encounter—the way some liquids mix or smells spread. In fact, experiments such as those described in Table 12.1 could have led the Greek philosopher Democritus (460–370 B.C.) to the *atomistic* model. *Atomos* in Greek means *indivisible*. According to Democritus, matter was composed of small, indivisible pieces with different shapes and properties. Democritus also suggested that the pieces were separated by tiny regions of completely empty space, which is a crucial idea as we understand now. Democritus proclaimed, “There is nothing in the world but atoms and empty space.”

In 1827, Scottish botanist Robert Brown was observing pollen in a droplet of water through a microscope. To his surprise, the pollen granules were moving randomly, stopping, and then continuing in a different direction after each stop. To explain his observations, Brown used Mechanism 3—that water itself is composed of particles smaller than the granules and these particles move randomly between frequent collisions (Figure 12.2a). The water particles randomly hit the pollen granules from all directions and caused random changes in the position of a granule (Figure 12.2b). This experiment supported the model of water composed of invisible particles that were in continual random motion.

In 1905, Albert Einstein constructed a quantitative model to describe the phenomenon observed by Brown, called *Brownian motion*. He predicted the average distance that a granule of a certain size would move in a given time interval under specific conditions. Later, Jean Perrin conducted experiments whose outcomes matched Einstein's predictions. Physicists consider Perrin's experiments strong support for the *particle structure of matter*.

We now know that **atoms** are the smallest objects that still retain the chemical properties of a particular element (hydrogen, oxygen, carbon, iron, gold, etc.). Atoms of these various elements combine to build solids, liquids, and gases. A **molecule** is a certain combination of atoms that bond together in a particular arrangement. Molecules may consist of two atoms (such as oxygen, O_2 , and nitrogen, N_2), three atoms (such as water—two hydrogen atoms and one oxygen atom, H_2O), or many atoms. Protein molecules can consist of thousands of atoms. Atoms are composed of fundamental particles—protons, neutrons, and electrons. We will discuss these fundamental particles later in the book (Chapters 28 and 29). For now, we will use the word *particle* to indicate an object approximately the size of a molecule or smaller. Our investigations and the history of physics bring us to a *particle model* of the internal structure of matter. In this model the matter is made of small particles that move randomly. The space between the particles is completely empty. We will learn later what conditions affect particle motion. We will also learn how these particles interact with each other.

The particle model explains how we can smell things even when we are not near them. Suppose we open a bottle of perfume while standing in the middle of a room. Several minutes later, people all over the room can smell the perfume. According to the particle model, the little particles of perfume leave the bottle and gradually disperse, eventually arriving at our nostrils. Since everyone in the room eventually smells the perfume, the perfume particles must move in all directions. However, it takes time to smell the perfume if you are far away from the bottle. Why is that? Perfume particles leaving the bottle move quickly but collide with air particles along the way and reach your nose only after many collisions.

If air is composed of tiny particles, and those particles have mass, then air must have mass. We can test this hypothesis with another simple experiment. Take two identical rigid metal cylinders—one that has had the air evacuated with a vacuum pump, and the other filled with air. Then weigh them. If the hypothesis that air has mass is correct, the evacuated cylinder should weigh less than the unevacuated cylinder. The cylinder filled with air is indeed heavier than the evacuated cylinder (Figure 12.3). However, the difference in masses is small, about 7 g for cylinders of about $6 \times 10^{-3} \text{ m}^3$ (1.5 gallons). Air particle composition can also help us explain the propagation of sound. The propagating disturbances are the regions of air particles closer together or farther apart.

Gases, liquids, and solids

We know from experience that gases are easy to compress, while liquids and solids are almost incompressible (Figure 12.4). The particle model helps us explain this difference: we assume that matter in all states is composed of small particles, but the amount of empty space between the particles is different in solids, liquids, and gases. In solids and liquids, the particles are closely packed (almost no empty space between them), while in gases the particles are packed more loosely (lots of empty space).

Gases tend to occupy whatever volume is available. If you take the air that fills a small cylinder and move it to a much larger cylinder, the air also fills the large cylinder (Figure 12.5a). In contrast, if we move the liquid filling a small container to a much larger container, the liquid volume remains the same independent of the container's shape (Figure 12.5b). Solids maintain not only their volume but also their shape. To distinguish them from solids, gases and liquids are sometimes called *fluids*. We will learn more about fluid behavior in Chapters 13 and 14.

In order for particles to form a substance, the particles must somehow be attracted to each other. We will learn later (in Chapters 17 and 28) that although the nature of this attractive force between particles is different from their gravitational attraction for each other, there are some similarities between the two types of attraction. For instance,

FIGURE 12.3 Air has mass.

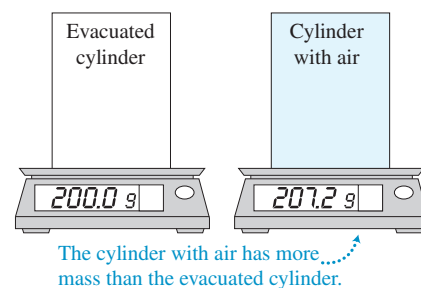


FIGURE 12.4 A gas can be compressed, but a liquid cannot.

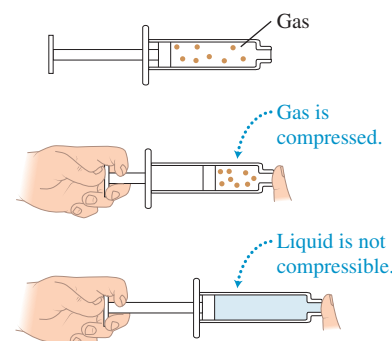
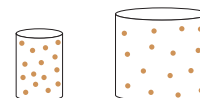


FIGURE 12.5 Although gases and liquids are both fluids, they do not share all the same properties.

- (a) The same gas completely fills a different volume.



- (b) The liquid volume remains the same regardless of the container.



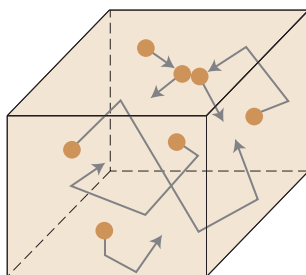
in both cases the particles are attracted to each other at a distance, without any direct contact. The behavior of gases indicates that particles in gases must be so far from each other that such attractive forces are very small or even negligible. In liquids, the particles are closer together, and these new attractive forces between them are strong. In solids they are even stronger.

Ideal gas model

From the observed behavior of gases and our explanations based on the particle nature of matter, we can conclude that gases will be the easiest to study, as we can neglect the interactions of the particles at a distance. We start by constructing a simplified model of a gas as a system.

In this model, we assume that the average distance between the particles (molecules or atoms) is much larger than the size of the particles. We assume that the gas particles are point-like objects that only interact with each other and with any surfaces in their containers during collisions (Figure 12.6), similar to the impulsive interactions of a billiard ball with a side of a pool table. In our model the particles do not attract each other at a distance the way they do in solids and liquids. We also assume that the motions of these particles obey Newton's laws. Thus, between collisions the particles move in straight lines at constant velocity and only change their velocities during collisions. Altogether, these simplifying assumptions make up the **ideal gas model**.

FIGURE 12.6 Ideal gas model.



Ideal gas model A model of a system in which gas particles are considered point-like and only interact with each other and the walls of their container through collisions. This model also assumes that the particles and their interactions are accurately described using Newton's laws.

Ideal in this context does not mean perfect; it means *simplified*. This is a simplified model with certain assumptions. Whether or not this model can be used to represent a real gas remains to be seen. Only testing experiments can resolve that issue.

How useful is this new model of a gas? As with our previous models of objects (point-like objects, rigid bodies), we will use it to describe and explain known phenomena and then to predict new phenomena. However, so far our model is only qualitative. We need to devise physical quantities to represent the features and behavior of the model. This process will then allow us to construct a mathematical description of an ideal gas and predict its behavior.

REVIEW QUESTION 12.1 Use the particle model to explain how moist objects dry out.

12.2 Pressure, density, and the mass of particles

In this section we will identify several new physical quantities that are useful for describing the properties of the model of an ideal gas. These quantities are also applicable to the properties of real gases, liquids, and solids.

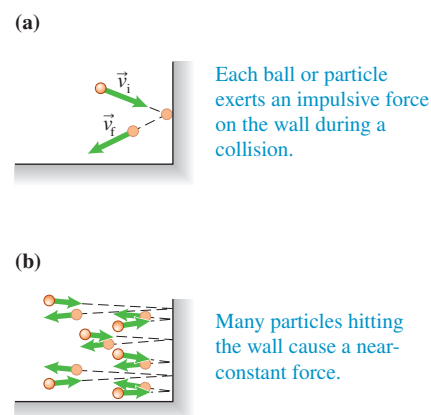
Gas pressure

Imagine you are holding an air-filled balloon. Try to crush it a little bit. You feel the balloon resisting the crushing, as if something inside it pushes back on your fingers. You might think it is the rubber resisting your squeeze, but what makes the rubber bulge out

in the same way in all directions? The balloon is filled with air. As we will learn later in this section, the particles of matter (atoms and molecules) are rather small. As air particles move randomly in space, they eventually collide with the solid surfaces of any objects in that space. In each of these collisions, the particle exerts an impulsive force on the object—like a tennis ball hitting a practice wall (see **Figure 12.7a**). However, when a huge number of particles bombard a solid surface at a constant rate, these collisions collectively exert an approximately constant force on the object (Figure 12.7b). This impulsive force must be what we feel when we are trying to squeeze the balloon. Notice that we have now constructed a *model of a process* that explains how the motion of the particles of gas inside the balloon accounts for the observational evidence—the apparent resistance of the balloon to squeezing. This ideal process that we imagined serves as a mechanism for what we observe.

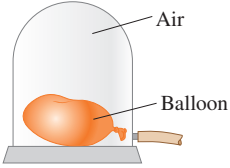
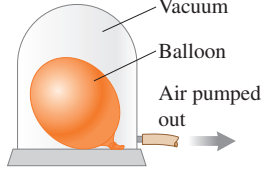
We can test this process model with a simple experiment. When we blow up a balloon, filling it with air, we should not forget that there is also air outside the balloon. It is this outside air that will help us do the testing experiment described in Testing Experiment **Table 12.2**.

FIGURE 12.7 Impulsive forces during collisions cause an approximately constant force against a wall.



TESTING EXPERIMENT TABLE 12.2 Testing the model of moving gas particles pushing on the surface



Testing experiment	Prediction	Outcome
Place a partially inflated balloon inside a vacuum jar. Seal the jar.  <p>What happens to the balloon's shape when you start pumping air out of the jar?</p>	As we remove air particles from outside the balloon, the collisions of particles on the outside of the balloon are less frequent and exert less force on each part of the balloon's outer surface. The collision rate of the particles inside the balloon does not decrease. Therefore, the balloon should expand.	As air outside of the balloon is removed, the balloon expands. 

Conclusion

The model of air consisting of moving particles colliding with objects exposed to the air has not been disproved.

In normal situations, an extremely large number of gas particles collide each second with the surface that is in contact with the gas. For example, about 10^{23} particles of air collide each second with each square centimeter of your skin. Making a force diagram for the skin by including these individual particle collision forces is not practical. A different approach is needed.

As we have discussed, although each particle collision is impulsive, the forces are so small and so frequent that the force exerted by the gas on the walls of the container can be modeled as a single constant force. The force also depends on the area of that surface—the bigger the area, the more particles push on it during collisions. Thus, instead of using force to describe gas processes, we use the physical quantity of **pressure**, which we learned about in Chapter 11.

Pressure P Pressure is a physical quantity equal to the magnitude of the perpendicular component of the force, F_{\perp} , that a gas, liquid, or solid exerts on a surface divided by the contact area A over which that force is exerted (see **Figure 12.8**):

$$P = \frac{F_{\perp}}{A} \quad (12.1)$$

FIGURE 12.8 The quantity pressure.

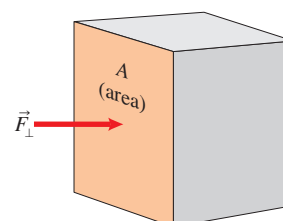


TABLE 12.3 Pressure units

1 atm = $1.01 \times 10^5 \text{ N/m}^2 = 14.7 \text{ lb/in.}^2$
= 760 mm Hg
1 Pa = $1 \text{ N/m}^2 = 1.45 \times 10^{-4} \text{ psi}$

The SI unit of pressure is the pascal (Pa), where $1 \text{ Pa} = 1 \text{ N/m}^2$. In British units, pressure is measured in pounds per square inch (psi). **Table 12.3** gives common pressure unit conversions.

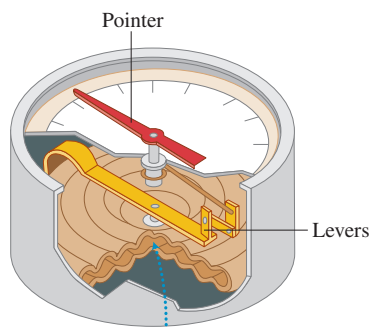
Pressure is easy to visualize when you think about two solid surfaces that contact each other. Compare the magnitude of force you exert on soft snow when wearing street shoes versus the magnitude of force that you exert on the snow when wearing snowshoes. The magnitude of force that you exert on the snow is the same, but the corresponding pressure is not. The snowshoes decrease the pressure you exert on the snow, since the force is spread over a much larger area. Similarly, when you decrease the area over which the force is spread, the pressure increases. Scissors and knives increase pressure on a surface because they decrease the area over which they exert a force.

Does a gas exert pressure inside itself, or does it only press on the walls of its container? Let us think about the air in a room. The air particles hitting one wall in the room collectively exert an average force on the wall, and therefore a pressure. However, the air particles in the center of the room are not interacting with the wall at all, at least not for the moment. Yet the air there has a pressure as well. If a table were placed in the center of the room, the air particles would exert an average force, and therefore a pressure, on the top, bottom, and sides of the table. Thus, air has a pressure whether or not a solid object is present.

Measuring pressure

Many instruments are used to measure pressure. An aneroid barometer is used to measure gas pressure directly. The barometer contains a small aneroid cell (**Figure 12.8**). Inside, the cell has almost no air. A lever is attached to the cell's moveable wall. As the outside air pressure on this wall changes, the cell thickness changes, and the lever causes a pointer needle on the aneroid barometer to turn, indicating the outside air pressure. Measurements show that the pressure of the atmospheric air at sea level is on average 10^5 N/m^2 , or 10^5 Pa . This atmospheric pressure defines yet another unit of pressure, called an atmosphere: $1.0 \text{ atm} = 1.0 \times 10^5 \text{ Pa} = 1.0 \times 10^5 \text{ N/m}^2$.

Another way to measure gas pressure is to compare the pressure of a gas in contact with a gauge to the atmospheric pressure, as we did in Chapter 11 when studying sound waves. For example, when you use a tire gauge to measure the air pressure in a car tire, you are comparing the air pressure inside the tire to that of the atmosphere outside the tire. The pressure in the tire is called **gauge pressure**. If the pressure in a container is 1.0 atm , its gauge pressure is zero, because there is no difference between the pressure inside the container and outside of it.

FIGURE 12.9 An aneroid barometer.

An evacuated aneroid cell gets thicker or thinner depending on the outside air pressure.

Gauge pressure P_{gauge} Gauge pressure is the difference between the pressure in some container and the atmospheric pressure outside the container:

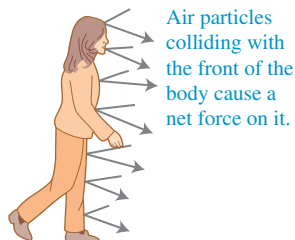
$$P_{\text{gauge}} = P - P_{\text{atm}} \quad (12.2)$$

where $P_{\text{atm}} = 1.0 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2$.

We are continually immersed in a fluid—the gaseous atmosphere. What is the magnitude of the force that the air exerts on one side of your body?

QUANTITATIVE EXERCISE 12.1 The force that air exerts on your body

Estimate the total force that air exerts on the front side of your body, assuming that the pressure of the atmosphere is constant.



Represent mathematically To calculate the total force exerted by air on the front of your body, multiply your front surface area by 10^5 N/m^2 , the atmospheric air pressure. To estimate the surface area of the front of your body, model your body as a rectangular box—your height times your width. Assume a height of 1.8 m and a width of 0.3 m. Use Eq. (12.1) to estimate the force:

$$F_{\text{A on F}} = P \cdot A$$

Solve and evaluate $F_{A \text{ on } F} = P \cdot A = (10^5 \text{ N/m}^2)[(1.8 \text{ m})(0.3 \text{ m})] \approx 5 \times 10^4 \text{ N}$. That's $5 \times 10^4 \text{ N} \times 0.22 \text{ lb/N} = 1.1 \times 10^4 \text{ lb}$, or 10,000 lb. So why aren't you immediately thrown backward?

(Hint: Think about the force that the air behind you is exerting on you as well.)

Try it yourself What is the minimum force that you must exert to lift a $10 \text{ cm} \times 20 \text{ cm}$ 0.10-kg rubber sheet that is stuck flat to a tabletop? Assume that there is no air below the sheet.

Answer $F_{y \text{ on } s} = (P \cdot A) + mg = (10^5 \text{ N/m}^2)(0.10 \text{ m})(0.20 \text{ m}) + (0.10 \text{ kg})(9.8 \text{ N/kg}) = 2000 \text{ N}$, or about 450 lb. Since it is not really this difficult (although it is not easy!) to lift a rubber sheet off the table, under real circumstances there must be a small amount of air trapped between the rubber sheet and the table, exerting an additional upward force on the sheet.

Density

The quantity *mass* helps describe solid objects that have discrete real boundaries—a person, a car, or a ball. However, air is all around us. We can't see it, and it doesn't have well-defined boundaries. If we used the quantity mass to describe air, would it be the mass of one molecule of air, the mass of the air in a room, the mass of air over the street to a certain height, or something else? It's difficult to visualize air as a macroscopic object. For gases, a much more useful physical quantity is the mass of one unit of volume—volumetric mass density, or simply **density**.

Density measures the mass of one cubic meter of a substance. For example, at sea level and 0°C , the mass of 1.0 m^3 of air is 1.3 kg. We say that the density of air is 1.3 kg/m^3 . If we had 2.0 m^3 of air at sea level, its mass would be 2.6 kg. Its density is still 1.3 kg/m^3 , since

$$\frac{2.6 \text{ kg}}{2.0 \text{ m}^3} = 1.3 \text{ kg/m}^3$$

Density ρ The density ρ (lowercase Greek letter “rho”) of a substance or of an object equals the ratio of the mass m of a volume V of the substance (for example, air or water) divided by that volume V :

$$\rho = \frac{m}{V} \quad (12.3)$$

The unit of density is kg/m^3 .

TIP Density is different from mass. Air in a room has a particular mass and density. If you divide the room into two equal parts using a screen, the mass of air in each part will be half the total mass, but the density in each part will remain the same.

QUANTITATIVE EXERCISE 12.2 The density of a person

Estimate the average density of a person.

Represent mathematically Assume the following about the person: mass is 80 kg; dimensions are 1.8 m tall, 0.3 m wide, and 0.1 m thick; and volume is $V \approx 1.8 \text{ m} \times 0.3 \text{ m} \times 0.1 \text{ m} = 0.054 \text{ m}^3$.

The person's average density is

$$\rho = \frac{m}{V}$$

Solve and evaluate Substitute the person's mass and volume into the above to get

$$\rho = \frac{80 \text{ kg}}{0.054 \text{ m}^3} = 1500 \text{ kg/m}^3$$

This is the correct unit for density. In Chapter 13, you will learn how to decide whether this is a reasonable estimate.

Try it yourself An iron ball with radius 5.0 cm has a mass of 2.0 kg. Determine the ball's average density. Explain your answer.

Answer The density of the ball is 3800 kg/m^3 , much less than the 7860 kg/m^3 density of iron. The ball must be hollow.

Mass and size of particles

In 1811, an Italian scientist named Amedeo Avogadro proposed that equal volumes of different types of gas, when at the same temperature and pressure, contain the same number of gas particles. Using Avogadro's hypothesis, scientists could determine the relative masses of different types of particles by comparing the masses of equal volumes of the gases. Presently, scientists use Avogadro's number N_A to indicate the number of atoms or molecules present in 22.4 L ($22.4 \times 10^3 \text{ cm}^3$) of any gas at 0°C and standard atmospheric pressure.

FIGURE 12.10 A mole (6×10^{23} particles) of helium (4 g of helium in the balloon), water (18 g), and salt (58 g).



The mass in grams of any substance that has exactly Avogadro's number of particles is equal to the **atomic mass** listed in the periodic table of chemical elements. For example, the atomic mass of molecular hydrogen is 2, that of molecular oxygen is 32, and that of lead is 207; therefore, 2 g of molecular hydrogen, 32 g of molecular oxygen, and 207 g of lead all have the same number of particles—exactly $N_A = 6.02 \times 10^{23}$. This number of particles is called a **mole** (Figure 12.10).

Avogadro's number and the mole Avogadro's number $N_A = 6.02 \times 10^{23}$ particles is called a mole. The number of particles in a mole is the same for all substances and is the number of particles whose total mass in grams is equal in magnitude to the molecular mass as measured in atomic mass units.

The mass of one mole of particles of any substance is called **molar mass**. One mole of hydrogen (H_2) has a mass of 2.0 g, one mole of oxygen (O_2) has a mass of 32 g, and one mole of lead has a mass of 207 g.

We can now easily determine the mass of a single gas particle of any substance by dividing the molar mass of the substance by 6.02×10^{23} , the number of particles in one mole of the substance:

$$m_{\text{particle}} = \frac{m_{\text{mole}}}{N_A}$$

We find that air (typically 70% N_2 with $m_{\text{N}_2} = 28 \text{ g}$, 29% O_2 with $m_{\text{O}_2} = 32 \text{ g}$, and small percentages of other gases) has a molar mass of about 29 g. Thus, the mass per air particle is approximately

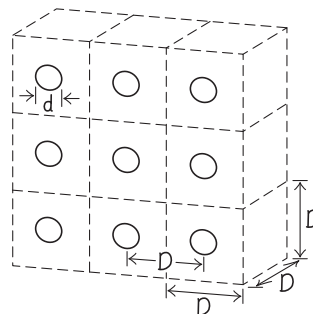
$$m_{\text{air particle}} = \frac{29 \times 10^{-3} \text{ kg}}{6.02 \times 10^{23} \text{ air particles}} = 4.8 \times 10^{-26} \text{ kg/air particle}$$

In addition to a particle's mass, its size is also important. The size of particles was estimated much later than the mass—in the 1860s by Josef Loschmidt. Loschmidt found the linear size of the particles that made up gases, liquids, and solids to be about $d \approx 10^{-9} \text{ m} = 10^{-7} \text{ cm} = 1 \text{ nm}$. Contemporary methods indicate that the size of nitrogen and oxygen particles is about 0.3 nm.

EXAMPLE 12.3 The average distance between air particles

What is the average separation between nearby gas particles in the air, and how does it compare to the size of the particles themselves?

Sketch and translate Sketch the situation as a mole of air divided into equal-sized cubes as shown at the right, with one particle located at the center of each cube. The diameter of an individual particle is d ; the average distance between them is D .



Simplify and diagram Assume standard conditions, with one mole of gas particles occupying a volume of $22.4 \times 10^3 \text{ cm}^3$. Now, imagine this volume divided equally between the particles (the cubes in the figure) and each particle being contained in a cube whose volume is D^3 .

Represent mathematically We determine the volume of the cube corresponding to each particle and then take the cube root of that volume to estimate the average distance between them.

$$V_{\text{per particle}} = \frac{V_{\text{one mole}}}{N_{\text{particles in a mole}}}$$

$$D = \sqrt[3]{V_{\text{per particle}}} = \sqrt[3]{\frac{V_{\text{one mole}}}{N_{\text{particles in a mole}}}}$$

Solve and evaluate

$$D = \sqrt[3]{\frac{V_{\text{one mole}}}{N_{\text{particles in a mole}}}} = \sqrt[3]{\frac{22.4 \times 10^3 \text{ cm}^3}{6.02 \times 10^{23} \text{ particles}}}$$

$$= \sqrt[3]{3.72 \times 10^{-20} \text{ cm}^3/\text{particle}} = \sqrt[3]{37.2 \times 10^{-21} \text{ cm}^3/\text{particle}}$$

$$= 3.34 \times 10^{-7} \text{ cm}$$

Recall that the average diameter of a single particle of air is $3 \times 10^{-8} \text{ cm}$. Thus, the approximate distance between the particles of air is on average

$$\frac{3.34 \times 10^{-7} \text{ cm}}{3 \times 10^{-8} \text{ cm}} \approx 10 \text{ times the size of the particles } (D/d = 10).$$

This is a lot of empty space. A macroscopic analogy would be two apples separated by about four feet. The empty space between the apples compared to the apple size is similar to the empty space between the particles of air compared to their size.

Try it yourself Estimate the distance between water molecules in liquid water and compare this distance to their dimensions. The density of water is $1.0 \times 10^3 \text{ kg/m}^3$. The molar mass of water is $18 \times 10^{-3} \text{ kg/mole}$.

Answer

The distance between particles is $D \approx 3 \times 10^{-8} \text{ cm}$. This is just a little larger than the size of the molecules, which means there is very little space between the water molecules.

$$V_{\text{one molecule}} = \frac{V_{\text{one mole}}}{N_{\text{particles in a mole}}} = \frac{18 \times 10^{-6} \text{ m}^3}{6.02 \times 10^{23}} \approx 3 \times 10^{-29} \text{ m}^3$$

$$V_{\text{one molecule}} = \frac{1000 \text{ kg/m}^3}{18 \text{ kg/mole}} \approx 55.6 \text{ mol/m}^3$$

$$V_{\text{one mole}} = \frac{1000 \text{ kg/m}^3}{18 \text{ kg/mole}} \approx 55.6 \text{ mol/m}^3$$

REVIEW QUESTION 12.2 The distance between air particles is very small—about $3 \times 10^{-7} \text{ cm}$. How can we say that there is considerable empty space in air?

12.3 Quantitative analysis of an ideal gas

We can use the quantities pressure, density, and the mole to construct a mathematical description of an ideal gas that will allow us to make predictions about new phenomena.

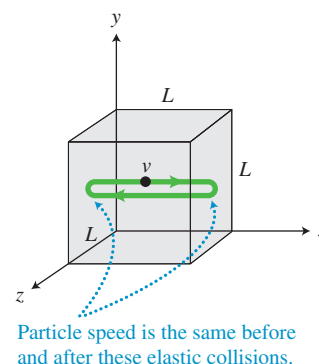
To start, we make a few more simplifying assumptions. First, in addition to modeling the particles (atoms or molecules) as point-like objects whose motion is governed by Newton's laws, assume that the particles do not collide with each other—they only collide with the walls of the container, exerting pressure on the walls (in other words, they move like the model depicted in Figure 12.6 but with no particle collisions). This is a reasonable assumption for a gas of low density. Second, assume that the collisions of particles with the walls are elastic. This makes sense, as the pressure of the gas in a closed container remains constant, which would not happen if the particles' kinetic energy decreased during inelastic collisions.

Now, let's construct a mathematical description of an ideal gas. Imagine the gas inside a cubic container with sides of length L (see Figure 12.11). A particle moves at velocity \vec{v} with respect to a vertical wall. When it hits the wall, the wall exerts a force on the particle that causes it to reverse direction. Since the kinetic energy of the particle is the same before and after the collision, the same is true for its speed. The wall exerts a force on the particle, and the particle in turn exerts an equal-magnitude and oppositely directed force on the wall (Newton's third law).

Let's use impulse-momentum ideas to analyze the particle-wall collision. During the collision with the right wall, it exerts a normal force on the particle in the negative x -direction. Before the collision, the particle has a positive x -component of velocity v_{xi} . After the collision, the particle has a negative x -component of velocity v_{xf} . The impulse-momentum equation gives

$$mv_{xi} + F_{W \text{ on } P} \Delta t_c = mv_{xf}$$

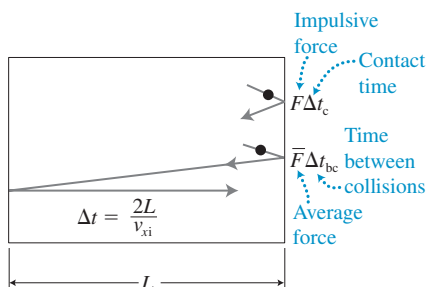
FIGURE 12.11 A gas particle bouncing back and forth between the walls of a container.



Because the particle's speed is the same before and after the collision, $v_{xf} = -v_{xi}$. In addition, because of Newton's third law, $F_{W \text{ on } P_x} = -F_{P \text{ on } W_x}$. Thus,

$$\begin{aligned} mv_{xi} + (-F_{P \text{ on } W_x} \Delta t) &= -mv_{xi} \\ 2mv_{xi} &= F_{P \text{ on } W_x} \Delta t_c \end{aligned}$$

FIGURE 12.12 A method to find the average force of particles colliding with the container wall.



The impulsive force exerted by a particle on the wall during the short contact time interval equals the average force exerted by the particle against the wall during the long time interval between collisions: $F\Delta t_c = \bar{F}\Delta t_{bc}$.

In the above equation, $F_{P \text{ on } W_x}$ is the impulsive force that the particle exerts on the wall during the very short time interval Δt that the particle is actually touching and colliding with the wall. How can we determine an average effect of these impulsive collisions in order to determine the pressure of the gas on the wall? We rewrite the right side of the equation as the product of the *average force* exerted by the particle on the wall from one collision to the next, multiplied by the time that *passes between collisions*. Note that this average force is much smaller than the impulsive force, since most of the time the particle is flying through the container and is not in contact with the wall (**Figure 12.12**). However, the time interval between collisions is longer than the impulsive time interval. The product of the big impulsive force and short time interval equals the product of the small average force and the long time interval between collisions.

Looking at the x -component of the motion of the particle, we see that the time interval between collisions with the wall is $\Delta t_{bc} = (2L/v_{xi})$ since the particle must travel a distance $2L$ in the x -direction before colliding with the same wall again. So our equation becomes

$$2mv_{xi} = F_{P \text{ on } W_x} \Delta t_c = \bar{F}_{P \text{ on } W_x} \Delta t_{bc} = \bar{F}_{P \text{ on } W_x} \frac{2L}{v_{xi}}$$

The bar above the force in the last two expressions indicates that it is the *average force* exerted over the time interval between collisions. Multiplying by v_{xi} and dividing by 2 gives

$$mv_{xi}^2 = \bar{F}_{P \text{ on } W_x} L$$

To relate this microscopic relationship to macroscopic quantities, such as the pressure of the gas and its volume, multiply both sides of the equation by N , the number of particles of gas in the container:

$$Nm v_{xi}^2 = N \bar{F}_{P \text{ on } W_x} L$$

Because the particles do not all move with the same speed, we also replace the quantity v_{xi}^2 (the square of the x -component of the velocity of an individual particle) with its average value for all the particles:

$$Nm \bar{v}_x^2 = N \bar{F}_{P \text{ on } W_x} L$$

and then multiply both sides of the equation by L^2/L^2 :

$$Nm \bar{v}_x^2 = \left(N \frac{\bar{F}_{P \text{ on } W_x}}{L^2} \right) L^3$$

The term in parentheses is the pressure exerted by the gas on the wall (the force exerted by all N particles divided by the wall area L^2 .) The L^3 outside the bracket is the volume occupied by the gas. Thus, the above equation becomes

$$Nm \bar{v}_x^2 = PV$$

Note that this equation refers only to the motion of the particles along the x -direction. The particles also move in the y - and z -directions. Those equations are

$$Nm \bar{v}_y^2 = PV$$

$$Nm \bar{v}_z^2 = PV$$

To make an equation that simultaneously takes into account the motion of the particles in all three directions, we just add these three equations:

$$Nm(\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2) = 3PV$$

How does the sum $\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$ relate to \bar{v}^2 , the average of the squared speed of a particle? We can think of the square of the speed of any particle at any time as the sum of components of velocity along the three axes: $v^2 = v_x^2 + v_y^2 + v_z^2$ (using the

geometrical argument in **Figure 12.13**). These components are independent of each other; this means that the average of the right side of the above v^2 equation over time is simply the sum of the averages: $\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$. Because all directions are equivalent, the average of the square of a particle's velocity in the x -direction is the same as in any other direction: $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$. Therefore, $\bar{v}_x^2 = \frac{1}{3}\bar{v}^2$. Using this new average of the squared speed, we can write

$$Nm\bar{v}^2 = 3PV$$

The left side of the equation is the average kinetic energy of the particles if we multiply by $\frac{1}{2}$. After multiplying by $\frac{1}{2}$ and rearranging the equation, we get

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\bar{v}^2\right) = \frac{2}{3}N\bar{K}$$

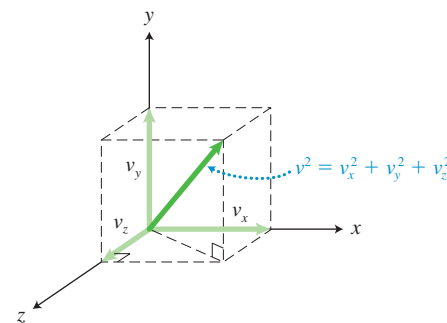
Dividing by V , we get

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\bar{v}^2\right) \quad (12.4)$$

When the particles have high speed, they (1) hit the walls of the container more frequently and (2) exert a greater force during the collisions. Both factors lead to a greater pressure. Thus, it is the speed squared and not just the speed of the particles that affects the pressure.

Suppose the number of particles N and the average of the squared speed of a particle \bar{v}^2 remain the same, but the volume of the box decreases. Then the pressure must increase. This seems reasonable. The smaller the container, the more frequently particles collide with the walls, and the greater the pressure. A unit analysis with a little manipulation indicates that both sides of Eq. (12.4) have the units $\text{kg}/(\text{m} \cdot \text{s}^2)$, so the equation checks out from the point of view of dimensional analysis.

FIGURE 12.13 Using the Pythagorean theorem, we can relate the square of the speed of an individual particle at an arbitrary time to the squares of the x -, y -, and z -components of velocity.



TIP Every time you derive a new equation, ask yourself whether it makes sense. In Eq. (12.4), the pressure is proportional to the average squared speed of the particles. This means that doubling the speed of all the particles quadruples the pressure. Try to explain this dependence before you read on.

QUANTITATIVE EXERCISE 12.4 How fast do they move?

Estimate the average speed of air particles at standard conditions, when the air is at atmospheric pressure ($1.0 \times 10^5 \text{ N/m}^2$), and one mole of the air particles (6.02×10^{23} molecules) occupies 22.4 L or $22.4 \times 10^{-3} \text{ m}^3$. Although air is composed of many types of particles, we will assume that the air particles have an average mass $m_{\text{air}} = 4.8 \times 10^{-26} \text{ kg/particle}$.

Represent mathematically We can estimate the average speed using Eq. (12.4):

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m_p\bar{v}^2\right)$$

Solve and evaluate Multiply both sides by $3V$, divide by Nm_p , and take the square root to get

$$\bar{v} = \sqrt{\frac{3PV}{Nm_p}}$$

Inserting the appropriate values gives

$$\begin{aligned} \bar{v} &= \sqrt{\frac{3PV}{Nm_p}} = \sqrt{\frac{3(1.0 \times 10^5 \text{ N/m}^2)(22.4 \times 10^{-3} \text{ m}^3)}{(6.02 \times 10^{23})(4.8 \times 10^{-26} \text{ kg})}} \\ &= 480 \text{ m/s} \end{aligned}$$

This number seems very high. We know that it takes several minutes for the smell of perfume to propagate across a room. How could it take so long if the perfume molecules move at hundreds of meters per second (about 1000 mph)? We will find out in the next section.

Try it yourself The pressure in a diver's full oxygen tank is about $4 \times 10^7 \text{ N/m}^2$. What happens to the average speed of the particles when some of the oxygen is used up and the pressure in the tank drops to half this value? What assumptions did you make?

Answer If we assume that the only change is in the number of particles in the tank, then their average speed should stay the same. However, when gas expands (as this one does when the valve to the tank is opened), the gas inside the tank has to push outward against the environment in order to leave the tank. This requires energy, which lowers the average kinetic energy (and therefore the speed) of the particles inside.

We will learn later that the speed we calculated in Exercise 12.4 has a special name—it is called the *root mean square* speed, or *rms* speed.

Time interval between collisions

We estimated that the average speed of air particles is $v = 480 \text{ m/s}$, or about 1000 mph. Then why does it take 5 to 10 minutes for the smell of perfume to travel across a room? Remember that we estimated that the average distance between particles in a gas at normal conditions is about $D = 3.3 \times 10^{-7} \text{ cm}$. While deriving our mathematical descriptions of the ideal gas model, we assumed that the particles do not collide with each other. Perhaps the gas particles actually do collide with neighboring particles and change direction due to each collision, thus making little progress in crossing the room. More detailed estimates show that particles collide about 10^9 times a second under typical atmospheric conditions. They change direction at each collision, and even though they are moving very fast, their migration from one place to another is very slow.

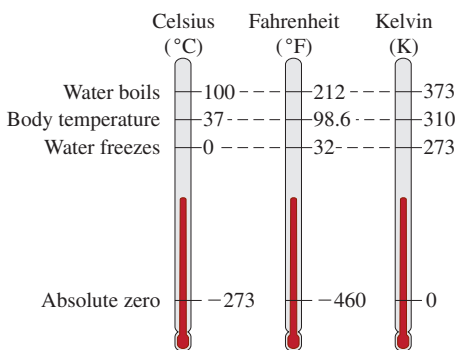
REVIEW QUESTION 12.3 In the expression $PV = \frac{1}{3}N(m_p\bar{v}^2)$, pressure is proportional to the particle mass times the average squared speed of the particles. Thus, doubling the average speed of particles leads to quadrupling the pressure. Explain why this makes sense.

12.4 Temperature

So far, we have said nothing about the temperature of the gas. Could there be a connection between the temperature of a gas and the average kinetic energy of its particles? We now consider this idea.

As with other physical quantities, temperature can be measured. A common way to measure temperature is with a liquid thermometer. A liquid thermometer consists of a narrow tube connected to a bulb at the bottom (Figure 12.14). The bulb and part of the tube are filled with a liquid that expands predictably when heated and shrinks when cooled. To calibrate a thermometer, one marks the height of the liquid at the freezing and boiling conditions of water and then divides this interval by a set number of degrees. On the Celsius scale, 100°C separates the boiling point and freezing point of water. On the Fahrenheit scale, 180°F separates these same points (212°F for boiling and 32°F for freezing). We'll discuss the Kelvin scale shortly.

FIGURE 12.14 Thermometers calibrated for (a) Celsius scale, (b) Fahrenheit scale, and (c) absolute (Kelvin) scale.



What does temperature really quantify?

We measure the temperature of an object indirectly by measuring the changing volume of a liquid that contacts the object. But what is actually different about an object at higher temperature compared to an object at lower temperature? Consider the following experiments.

Imagine you have kept your inflated beach ball in a cold garage in the winter. When you pick it up, it feels soft and looks a little wrinkly. You then bring it into your warm house, and it suddenly becomes firmer and bouncier. If you take it back to the garage, it becomes soft and wrinkly again. Assuming that the air pressure inside the house and garage is the same, the particles of air inside the ball must exert a higher pressure when the gas inside the ball gets warmer and a lower pressure when the gas inside the ball gets colder. Changing the temperature of the gas therefore seems to change its pressure.

How can we explain that a warmer gas seems to exert a greater pressure on its container? We can hypothesize that the particles in the warm gas exert a greater impulsive force on the container because they are moving faster, and therefore in addition to exerting a greater force during every collision, they collide more frequently with the container walls. Conversely, the particles in a cooler gas exert a smaller force on their container's walls because they are moving more slowly and colliding less frequently. Based on this reasoning, we can hypothesize that the temperature of a gas is related to

the speed of the random motion of its particles. Is the temperature of the gas related to any other properties of the gas (the pressure or volume of the gas, or perhaps how many particles comprise the gas and how massive they are)?

Let's do more formal observational experiments. We place three different gases in three containers of different but known volumes. A pressure gauge measures the pressure inside each container. The number N of particles (atoms or molecules) in each container is determined by measuring the mass of the gas m_{gas} in each container and then calculating

$$N = \frac{m_{\text{gas}}}{m_{\text{molar mass}}} N_A$$

where N_A is Avogadro's number. Each container, with known V , P , and N for each gas, is placed first in an ice water bath and then in boiling water, as depicted in **Figure 12.15**. Notice that the volume, pressure, and the number of particles in each container are different, but the temperature of the matter in the three containers is the same. Collected data show the following pattern: independently of the type of gas in a container, the ratio PV/N is identical for all of the gases in the containers when they are at the same temperature:

$$\frac{P_N V_N}{N_N} = \frac{P_O V_O}{N_O} = \frac{P_{\text{He}} V_{\text{He}}}{N_{\text{He}}}$$

The ratio for the gases in the containers at $T_1 = 0^\circ\text{C}$ is smaller than that for the gases in the containers at $T_2 = 100^\circ\text{C}$.

From these experiments we can conclude that if you have any amount of a particular type of gas and know its pressure, volume, and the number of particles, then the ratio PV/N only depends on the temperature of the gas. Maybe it equals the temperature? Consider the units of this quantity:

$$\frac{(\text{N}/\text{m}^2)\text{m}^3}{\text{particle}} = \frac{\text{N} \cdot \text{m}}{\text{particle}} = \text{J}/\text{particle}$$

The joule is a unit of energy, not temperature! Perhaps gas particles at the same temperature have the same average energy per particle. Remember that in the ideal gas model, the particles do not have any potential energy between them. This suggests that temperature is related to the average kinetic energy per particle of the gas.

Can we mathematically relate the energy per particle of the gas molecules to the temperature of the gas? The simplest relationship is a direct proportionality to the temperature:

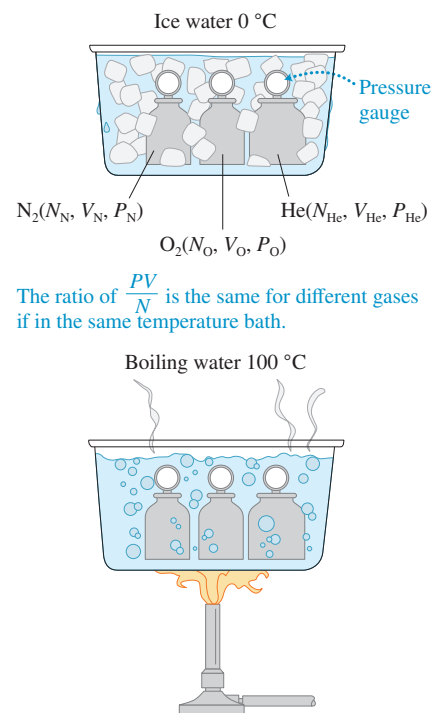
$$\frac{PV}{N} = k_B T \quad (12.5)$$

where k_B is a proportionality constant whose value we need to determine (you'll see shortly why we are using the subscript B). Notice that this relationship immediately leads to a difficulty. The kinetic energy per particle is always a positive number. But in the Celsius and Fahrenheit scales, temperatures can have negative values. So the particle energy cannot be proportional to temperature if measured using either of those scales.

Absolute (Kelvin) temperature scale and the ideal gas law

We need a scale in which the zero point is the lowest possible temperature. That way, all temperatures will be positive. This lowest possible temperature can be found by applying Eq. (12.5) to measurements with a container of gas at two different reference temperatures—the freezing and boiling temperatures of water. The data in **Table 12.4** on the next page were collected when a constant volume metal container with 1 mol

FIGURE 12.15 The ratio PV/N seems to depend only on the gas temperature.



The ratio of $\frac{PV}{N}$ is the same for different gases if in the same temperature bath.

of nitrogen ($N = N_A = 6.02 \times 10^{23}$ particles) was placed in baths at two different temperatures. If we assume that the ratio PV/N is proportional to the absolute temperature of the gas, we can find the coefficient of proportionality.

TABLE 12.4 PV/N for one mole of gas in a 22.4-L container at two different temperatures

Conditions in the bath	Pressure	Volume	$\frac{PV}{N} = k_B T$
Ice water (T)	$1.013 \times 10^5 \text{ N/m}^2$	$22.42 \times 10^{-3} \text{ m}^3$	$3.773 \times 10^{-21} \text{ J}$
Boiling water ($T + 100$)	$1.384 \times 10^5 \text{ N/m}^2$	$22.42 \times 10^{-3} \text{ m}^3$	$5.154 \times 10^{-21} \text{ J}$

We now have two equations with two unknowns: the constant k_B and the water temperature T on the new scale.

$$\begin{aligned} 3.773 \times 10^{-21} \text{ J} &= k_B T \\ 5.154 \times 10^{-21} \text{ J} &= k_B (T + 100) \end{aligned}$$

We subtract the first equation from the second to get $k_B = 1.38 \times 10^{-23} \text{ J/degree}$. The freezing temperature T of the water is then

$$T = \frac{3.773 \times 10^{-21} \text{ J}}{k_B} = \frac{3.773 \times 10^{-21} \text{ J}}{1.381 \times 10^{-23} \text{ J/degree}} = 273.2 \text{ degrees}$$

On this new scale, water freezes at $T = 273.2$ degrees above the lowest possible temperature. The lowest possible temperature on the new scale is 0 and on the Celsius scale should be -273.2°C . Considerable modern research has refined this value to -273.15°C . This temperature scale is called the absolute temperature scale or the Kelvin scale (because it was invented by William Thomson (Lord Kelvin) in 1848). Temperatures are described in kelvin (see the right scale in Figure 12.14). Temperature intervals on the Kelvin scale are the same as on the Celsius scale: a change in temperature of 1°C is equivalent to a change in temperature of 1 K. Celsius temperatures are related to kelvin as follows:

Temperature conversions

$$T_F = (9/5)T_C + 32^\circ$$

$$T_C = (5/9)(T_F - 32^\circ)$$

$$T_K = T_C + 273.15^\circ$$

$$T_K = T_C + 273.15 \quad (12.6)$$

In other words, a temperature of 273.15 K is equivalent to 0°C .

The constant k_B that we determined using the data in Table 12.4 is called **Boltzmann's constant** after the German physicist Ludwig Boltzmann (1844–1906):

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

We can now rewrite Eq. (12.5) using the absolute temperature T and the value of the constant k_B :

$$PV = Nk_B T \quad (12.7)$$

Equation (12.7) is called **the ideal gas law**. It is also commonly used in a slightly different form. Rather than referring to the number of particles that comprise the gas (typically an extremely large number), we refer to the number n of moles of the gas. Since one mole has Avogadro's number of particles, $N = nN_A$. Substituting this into Eq. (12.7), we get

$$PV = nN_A k_B T$$

The product of the two constants $N_A k_B$ is another constant called the **universal gas constant** R . Equation (12.7) then becomes

$$PV = nRT \quad (12.8)$$

$$\text{where } R = N_A k_B = \left(6.02 \times 10^{23} \frac{\text{particles}}{\text{mole}} \right) \left(1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} \right) = 8.3 \frac{\text{J}}{\text{K} \cdot \text{mole}}$$

This is the more common form of the ideal gas law.

TIP Note that Eq. (12.7) implies that when the absolute temperature of the ideal gas is zero, its pressure must be zero.

Ideal gas law For an ideal gas, the quantities pressure P , volume V , number of particles N , temperature T (in kelvins), and Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K are related in the following way:

$$PV = Nk_B T \quad (12.7)$$

The law can also be written in terms of the number of moles n , and the universal

gas constant $R = 8.3 \frac{\text{J}}{\text{K} \cdot \text{mole}}$:

$$PV = nRT \quad (12.8)$$

Temperature and particle motion

Let us look back at what we have done so far. First, we found the relation between the pressure and volume of an ideal gas and the average kinetic energy of the particles that comprise it, $PV = \frac{2}{3}N\bar{K}$ [Eq. (12.4)]. This was a reasonable finding: the faster the particles move inside the gas, the more often and the harder they hit the walls. The particle mass and the number of them per unit volume N/V also affect the pressure.

Next, we found that the product of the pressure and volume of a gas is related to the temperature of the gas, $PV = Nk_B T$ [Eq. (12.7)]. We can now connect the average kinetic energy of the gas particles to the absolute temperature of the gas. Rearrange Eqs. (12.4) and (12.5) so they each have PV/N on the left side. Insert the average kinetic energy $\bar{K} = \frac{1}{2}m\bar{v}^2$ in the right side of Eq. (12.4) and then set the right sides of the two equations equal to each other to get

$$\bar{K} = \frac{3}{2}k_B T \quad (12.9)$$

The temperature of a gas is an indication of the average random translational kinetic energy of the particles in the ideal gas. Note that temperature is an indication of not only the particle's speed but also its kinetic energy—the mass of the particle also matters. One implication of this discovery is that when you have a mixture of particles of different gases in one container (for example, in air there are nitrogen molecules, oxygen molecules, carbon dioxide molecules, etc.), the lighter molecules move faster than the heavier ones, though each species of particles has the same average kinetic energy (since each species will have the same temperature once the gas has mixed together thoroughly).

Before we move to an example, let's think more about temperature. Imagine that you have two metal containers with identical gases that have been sitting in the same room for a long time. One container is large and the other one is small. Which one has the higher temperature? Since the average kinetic energy per particle is the same in each container, the temperatures of the two gases are the same. However, the total kinetic energy of the particles in the large container is larger because it contains more particles.

Imagine another scenario: you have two containers with the same type of gas. In one container the gas is hot, and in the other it is cold. What will happen if you mix those two gases together? The faster moving particles of the hot gas will collide with the slower moving particles of the cold gas. If we use the laws of momentum and kinetic energy conservation (assuming that collisions are elastic), we find that following a collision, the faster moving particle on average is moving slower than before the collision, and the slower particle is on average moving faster than before. Eventually, the particles of the two gases have the same average kinetic energy and therefore the same temperature. Physicists say that the gases are in *thermal equilibrium*.

TIP Only when temperature is measured in kelvins can Eq. (12.9) be used to calculate the average kinetic energy of the particles.

Using Eq. (12.9) we can determine the square root of the average speed squared of gas particles $\sqrt{\bar{v}^2}$, called the root-mean-square speed, or **rms speed**:

$$\begin{aligned}\bar{K} &= \frac{1}{2}m\bar{v}^2 = \frac{3}{2}k_B T \\ \Rightarrow \sqrt{\bar{v}^2} &= \sqrt{\frac{3k_B T}{m}}\end{aligned}\quad (12.10)$$

Calculating rms speeds of particles allows us to estimate quantities involved in gas processes.

QUANTITATIVE EXERCISE 12.5 Comparing speeds for the same temperature

Two different gases (gas 1 has particles of mass m_1 , and gas 2 has less massive particles, of mass m_2) are at the same temperature. How do the rms speeds of their particles compare?

Represent mathematically Because the gases are at the same temperature, their particles have the same average kinetic energy (Eq. 12.9):

$$\bar{K} = \frac{3}{2}k_B T$$

Therefore, their rms speeds are $\sqrt{\bar{v}^2} = \sqrt{\frac{3k_B T}{m}}$.

Solve and evaluate For gas 1, $\sqrt{\bar{v}_1^2} = \sqrt{\frac{3k_B T}{m_1}}$; for gas 2, $\sqrt{\bar{v}_2^2} = \sqrt{\frac{3k_B T}{m_2}}$. The ratio of the rms speeds is

$$\frac{\sqrt{\bar{v}_1^2}}{\sqrt{\bar{v}_2^2}} = \frac{\sqrt{m_2}}{\sqrt{m_1}}$$

The ratio of the rms speeds is the inverse ratio of the square roots of their masses. Therefore, the molecules of smaller mass move faster at the same temperature.

REVIEW QUESTION 12.4 Ken says that the temperature of a gas measured in kelvins is the average kinetic energy of the gas particles. Do you agree with him? Support your answer with an explanation.

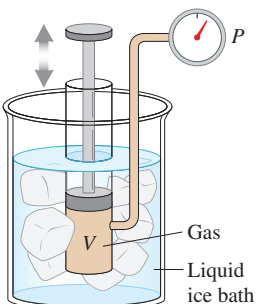
12.5 Testing the ideal gas law

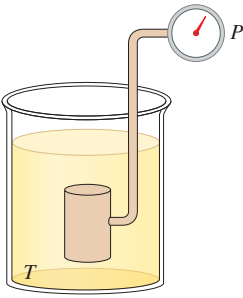
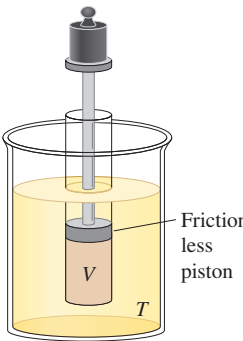
In order to determine if the ideal gas law describes the behavior of real gases, we will use the law to predict the outcomes of some testing experiments. If the predictions match the outcomes, we gain confidence in the ideal gas law. In the experiments below we will keep one of the variables (T , V , or P) constant and predict the relation between the two other variables. Processes in which T , V , or P are constant are called **isoprocesses**. The three types of isoprocesses we will investigate are **isothermal** ($T = \text{constant}$), **isochoric** ($V = \text{constant}$), and **isobaric** ($P = \text{constant}$).

TESTING EXPERIMENT TABLE 12.5



Does the ideal gas law apply to real gases?

Testing experiment	Prediction	Outcome								
<p>Experiment 1: Isothermal process. n moles of gas are in a variable volume V container that is held in an ice bath at constant 0°C (273 K) temperature T. How does the pressure of the gas change as we change the volume of the container? We push the piston slowly so that the temperature of the gas is always the same as the ice bath.</p> 	<p>According to the ideal gas law $PV = nRT$, during a constant temperature process, the product of PV should remain constant. We predict that as the volume decreases, the pressure will increase so that the product remains constant.</p>	<p>Data collected:</p> <table border="1"> <thead> <tr> <th>$V(\text{m}^3)$</th> <th>$P(\text{N/m}^2)$</th> </tr> </thead> <tbody> <tr> <td>3.0×10^{-4}</td> <td>2.0×10^5</td> </tr> <tr> <td>6.0×10^{-4}</td> <td>1.0×10^5</td> </tr> <tr> <td>9.0×10^{-4}</td> <td>0.67×10^5</td> </tr> </tbody> </table> <p>The product of volume and pressure remains constant in all experiments.</p>	$V(\text{m}^3)$	$P(\text{N/m}^2)$	3.0×10^{-4}	2.0×10^5	6.0×10^{-4}	1.0×10^5	9.0×10^{-4}	0.67×10^5
$V(\text{m}^3)$	$P(\text{N/m}^2)$									
3.0×10^{-4}	2.0×10^5									
6.0×10^{-4}	1.0×10^5									
9.0×10^{-4}	0.67×10^5									

Testing experiment	Prediction	Outcome								
<p>Experiment 2: Isochoric process. n moles of gas and the gas volume V are kept constant. The container is placed in different-temperature baths. How does the gas pressure change as the temperature changes?</p> 	<p>According to the ideal gas law $PV = nRT$, during a constant volume process, the ratio $\frac{P}{T} = \frac{nR}{V}$ should remain constant. We predict that the pressure should increase in proportion to the temperature.</p>	<p>Data collected:</p> <table border="1" data-bbox="1298 275 1532 422"> <thead> <tr> <th>T (K)</th> <th>P (N/m²)</th> </tr> </thead> <tbody> <tr> <td>300</td> <td>1.00×10^5</td> </tr> <tr> <td>350</td> <td>1.17×10^5</td> </tr> <tr> <td>400</td> <td>1.33×10^5</td> </tr> </tbody> </table> <p>The ratio of pressure and temperature is constant in all experiments.</p>	T (K)	P (N/m ²)	300	1.00×10^5	350	1.17×10^5	400	1.33×10^5
T (K)	P (N/m ²)									
300	1.00×10^5									
350	1.17×10^5									
400	1.33×10^5									
<p>Experiment 3: Isobaric process. n moles of gas and the gas pressure P are held constant, as a frictionless piston in the gas container can move freely up and down keeping the pressure constant. The pressure inside the container is the sum of the constant atmospheric pressure and the pressure exerted by the object on top of the piston. How does the gas volume change as the temperature changes?</p> 	<p>According to the ideal gas law $PV = nRT$, during a constant pressure process, the ratio $\frac{V}{T} = \frac{nR}{P}$ should remain constant. We predict that the volume should increase in proportion to the temperature.</p>	<p>Data collected:</p> <table border="1" data-bbox="1298 615 1532 762"> <thead> <tr> <th>T (K)</th> <th>V (m³)</th> </tr> </thead> <tbody> <tr> <td>300</td> <td>3.0×10^{-4}</td> </tr> <tr> <td>350</td> <td>3.5×10^{-4}</td> </tr> <tr> <td>400</td> <td>4.0×10^{-4}</td> </tr> </tbody> </table> <p>The ratio of volume and temperature remains constant in all experiments.</p>	T (K)	V (m ³)	300	3.0×10^{-4}	350	3.5×10^{-4}	400	4.0×10^{-4}
T (K)	V (m ³)									
300	3.0×10^{-4}									
350	3.5×10^{-4}									
400	4.0×10^{-4}									
Conclusion										

The outcomes of all three experiments are consistent with the predictions.

- In the first experiment, the product of pressure and volume remains constant, as predicted.
- In the second experiment, the pressure increases in direct proportion to the temperature, as predicted.
- In the third experiment, the volume increases in direct proportion to the temperature, as predicted.

The outcomes of the experiments in Testing Experiment [Table 12.5](#) were consistent with predictions based on the ideal gas law, giving us increased confidence that the law applies to real gases in the range of temperatures and pressures used in the experiments (however, we cannot say that we proved it). A summary of gas processes (some of which are not isoprocesses) is provided in [Table 12.6](#), on the next page.

Reflection on the process of construction of knowledge

Let's pause here and reflect on the process through which we arrived at the mathematical version of the ideal gas law. The first step was to construct a simplified model of a system that could represent a real gas—the ideal gas model. This involved making assumptions about the internal structure of gases. This model was based on some observations and also on the knowledge of particle motion and interactions developed earlier—Newton's laws of motion. We used this model to devise a mathematical description of the behavior of gases, the ideal gas law. We then tested its applicability to real gases by using it to predict how macroscopic quantities describing the gas (temperature, pressure, volume, and the amount of gas) would change during specific processes (isothermal, isobaric, and isochoric) and used the ideal gas law to construct equations that described those processes. These predictions were consistent with the outcomes of the new testing experiments.

TABLE 12.6 A summary of ideal gas law processes

Name	Constant quantities	Changing quantities	Equation	Graphical representations		
Isothermal	N or n , T	P , V	$PV = \text{constant}$ $P_1V_1 = P_2V_2$			
Isochoric	N or n , V	P , T	$\frac{P}{T} = \text{constant}$ $\frac{P_1}{T_1} = \frac{P_2}{T_2}$			
Isobaric	N or n , P	V , T	$\frac{V}{T} = \text{constant}$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$			
[No name]	N or n	P , V , T	$\frac{PV}{T} = \text{constant}$ $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$			
[No name]		P , V , T , N or n	$\frac{PV}{NT} = k_B$ $\frac{PV}{nT} = R$			

The process of constructing the knowledge described above looks relatively smooth and straightforward—observe, simplify, explain, test. However, in real physics, knowledge construction is not that simple and straightforward. For example, the isoprocesses mentioned above were known in physics long before the ideal gas model was constructed. They were discovered in the 17th and 18th centuries and carried the names of the people who discovered them through patterns found in observational experiments. The relation $PV = \text{constant}$ for constant temperature processes is called **Boyle's law** and was discovered experimentally by Robert Boyle in 1662. The relation $V/T = \text{constant}$ for constant pressure processes is called **Charles's law** and was discovered by Jacques Charles in 1787, though the work was published by Joseph Gay-Lussac only in 1802. Gay-Lussac also discovered the relation $P/T = \text{constant}$, now called **Gay-Lussac's law**. When these relations were discovered empirically, there was no explanation for why gases behaved in these ways. The explanations arrived much later via the ideal gas model. The real process of knowledge construction is often more complicated and nonlinear than how it is presented in a textbook. The skills you are learning by constructing knowledge through experimentation will prepare you for those more complicated situations.

Applications of the ideal gas law

The ideal gas law has numerous everyday applications. Try to explain the following phenomena using the ideal gas law. Clearly state your assumptions.

- A sealed, half-full plastic bottle of water shrinks when placed in the refrigerator.
- A refrigerator's door is difficult to open again right after you opened and closed it the first time.
- A bubble of gas expands as it rises from the bottom of a lake.
- Air rushes into your lungs when your diaphragm, a dome-shaped membrane, contracts.

Below we consider in greater detail two of these phenomena.

Breathing During inhalation, our lungs absorb oxygen from the air, and during exhalation, they release carbon dioxide, a metabolic waste product. Yet the lungs have no muscle to push air in or out. The muscle that makes inhaling and exhaling possible, the diaphragm, is not part of the lungs. The diaphragm is a large, dome-shaped muscle that separates the rib cage from the abdominal cavity (see **Figure 12.16**).

The diaphragm works like a bellows. As the diaphragm contracts and moves down from the base of the ribs, the volume of the chest cavity and lungs increases. If we assume that for a brief instant both the temperature and the number of particles are constant (an isothermal process),

$$P = \frac{nRT}{V}$$

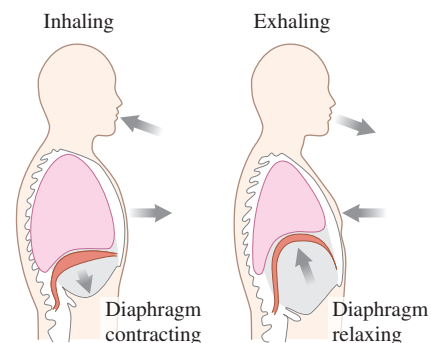
then the pressure in the cavity and in the lungs will decrease. The pressure of the outside air is greater than that of the inside. Because of this, the outside air at normal atmospheric pressure enters the mouth or nostrils and fills the lungs with fresh new air (an increase $\Delta n > 0$ of the amount of air). During exhalation, the opposite occurs. The diaphragm relaxes and rises, decreasing the volume of the chest cavity and the lungs, thus increasing the pressure inside (again assuming that for a brief instant both the temperature and the number of particles are constant),

$$P = \frac{nRT}{V}$$

Because the inside pressure is higher than the outside pressure, air is then forced out of the lungs.

Your water bottle on an airplane The behavior of an empty plastic water bottle on an airplane is another example of an isothermal process.

FIGURE 12.16 The diaphragm is a large, dome-shaped muscle that separates the rib cage from the abdominal cavity. Relaxing or contracting the diaphragm changes the volume of the lungs and chest cavity.



Diaphragm contraction and relaxation cause air to enter and leave the lungs.

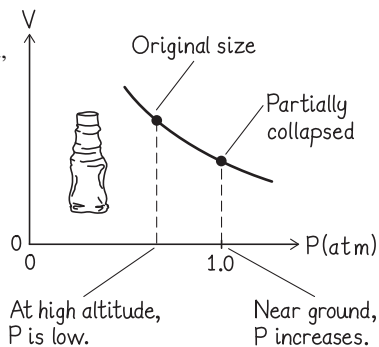
CONCEPTUAL EXERCISE 12.6 A shrunk bottle

On a flight from New York to Los Angeles you drink all of a bottle of water, close the cap, and then store it in your seat pocket. You take it out right after landing. The bottle has changed shape; it looks like someone crushed it. How can we explain this shape change?

Sketch and translate The bottle's volume decreased, though the temperature in the cabin didn't change much during the flight.

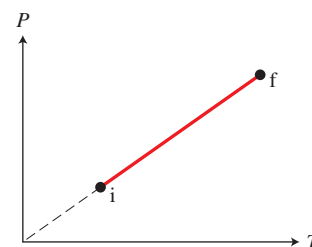
Simplify and diagram If we assume the bottle was perfectly sealed, and the temperature of the gas inside the bottle remained constant, then we can model the process as an isothermal process, represented graphically in the figure below.

Although the cabin is pressurized, at higher elevations air pressure and air density inside the cabin are less than at lower elevations. You closed the bottle when it was filled with low-density air at high elevation. As the plane descended, the air density and pressure inside the cabin increased. More air particles



were hitting the outside walls of the empty bottle than were hitting the inside. The higher pressure from outside partially crushed the bottle. If you open the cap after landing, the bottle will pop back to its original shape.

Try it yourself A different process is represented on a graph at right. Describe the process in words. Assume the mass to be constant.



Answer

The graph represents an isochoric process. The graph line, if extended, passes through the origin; thus the pressure is directly proportional to temperature. The gas was in a sealed container. The gas container was first placed in a water bath at low temperature and then transferred to a water bath at high temperature. The volume of the gas is constant. The pressure increases as the particles move faster and collide with the walls of the container more often.

Comparing different gases

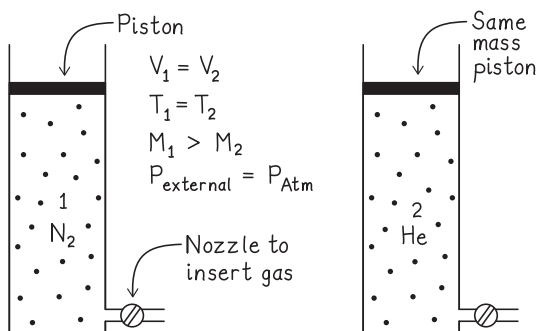
How does the type of gas that is in a container affect the pressure of the gas? Consider the next Conceptual Exercise.

CONCEPTUAL EXERCISE 12.7 Analyzing two types of gas

You have two containers, both with pistons of equal mass that can move up and down depending on the pressure of the gas below. Each container has a nozzle that allows you to add gases to the containers. Container 1 holds nitrogen (molar mass M_1); we label its volume V_1 . Container 2 (V_2) holds helium (molar mass $M_2 < M_1$). The volumes are the same ($V_1 = V_2$), and the containers sit in the same room. What physical quantities describing the gases inside the two containers can we compare?

Sketch and translate We sketch the situation in the figure below. Quantities that we can try to compare between the two containers are the following:

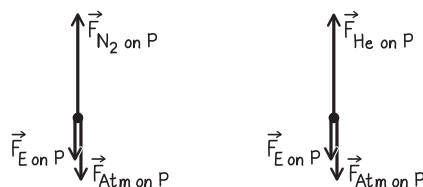
- the pressure inside the containers
- the average kinetic energy of a particle of each type of gas
- the mass of individual gas particles
- the rms speed of the particles in each container
- the number of particles in each container
- the mass of the gas in each container
- the density of the gas in the containers



Simplify and diagram Assume that the ideal gas model accurately describes the gases.

(a) Consider the pressures inside the containers by analyzing the identical movable pistons above the gases. Force diagrams for both pistons are shown at above right. Earth exerts a downward force $\vec{F}_{E \text{ on } P}$ on the piston, the atmospheric gas above a piston pushes down on the piston $\vec{F}_{\text{Atm on } P}$, and the gases inside the containers push up on their pistons $\vec{F}_{N_2 \text{ on } P}$ or $\vec{F}_{He \text{ on } P}$. As the pistons are not accelerating, the net force exerted on each is zero. Because the downward forces exerted by Earth and by the outside atmospheric air on each piston are the same, the upward forces exerted by the gases in each container on the piston

must also be the same. Since the surface areas of the two pistons are the same, the pressure of the gas inside each container must be the same, $P_{N_2} = P_{He}$.



The downward forces on each piston are the same, so the upward force is also the same.

(b) Since the temperature of a gas depends only on the average kinetic energy of the particles, and the temperature of the two gases is the same, the average kinetic energy of each particle is the same:

$$\frac{3}{2}k_B T = \frac{1}{2}m_{N_2 \text{ in } 1}\bar{v}_{N_2 \text{ in } 1}^2 = \frac{1}{2}m_{He \text{ in } 2}\bar{v}_{He \text{ in } 2}^2$$

(c) and (d) Since the N_2 particles in gas 1 have a higher molar mass than the He particles in gas 2, the N_2 molecular mass $m = M_1/N_A$ is also the higher mass ($m_{N_2 \text{ in } 1} > m_{He \text{ in } 2}$). Using this result and that from (b), we find that $\bar{v}_{N_2 \text{ in } 1} < \bar{v}_{He \text{ in } 2}$. The more massive particles move slower.

(e) The pressure, volume, and temperature are the same for both gases. Thus, according to the ideal gas law, both gases have the same number of particles and the same number of moles of gas in their containers ($PV/k_B T = N$; $PV/RT = n$).

(f) and (g) Since the mass of a nitrogen molecule is greater than the mass of a helium atom, and there are equal numbers of particles in each container, the total mass of the gas in the nitrogen container must be greater than the total mass of the gas in the helium container ($M_{N_2 \text{ in } 1} > M_{He \text{ in } 2}$). Since the volumes of the containers are equal, the density $\rho = M/V$ of the gas in the nitrogen container must be greater than that in the helium container ($\rho_{N_2 \text{ in } 1} > \rho_{He \text{ in } 2}$).

Try it yourself Why are the gases at the same temperature?

Answer

Both containers sit in the same room temperature environment.

REVIEW QUESTION 12.5 What is the difference between the following two equations: $PV = \frac{1}{3}Nm\bar{v}^2$ and $PV = nRT$?

12.6 Speed distribution of particles

In our previous analysis, we defined the rms speed of gas atoms or molecules:

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_{\text{B}}T}{m}} \quad (12.10)$$

We found that for air molecules at room temperature, the rms speed was about 500 m/s. When we derived relationships such as these, we assumed for simplicity that gas particles do not collide with each other, just with the walls of a container. However, we know that if gas particles did not collide, the smells of food and perfume would spread at hundreds of meters per second—almost instantly. The smells spread slowly, so the particles must be colliding. What happens if we no longer ignore collisions of particles with each other?

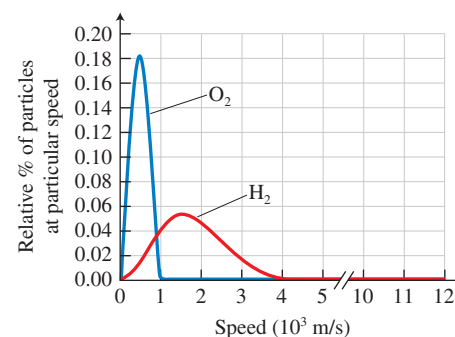
Maxwell speed distribution

In 1860 James Clerk Maxwell included the collisions of the particles in his calculations involving an ideal gas. This inclusion led to the following prediction: at a particular temperature, the collisions of gas particles with each other cause a very specific distribution of speeds. When we were deriving Eq. (12.4) we assumed that the speeds of the particles were different, but we did not have any idea of why they were different. Maxwell's work explained this variability of speeds by the collisions of the particles with each other. Consider the red H_2 line (hydrogen atoms) in **Figure 12.17**. On the vertical axis, we plot the percentage of particles that have a particular speed. According to Maxwell, a certain percentage of the particles should have speeds around 1000 m/s, but the most around 1500 m/s, and fewer at higher speeds. Very few particles have extremely low or extremely high speeds. Most should have intermediate speeds. The most probable speed is at the highest point on the curve in **Figure 12.17**. Surprisingly, the rms speed of the particles at a particular temperature is the same as that predicted by the model with no collisions—Eq. (12.10).

Maxwell's distribution of molecular speeds allows us to explain some very interesting observational evidence. You know that our atmosphere contains molecules of oxygen, nitrogen, carbon dioxide, water, and so forth, but it has almost no hydrogen molecules. Why is there so little free hydrogen in the Earth's atmosphere? Recall from Chapter 7 that every planet has a specific value of escape speed—the speed that an object needs to have to escape the gravitational pull of that planet. For Earth, this speed is about $11 \text{ km/s} = 11,000 \text{ m/s}$. In Quantitative Exercise 12.4 we estimated the rms speed of an air molecule to be about 480 m/s. The most probable molecular speed according to Maxwell's distribution is a little smaller. We also know that the lighter the molecules, the faster they move at the same temperature. Assuming that a hydrogen molecule is about 15 times less massive than an average air molecule, we can say that its rms speed at the same temperature should be about 4 times higher (see the reasoning in Quantitative Exercise 12.5), but it is still much smaller than the escape speed. Based on this estimate, we should have an atmosphere full of hydrogen molecules. What is wrong with our reasoning?

We did not take into account the *shape* of Maxwell's distribution of molecular speeds (**Figure 12.17**). The tail of the distribution shows that there are molecules with a speed that is much faster than the most probable speed. These molecules might escape. As soon as the fastest molecules leave, more molecules, colliding with other molecules, might acquire the speeds in the tail of the distribution and leave, too. This process will repeat until almost all light molecules are gone. Is this reasoning correct? Surely there are some oxygen or nitrogen molecules that have very high speeds—why don't they eventually escape? It turns out that the answer lies in the word *probability*. The likelihood (or probability) of the escape of a hydrogen molecule at 10°C (50°F) is 300 orders of magnitude higher than that for the molecules of oxygen or nitrogen. It is the interplay between escape speed (gravitational interaction) and the Maxwell distribution of molecular speeds (gas processes) during the 4.6-billion-year history of our solar system that determines what kind of atmosphere a particular planet has.

FIGURE 12.17 The Maxwell particle speed distributions at a particular temperature for two gases.



Limitations of the ideal gas law

Isoprocesses were discovered empirically with experiments long before the model of an ideal gas was constructed. However, the fact that the ideal gas law predicts those empirical results is evidence that the law itself and the ideal gas model on which the law is based can be applied to real gases in certain conditions (although real gases are made of molecules that have internal structure and the ideal gas model assumes point-like particles). Nevertheless, the ideal gas law has limitations.

For real gases such as air, the measurements of pressure and volume at the conditions of normal pressure ($1.0 \times 10^5 \text{ N/m}^2$, as elsewhere) and temperature (room temperature) are consistent with predictions of the ideal gas law. But at very high pressures or very low temperatures, real measurements differ from those predictions (for example, the ideal gas law does not predict that one can turn a gas into a liquid). The ideal gas law describes gases accurately only over certain temperature and pressure ranges.

REVIEW QUESTION 12.6 Why do we need the term “probability” when we explain the absence of free hydrogen in Earth’s atmosphere?

12.7 Skills for analyzing processes using the ideal gas law

In this section, we adapt our problem-solving strategy to analyze gas processes. A general strategy for analyzing such processes is described on the left side of the table in Example 12.8 and illustrated on the right side for a specific process.

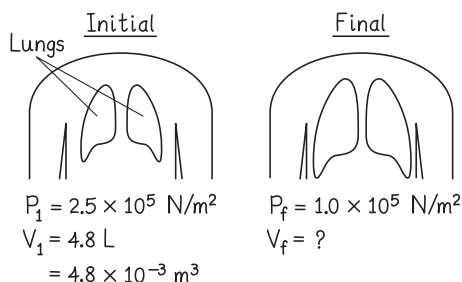
PROBLEM-SOLVING STRATEGY 12.1

Applying the ideal gas law

EXAMPLE 12.8 Scuba diver returns to the surface

The pressure of the air in a scuba diver’s lungs when she is 15 m under the water surface is $2.5 \times 10^5 \text{ N/m}^2$ (equal to the pressure of the water at that depth), and the air occupies a volume of 4.8 L. Determine the volume of the air in the diver’s lungs when she reaches the surface, where the pressure is $1.0 \times 10^5 \text{ N/m}^2$.

The gas inside the diver’s lungs is the system. The initial state of the system is when the diver is underwater; the final state is when she is at the surface. As the diver swims upward, the pressure of the system decreases and its volume increases.



Sketch and translate

- Sketch the process. Choose a system and describe the initial and final states of the system.
- Label the knowns and the unknowns on the sketch.
- Describe the process (a word description of the changes in the system between the initial and final states) in terms of macroscopic quantities (pressure, volume, temperature, moles of gas).

Simplify and diagram

- Decide if the system can be modeled as an ideal gas.
- Decide which macroscopic quantities remain constant and which do not.
- If helpful, draw P vs. V , P vs. T , and/or V vs. T graphs to represent the process.

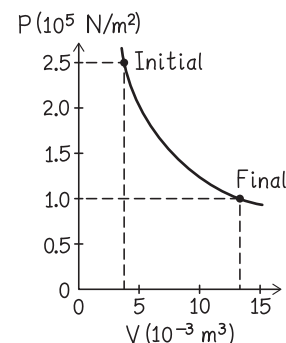
Represent mathematically

Use your sketch or the graphs to help construct a mathematical description—some version of the ideal gas law.

Solve and evaluate

- Solve for the unknowns.
- Evaluate the answer: is it reasonable? (For example, evaluate the magnitude of the answer, its units, and how the solution behaves in limiting cases.)

Assume that the gas inside the lungs can be modeled as an ideal gas. Assume that the diver does not exhale, which means that the moles of gas remain constant. Assume the temperature of the gas is constant at body temperature. A pressure-versus-volume graph for the process is shown at right.



$$P_i V_i = P_f V_f \Rightarrow V_f = \frac{P_i V_i}{P_f}$$

$$V_f = \frac{P_i V_i}{P_f} = \frac{(2.5 \times 10^5 \text{ N/m}^2)(4.8 \times 10^{-3} \text{ m}^3)}{(1.0 \times 10^5 \text{ N/m}^2)} = 12 \times 10^{-3} \text{ m}^3$$

We found the lung volume to be 12 L, much larger than seems possible. Is the answer realistic? Remember that we assumed that the mass of the gas inside remains constant—the diver does not exhale. As we see from the solution, it is important for divers to exhale as they are ascending and the gas expands. If not, they can suffer severe internal damage.

Try it yourself How many moles of gas are in the diver's lungs and how many should she exhale so the final volume is only 6 L instead of 12? Assume that the gas temperature is 37 °C.

Answer 0.47 moles; 0.23 moles

TIP Notice that the graph lines for isobaric processes (constant pressure) pass through the origin in V -versus- T graphs and those for isochoric (constant volume) processes pass through the origin in P -versus- T graphs.

EXAMPLE 12.9 Further investigation of an airplane bottle

In Conceptual Exercise 12.6 you qualitatively investigated the behavior of a plastic bottle on a long flight. On your next long flight, you decide to investigate the shrinking bottle phenomenon quantitatively. In the middle of the flight you open a bottle (Figure a) and then close it again. When you land, you observe that the bottle shrinks (Figure b). You take

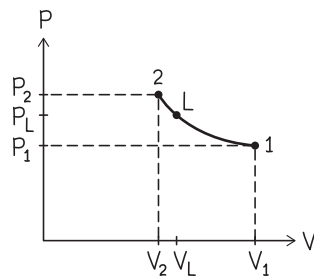
the closed bottle home, turn it upside down into a bowl of water (Figure c), and open it. The bottle expands to its normal size, letting 150 mL of water into the bottle (Figure d). You measure the total volume of the bottle to be 550 mL. Use this information to estimate the pressure in the airplane cabin.



(CONTINUED)

Sketch and translate The air inside the bottle is the system. The initial state is when you open the bottle on the cruising airplane. The air fills the whole bottle ($V_1 = 550 \text{ mL}$) at the pressure inside the airplane (P_1). When the plane lands, the volume decreases to V_L (the bottle is crushed) and the pressure inside the bottle increases to P_L . The L state is an intermediate state in the process. The final state is when you open the bottle at home and water is let in. The volume of air in the bottle is now $V_2 = 550 \text{ mL} - 150 \text{ mL} = 400 \text{ mL}$. The pressure inside is P_2 . We need to determine the pressure P_1 . We already know why the bottle shrinks when the plane lands. But why does the water enter the bottle? Even though the atmospheric air crushes the bottle, the pressure inside is probably still less than atmospheric due to the semiflexible walls of the bottle that tend to return the bottle to the original shape. When the bottle is opened under the water, the pressure difference pushes water into the bottle. In addition, the bottle expands, returning to its original shape, which helps let water into the bottle. This process stops when the pressure of the air in the bottle is equal to the atmospheric pressure outside the bottle. Thus $P_2 = P_{\text{atm}}$.

Simplify and diagram We assume that the temperature in the airplane was the same as at your home ($T_1 = T_2$). Thus the process is isothermal. We can represent the process that the gas in the bottle went through with the P -versus- V graph at right.



Represent mathematically We can now express the graphical representation with an equation for an isothermal process: $P_1V_1 = P_2V_2$.

Dividing both sides by V_1 we get $P_1 = P_2 \frac{V_2}{V_1}$. Now we see that we do not need to convert ml of volume into m^3 because the units for volume cancel.

Solve and evaluate

$$P_1 = P_2 \frac{V_2}{V_1} = (1.0 \times 10^5 \text{ N/m}^2) \frac{400 \text{ mL}}{550 \text{ mL}} = 0.73 \times 10^5 \text{ N/m}^2$$

The result seems reasonable: the pressure during flight is lower than the atmospheric pressure at the ground.

Try it yourself Your sister happens to measure the temperature while you are performing the experiments on the airplane and at home. Her data are $T_{\text{flight}} = 80^\circ\text{F}$ and $T_{\text{home}} = 67^\circ\text{F}$. Use her data to improve your previous estimation of the pressure during the flight.

Answer $0.73 \times 10^5 \text{ N/m}^2$

REVIEW QUESTION 12.7 Why is it helpful to know whether the mass of the gas is constant during a particular process?

12.8 Thermal energy, the Sun, and diffusion

We've seen that the ideal gas model can be applied to many real-world situations as long as the gas particles do not interact with each other (they are far apart and/or the pressure and temperature are not extreme). In this section we will look at two more everyday phenomena: the Sun shining and gas diffusion. In order to get started, we need to quantify something for gases that we've only discussed qualitatively in the textbook so far: thermal energy.

We learned in Chapter 7 that thermal energy is the component of the internal energy of a system that is associated with temperature. We know from Eq. (12.9) that the temperature of an ideal gas is determined by the average kinetic energy \bar{K} of its randomly moving particles. We can now therefore define the gas's **internal thermal energy** or just **thermal energy**, U_{th} , as the sum of all the individual particles' kinetic energies. This is equal to $N\bar{K}$, where N is the number of particles. Using Eq. (12.9), this gives

$$U_{\text{th}} = N \left(\frac{3}{2} k_B T \right)$$

How long can the Sun shine?

The thermal energy of gases can be used to help analyze many kinds of systems, such as automobile engines, Earth's atmosphere, and the Sun. The mass of the Sun is about $2 \times 10^{30} \text{ kg}$, its radius is about $7 \times 10^8 \text{ m}$, and the temperature of the Sun's surface is

about 6×10^3 K. Its core temperature is about 10^7 K. Every second, the Sun radiates about 4×10^{26} J as visible light and other forms of radiation. How long can our Sun shine if its particles possess *only* thermal energy?

We first need to determine how much thermal energy the Sun possesses. Then we can divide this amount of energy by the amount it loses every second from the emission of visible light and other forms of radiation to find the number of seconds the Sun can shine if its thermal energy is the only source of this radiative energy. Although in reality there are some complications with modeling the material of the Sun as an ideal gas, we will do so in this case for simplicity.

To estimate the thermal energy of the Sun, we need to know the number of particles and the average temperature of these particles. The Sun consists mostly of hydrogen atoms. To find the number of hydrogen atoms in the Sun, we estimate the number of moles of hydrogen gas and then multiply by the number of particles in one mole (note that 1 mole of hydrogen has a mass of $1 \text{ g} = 10^{-3} \text{ kg}$):

$$\begin{aligned} N &= \frac{m_{\text{Sun}}}{m_{\text{hydrogen atom}}} = \frac{m_{\text{Sun}}}{M_{\text{molar mass hydrogen}}/N_A} \\ &= \frac{(2 \times 10^{30} \text{ kg})}{(10^{-3} \text{ kg/mole})/(6 \times 10^{23} \text{ particles/mole})} \\ &= \frac{(2 \times 10^{30} \text{ kg})}{(1.7 \times 10^{-27} \text{ kg/particle})} = 12 \times 10^{56} \text{ particles} \end{aligned}$$

At the very high temperatures within the Sun, each hydrogen atom separates into two smaller particles called an electron and a proton—subjects of later study. This separation of hydrogen atoms doubles the number of particles to 24×10^{56} . We can show that the average distance between these particles is much greater than their dimensions, and thus we can apply the ideal gas model to the Sun's material.

These particles do not spread out into space or collapse in toward the center of the Sun because two competing forces remain in balance. All parts of the Sun exert a gravitational force on all other parts. If we select a small volume inside the Sun as the system of interest and add the forces that all other particles of the Sun exert on the system (see **Figure 12.18**), the net gravitational force points toward the center of the Sun. The second force exerted on the system is the pressure force exerted by other particles on the system. This force points outward and balances the gravitational force. As long as these forces balance each other, the Sun is in equilibrium and does not expand or collapse.

Now consider the total thermal energy of the Sun's particles for two extreme cases. A lower bound for the Sun's thermal energy assumes its temperature throughout equals its surface temperature. An upper bound for the Sun's thermal energy assumes its temperature throughout equals its core temperature.

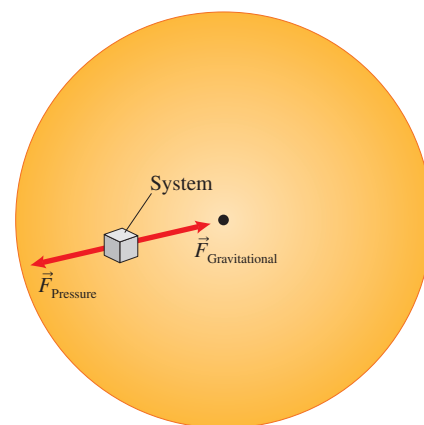
$$\begin{aligned} U_{\text{th min}} &= \frac{3}{2} N k_B T_{\text{min}} \\ &= \frac{3}{2} (24 \times 10^{56} \text{ particles}) (1.38 \times 10^{-23} \text{ J/K}) (6 \times 10^3 \text{ K}) = 3 \times 10^{38} \text{ J} \end{aligned}$$

$$\begin{aligned} U_{\text{th max}} &= \frac{3}{2} N k_B T_{\text{max}} \\ &= \frac{3}{2} (24 \times 10^{56} \text{ particles}) (1.38 \times 10^{-23} \text{ J/K}) (10^7 \text{ K}) = 5 \times 10^{41} \text{ J} \end{aligned}$$

Both numbers are so large that they imply that the Sun will shine for a long time. The life expectancy of the Sun in seconds (based on the thermal energy alone) equals the total thermal energy divided by the energy radiated per second, $L = 4 \times 10^{26}$ J/s. We can convert our result to years, noting that there are about 3×10^7 s in 1 year.

FIGURE 12.18 Gas in the Sun is in equilibrium due to two forces.

This small volume of gas (the system) is kept in equilibrium by the gravitational force and pressure force exerted on it by other particles in the Sun.



$$\Delta t_{\min} = \frac{E_{\min}}{L} = \frac{3 \times 10^{38} \text{ J}}{4 \times 10^{26} \text{ J/s}} = 7.5 \times 10^{11} \text{ s} = \frac{7.5 \times 10^{11} \text{ s}}{3 \times 10^7 \text{ s/year}} = 2 \times 10^4 \text{ years}$$

$$\Delta t_{\max} = \frac{E_{\max}}{L} = \frac{5 \times 10^{41} \text{ J}}{4 \times 10^{26} \text{ J/s}} = 1.2 \times 10^{15} \text{ s} = \frac{1.2 \times 10^{15} \text{ s}}{3 \times 10^7 \text{ s/year}} = 4 \times 10^7 \text{ years}$$

The maximum possible lifetime for the Sun if it simply converts its thermal energy into light and other forms of radiation is 4×10^7 years, or 40 million years. Yet according to astronomical knowledge of stars and our solar system, the Sun has been shining in the same way for over 4 billion years. Thus this simple estimate suggests that either the Sun's material cannot be modeled as an ideal gas or some source of energy other than the thermal energy of the Sun's particles has been supporting its existence for a time interval equal to the age of Earth. As the ideal gas model explains many phenomena occurring on the Sun, it means that there must be another energy source within the Sun that far exceeds the thermal energy present. (We will learn about it in Chapter 29.)

Diffusion

Imagine that you fill a balloon with air and leave it for one hour. The balloon looks the same; it did not change. You fill another balloon with carbon dioxide (CO_2); after one hour the balloon is significantly smaller. Why do the balloons look different?

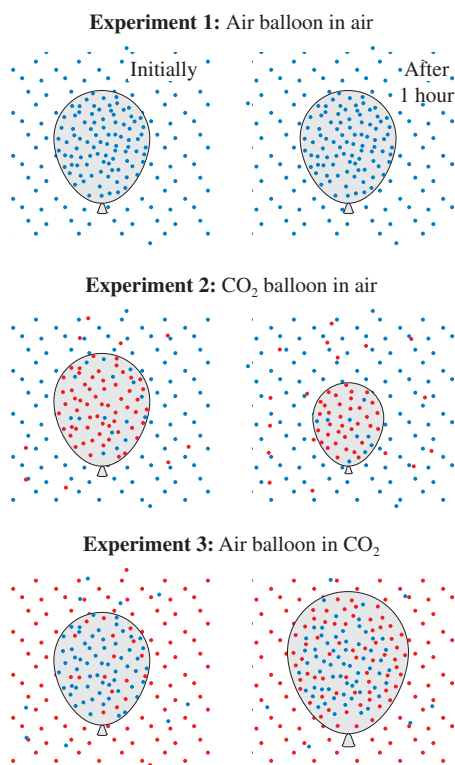
One possible explanation is that the balloon rubber interacts differently with different molecules. If we hypothesize that CO_2 molecules can easily pass through it, but other molecules in air cannot, then we can create a simple model of the process. To indicate that CO_2 molecules interact differently with the balloon than other air molecules, we will represent CO_2 molecules with red dots and other air molecules (mostly nitrogen and oxygen) with blue dots (Figure 12.19). Blue dots cannot pass through the wall. They bounce off the wall either immediately or after some time. This happens to air molecules inside the first balloon (Experiment 1) and outside both balloons. Red dots inside the balloon in Experiment 2 move randomly inside the balloon and slowly migrate through the wall to the outer surface, where their concentration is smaller. They then leave the surface, traveling randomly in all directions.

In our model, CO_2 molecules outside the balloon can as easily get into the balloon as get out of it, so why does CO_2 leak out? Remember that the concentration of CO_2 molecules inside the balloon is much higher than that outside. Many more CO_2 molecules leave the balloon than come back in (Experiment 2), while the air molecules from outside cannot get into the balloon at all. This imbalance in the behavior of different molecules explains why the balloon deflates.

We can test this explanation with another simple experiment. Place an air-filled balloon into a container filled with CO_2 . If our explanation is correct, then the CO_2 molecules from outside the balloon should pass through the rubber into the balloon. Air molecules cannot get out of the balloon (Experiment 3). The balloon should therefore expand and eventually burst. This is exactly what happens!

The process of molecules moving, due to their random motion, from a region of higher concentration to a region of lower concentration is called **diffusion**. The explanation of diffusion follows from the ideal gas model. Diffusion plays an important role in biological processes. Oxygen is carried by hemoglobin in the blood from the heart to the tiny capillary vessels spread throughout the body. Oxygen diffuses from the oxygen-rich blood inside the capillaries to the oxygen-poor cells near the capillaries. Some of these cells may be muscle fibers. Since a muscle fiber needs oxygen to twitch, the action of muscles may be limited by the rate of oxygen diffusion into the fiber. Thus, diffusion limits the rate of some processes in our bodies.

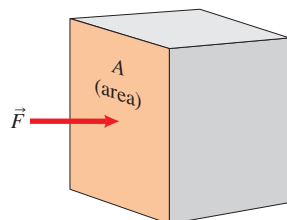
FIGURE 12.19 Testing the model of air molecule movement through semipermeable balloons.



REVIEW QUESTION 12.8 How do we know that the Sun's thermal energy is not the main source of the light energy it produces?

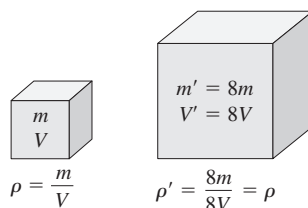
Summary

Pressure P The perpendicular component of the force F that another solid object or a gas or liquid exerts perpendicular to a surface of area A divided by that area. (Section 12.2)



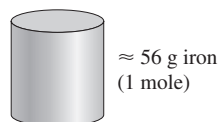
$$P = \frac{F_{\perp}}{A} \quad \text{Eq. (12.1)}$$

Density ρ The mass m of a substance divided by the volume V that the substance occupies. (Section 12.2)



$$\rho = \frac{m}{V} \quad \text{Eq. (12.3)}$$

Moles n and Avogadro's number N_A A mole of any type of particle equals Avogadro's number of that type of particle. Molar mass is the mass of one mole. (Section 12.2)



$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

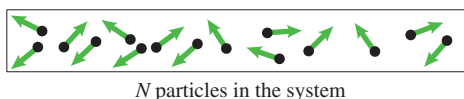
Temperature T and temperature scales

Temperature measures how hot or cool a substance is. When measured in kelvins, the temperature is directly proportional to the average random kinetic energy of a particle in that substance. (Section 12.4)

The average kinetic energy per atom or molecule in a gas is

$$\begin{aligned} \bar{K} &= \frac{3}{2} k_B T \\ T_F &= (9/5)T_C + 32^\circ \\ T_C &= (5/9)(T_F - 32^\circ) \\ T_K &= T_C + 273.15^\circ \end{aligned} \quad \text{Eq. (12.6)}$$

Thermal energy of an ideal gas U_{th} The sum of average kinetic energies of the particles of the ideal gas. (Section 12.8)



$$U_{\text{th}} = N \left(\frac{3}{2} k_B T \right)$$

Ideal gas model and ideal gas law The ideal gas model is a simplified model of gas in which atoms/molecules are considered to be point-like objects that obey Newton's laws. They only interact with each other and with the walls of the container during collisions exerting pressure. The ideal gas law relates the macroscopic quantities of such a gas. (Sections 12.1, 12.3, 12.4, and 12.5)

$$P = \frac{2}{3} \left(\frac{N}{V} \right) \left(\frac{1}{2} m \bar{v}^2 \right) \quad \text{Eq. (12.4)}$$

$$PV = Nk_B T \quad \text{Eq. (12.7)}$$

$$PV = nRT \quad \text{Eq. (12.8)}$$

Assume that N , n , and P are constant. Then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{(Isobaric)}$$

Assume that N , n , and V are constant. Then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{(Isochoric)}$$

Assume that N , n , and T are constant. Then

$$P_1 V_1 = P_2 V_2 \quad \text{(Isothermal)}$$

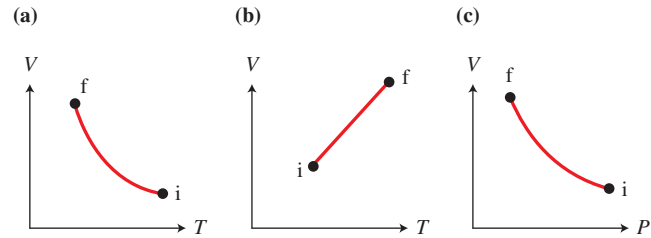
Questions

Multiple Choice Questions

- What experimental evidence rejects the explanation that wet clothes become dry because the air absorbs the water?
 - The clothes dry faster if you blow air across them.
 - They do not dry if you put the wet clothes in a plastic bag.
 - The clothes dry faster under a vacuum jar with the air pumped out.
- What is the difference between the words particle, molecule, and atom?
 - A particle is bigger than a molecule or an atom.
 - Particles can be microscopic and macroscopic, while atoms and molecules are only microscopic.
 - Molecules are made of atoms; both can be called particles.
 - All are correct.
 - Both b and c are correct.
- You have a basketball filled with gas. Which method below changes its volume because of a mass change of the gas inside?
 - Put it into a refrigerator.
 - Squeeze it.
 - Pump more gas into it.
 - Hold it under water.
 - Leave it in the sunshine.
- Choose the quantities describing the air inside a bike tire that do *not* change when you pump the tire.
 - Mass
 - Volume
 - Density
 - Pressure
 - Particle mass
 - Particle concentration
- Which answer below does *not* explain the decrease in size of a basketball after you take it outside on a cold day?
 - The pressure inside the ball decreases.
 - The temperature of the gas inside the ball decreases.
 - The volume of the ball decreases.
 - The number of gas particles inside the ball decreases.
- What causes balloons filled with helium to deflate as time passes?
 - The elasticity of the rubber decreases.
 - The temperature inside decreases.
 - The pressure outside the balloon increases.
 - The gas from inside diffuses into the atmosphere.
- From the list below, choose the assumption that we did not use in deriving the ideal gas law.
 - Gas particles can be treated as objects with zero size.
 - The particles do not collide with each other inside the container.
 - The particles collide partially inelastically with the walls of the container.
 - The particles obey Newton's laws.
- You have a mole of oranges. The mass of each orange is m . Imagine that you split each orange in half. What will be the molar mass of this pile of half oranges in kilograms?
 - $m/2$
 - $m/(6.02 \times 10^{23})$
 - $2m/(6.02 \times 10^{23})$
 - $(m/2)/(6.02 \times 10^{23})$
- How might physicists have come to know that at a constant temperature and constant mass, the pressure of an ideal gas is inversely proportional to its volume?
 - They could have conducted an experiment maintaining the gas as described above and made a pressure-versus-volume graph.
 - They could have derived this relationship using the equations describing the ideal gas model and the relationship between the speed of the particles and the gas temperature.
 - Both a and b are correct.
- A cylindrical container is filled with a gas. On the top of the container sits a heavy piston that is free to move (see the experimental setup in Table 12.5, Experiment 3). When the gas is heated, the piston moves up, causing the volume of the gas to increase. Which equation below best describes this process?
 - $V_{\text{initial}}/T_{\text{initial}} = V_{\text{final}}/T_{\text{final}}$
 - $P_{\text{initial}}V_{\text{initial}} = P_{\text{final}}V_{\text{final}}$
 - $P_{\text{initial}}V_{\text{initial}} = nRT_{\text{final}}$
 - $P_{\text{initial}}V_{\text{initial}} = Nk_{\text{B}}T_{\text{final}}$

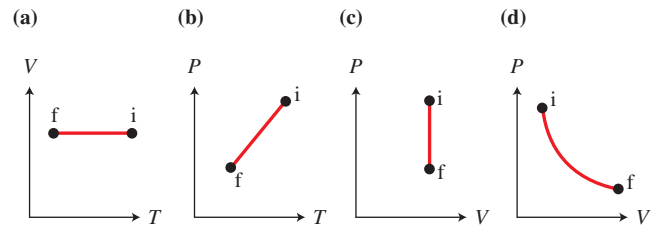
- A cylindrical container is filled with a gas. On the top of the container sits a heavy piston that is free to move. In a certain process, you observe that with constant external pressure pushing down on the piston, the piston moves up, causing the volume of the gas to increase. Which graph below best represents this process (Figure Q12.11)?

FIGURE Q12.11



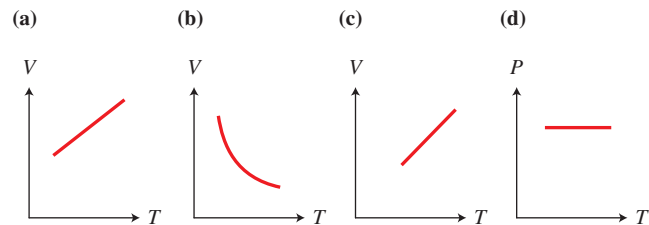
- A completely closed rigid container of gas is taken from the oven and placed in ice water. Which graph below does not represent this process (Figure Q12.12)?

FIGURE Q12.12



- None of them do.
- Which graph below does not represent a process described by the equation $V_{\text{initial}}/T_{\text{initial}} = V_{\text{final}}/T_{\text{final}}$ (Figure Q12.13)?

FIGURE Q12.13



- a and b
- What does contracting and relaxing the diaphragm allow?
 - Greater O_2 absorption by humans and mammals compared to other species
 - Greater pressure difference between air in the lungs and outside air
 - The only way to facilitate air intake into and out of the lungs
 - a and b
 - a, b, and c
 - When is air inhaled into the lungs?
 - When lung muscles cause them to expand
 - When the diaphragm contracts
 - When the chest expands
 - b and c
 - a, b, and c

16. Which of the following conditions are crucial for performing an isothermal compression of a gas (see the experimental setup in Table 12.5, Experiment 1)?
- The piston should be moved very slowly.
 - There should be no friction between the piston and the gas container.
 - The gas container should be kept in a vertical position.
 - The volume of the gas container should be much smaller than the volume of the ice bath.

Conceptual Questions

17. (a) How do we know if a real gas can be described as an ideal gas? (b) How can you decide if air in the physics classroom can be described using the ideal gas model?
18. Why does it hurt to walk barefoot on gravel?
19. In the magic trick in which a person lies on a bed of nails, why doesn't the person get hurt by the nails?
20. What does it mean if the density of a gas is 1.29 kg/m^3 ?
21. How many oranges would you have if you had two moles of oranges?
22. Imagine that you have an unknown gas. What experiments do you need to do and what real equipment do you need to determine the mass of one molecule of this gas?
23. Describe how temperature and one degree are defined on the Celsius scale.
24. Describe how temperature and one degree are defined on the Kelvin scale.
25. Why does sugar dissolve faster in hot tea than in cold water?
26. (a) Describe experiments that were used to test the predictions of the molecular kinetic theory. (b) What experiments revealed its limitations?
27. Give three examples of diffusion that are important for human life.
28. Why do very light gases such as hydrogen not exist in Earth's atmosphere but do exist in the atmospheres of giant planets such as Uranus and Saturn?
29. Why does the Moon have no atmosphere? Explain.
30. Explain why Earth has almost no free hydrogen in its atmosphere.

Problems

Below, **BIO** indicates a problem with a biological or medical focus. Problems labeled **EST** ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

12.2 Pressure, density, and the mass of particles

- What are the molar masses of molecular and atomic hydrogen, helium, oxygen, and nitrogen? What are their molecular masses?
- EST** Estimate the number of hydrogen atoms in the Sun. The mass of the Sun is $2 \times 10^{30} \text{ kg}$. About 70% of it by mass is hydrogen, 30% is helium, and there is a negligible amount of other elements.
- The average particle density in the Milky Way galaxy is about one particle per cubic centimeter. Express this number in SI units (kg/m^3). Indicate any assumptions you made.
- * (a) What is the concentration (number per cubic meter) of the molecules in air at normal conditions? (b) What is the average distance between molecules compared to the dimensions of the molecules? (c) Can you consider air to be an ideal gas? Explain your answer.
- * What is the mass of a water molecule in kilograms? What is the mass of an average air molecule in kilograms?
- You find that the average gauge pressure in your car tires is about 35 psi. (a) How many newtons per square meter is it? What is gauge pressure? (b) Determine the absolute pressure in the tires.
- BIO Forced vital capacity** Physicians use a machine called a spirometer to measure the maximum amount of air a person can exhale (called the forced vital capacity). Suppose you can exhale 4.8 L. How many kilograms of air do you exhale? What assumptions did you make to answer the question? How do these assumptions affect the result?
- A container is at rest with respect to a desk. Inside the container a particle is moving horizontally at a speed v with respect to the desk. It collides with a vertical wall of the container elastically and rebounds. Compare the direction and the speed of the particle if the wall is (a) at rest with respect to the desk; (b) moving in the same direction as the particle at a speed smaller than the particle's; and (c) moving in the direction opposite to the motion of the particle at a smaller speed.

12.3 Quantitative analysis of ideal gas

- Oxygen tank for mountains** Consider an oxygen tank for a mountain climbing trip. The mass of one molecule of oxygen is $5.3 \times 10^{-26} \text{ kg}$. What is the pressure that oxygen exerts on the inside walls of the tank if its concentration is $10^{25} \text{ particles/m}^3$ and its rms speed is 600 m/s ? What assumptions did you make?
- You have five molecules with the following speeds: 300 m/s , 400 m/s , 500 m/s , 450 m/s , and 550 m/s . (a) What is their average speed? (b) What is their rms speed? Compare it with the average speed.

- Two gases in different containers have the same concentration and same rms speed. The mass of a molecule of the first gas is twice the mass of a molecule of the second gas. What can you say about their pressures? Explain.
- Four molecules are moving with the following velocities: 300 m/s south, 300 m/s north, 400 m/s east, and 400 m/s west. (a) What is their average velocity? (b) What is their rms speed?
- * **Hitting tennis balls against a wall** A 0.058-kg tennis ball, traveling at 25 m/s , hits a wall, rebounds with the same speed in the opposite direction, and is hit again by another player, causing the ball to return to the wall at the same speed. The collision of the ball with the wall lasts 1 ms . The ball returns to the wall once every 0.60 s . (a) Determine the average force exerted on the ball during a single collision and the force that the ball exerts on the wall averaged over the time between collisions. State the assumptions that you made. (b) If 10 people are practicing against a wall with an area of 30 m^2 , what is the average pressure of the 10 tennis balls against the wall?
- * Friends throw snowballs at the wall of a $3.0 \text{ m} \times 6.0 \text{ m}$ barn. The snowballs have mass 0.10 kg and hit the wall moving at an average speed of 6.0 m/s . They do not rebound. Determine the average pressure exerted by the snowballs on the wall if 40 snowballs hit the wall each second. Which problem, this or the previous problem, resembles the actions of the molecules of an ideal gas hitting the walls of their container?
- * A ball moving at a speed of 3.0 m/s with respect to the ground hits a stationary wall at a 30° angle with respect to the surface of the wall. Determine the direction and the magnitude of the velocity of the ball after it rebounds. Explain carefully what physics principles you used to find the answer. What assumptions did you make? How will the answer change if one or more of them are not valid?
- * **Oxygen tank for mountain climbing** An oxygen container used by mountain climbers has a 90-min oxygen supply at a rate of 6 L/min (with the volume measured at atmospheric pressure). Determine everything you can about the gas in the container. Make reasonable assumptions.

12.4 Temperature

- Your temperature, when taken orally, is 98.6°F . When taken under your arm, it's 36.6°C . Are these results consistent? Explain.
- Air consists of many different molecules, for example, N_2 , O_2 , H_2O , and CO_2 . Which molecules are the fastest on average? The slowest on average? Explain.
- What is the average kinetic energy of a particle of air at standard conditions?
- Air is a mixture of molecules of different types. Compare the rms speeds of the molecules of N_2 , O_2 , and CO_2 at standard conditions. What assumptions did you make?
- * How many moles of air are in a regular 1-L water bottle when you finish drinking the water? What assumptions did you make? How do these assumptions affect your result?

22. * At approximately what temperature does the average translational kinetic energy of a N_2 molecule in an ideal gas equal the macroscopic translational kinetic energy of a copper atom in a penny that is dropped from the height of 1.0 m?
23. ** A molecule moving at speed v_1 collides head-on with a molecule of the same mass moving at speed v_2 . Compute the speeds of the molecules after the collision. What assumptions did you make? How does the answer to this problem explain why the mixing of hot and cold gases causes the cold gas to become warmer and the hot gas to become cooler?
24. * **Balloon flight** For a balloon ride, the balloon must be inflated with helium to a volume of 1500 m^3 at sea level. The balloon will rise to an altitude of about 12 km, where the temperature is about -52°C and the pressure is about 20 kPa. How many kg of helium should be put into the balloon? What assumptions did you make?
25. **BIO Ears pop** The middle ear has a volume of about 6.0 cm^3 when at a pressure of $1.0 \times 10^5 \text{ N/m}^2$. Determine the volume of that same air when the air pressure is $0.83 \times 10^5 \text{ N/m}^2$, as it is at an elevation of 1500 m above sea level (assume the air temperature remains constant). If the volume of the middle ear remains constant, some air will have to leave as the elevation increases. That is why ears “pop.”
26. * Even the best vacuum pumps cannot lower the pressure in a container below 10^{-15} atm . How many molecules of air are left in each cubic centimeter in such a “vacuum”? Assume that the temperature is 273 K.
27. **Pressure in interstellar space** The concentration of particles (assume neutral hydrogen atoms) in interstellar gas is 1 particle/ cm^3 , and the average temperature is about 3 K. What is the pressure of the interstellar gas? How does it compare to the best vacuum that can be achieved on Earth (see the previous problem)?

12.5, 12.6, and 12.7 Testing the ideal gas law, Speed distribution of particles, and Skills for analyzing processes using the ideal gas law

28. * Describe experiments to determine if each of the three gas isoprocess laws works. The experiments should be ones that you could actually carry out.
29. * The following data were collected for the temperature and volume of a gas. Can this gas be described by the ideal gas model? Explain how you know.

Temperature (°C)	Volume (mL)
11	95.0
25	100.0
47	107.5
73	116.0
159	145.1
233	170.0
258	177.9

30. * Explain the microscopic mechanisms for the relation of macroscopic variables for an isothermal process, an isobaric process, and an isochoric process occurring in an ideal gas.
31. ** **Scuba diving** The pressure of the air in a diver’s lungs when he is 20 m under the water surface is $3.0 \times 10^5 \text{ N/m}^2$, and the air occupies a volume of 4.8 L. How many moles of air should he exhale while moving to the surface, where the pressure is $1.0 \times 10^5 \text{ N/m}^2$?
32. * When surrounded by air at a pressure of $1.0 \times 10^5 \text{ N/m}^2$, a basketball has a radius of 0.12 m. Compare its volume at this condition with the volume that it would have if you took it 15 m below the water surface, where the pressure is $2.5 \times 10^5 \text{ N/m}^2$. What assumptions did you make?
33. * Some students are given the following problem: “A 5000-cm^3 cylinder is filled with nitrogen gas at $1.0 \times 10^5 \text{ Pa}$ and 300 K and closed with a movable piston. The gas is slowly compressed at constant temperature to a final volume of 5 cm^3 . Determine the final pressure of the gas.” (a) Explain why the ideal gas law cannot be applied to solve this problem. Present quantitative arguments. (b) Modify the problem so that it *can* be solved using the ideal gas law and give your solution.
34. ** You have gas in a container with a movable piston. The walls of the container are thin enough so that its temperature stays the same as the temperature of the surrounding medium. You have baths of water of different temperatures, different objects that you can place on top of the piston, etc. (a) Describe how you could make the gas undergo an isothermal process so that the pressure inside increases by 10%, then undergo an isobaric process so that the new volume decreases by 20%, and finally undergo an isochoric process so that the temperature increases by 15%. (b) Represent this three-step process in P -versus- T , V -versus- T , and P -versus- V graphs. (c) Express the new pressure, volume, and temperature in terms of their initial values.
35. * You want to determine the temperature in a freezer using the following equipment: an empty plastic bottle with a cap, a graduated measuring cylinder, and a bowl of water. You also know your room temperature T_R . (a) Which of the procedures described below will give data from which you can determine the temperature in the freezer? (b) For the procedure that you have chosen in part (a), derive the expression for the temperature in the freezer as a function of relevant physical quantities. Indicate any assumptions that you made.
- Put the open plastic bottle into the freezer for half an hour. Take the bottle out, turn it upside down into the water, and measure the volume of the water sucked into the bottle.
 - Put the open plastic bottle in the freezer for half an hour. Take the bottle out and close it tightly with a cap. Wait until the bottle’s shape stops changing. Turn the bottle upside down into the water, open the cap, and measure the volume of the water that entered the bottle.
 - Put the closed plastic bottle in the freezer for half an hour. Take the bottle out, turn it upside down into the water, and then open the cap. Measure the volume of the water sucked into the bottle.
 - Put the closed plastic bottle in the freezer for half an hour. Take the bottle out, open the cap, and turn the bottle upside down into the water. Measure the volume of the water sucked into the bottle.
36. * **Bubbles** While snorkeling, you see air bubbles leaving a crevice at the bottom of a reef. One of the bubbles has a radius of 0.060 m. As the bubble rises, the pressure inside it decreases by 50%. Now what is the bubble’s radius? What assumptions did you make to solve the problem?
37. ** **Diving bell** A cylindrical diving bell, open at the bottom and closed at the top, is 4 m tall. Scientists fill the bell with air at the pressure of $1.0 \times 10^5 \text{ N/m}^2$. The pressure increases naturally by $1.0 \times 10^5 \text{ N/m}^2$ for each 10 m that the bell is lowered below the surface of the water. If the bell is lowered 30 m below the ocean surface, how many meters of air space are left inside the bell? Why doesn’t water enter the entire bell as it goes under water? Draw several sketches for this problem. Assume the temperature remains constant.
38. * **Mount Everest** (a) Determine the number of molecules per unit volume in the atmosphere at the top of Mount Everest. The pressure is $0.31 \times 10^5 \text{ N/m}^2$, and the temperature is -30°C . (b) Determine the number of molecules per unit volume at sea level, where the pressure is $1.0 \times 10^5 \text{ N/m}^2$ and the temperature is 20°C .
39. * **EST Breathing on Mount Everest** Using the information from Problem 12.38, estimate how frequently you need to breathe on top of Mount Everest to inhale the same amount of oxygen as you do at sea level. The pressure is about one-third the pressure at sea level.
40. * **Capping beer** You would like to make homemade beer, but you are concerned about storing it. Your beer is capped into a bottle at a temperature of 27°C and a pressure of $1.2 \times 10^5 \text{ N/m}^2$. The cap will pop off if the pressure inside the bottle exceeds $1.5 \times 10^5 \text{ N/m}^2$. At what maximum temperature can you store the beer so the gas inside the bottle does not pop the cap? List the assumptions that you made.
41. **Car tire** With a tire gauge, you measure the pressure in a car tire as $2.1 \times 10^5 \text{ N/m}^2$. How can this be if you know that absolute pressure in the tire is three times higher than atmospheric? The tire looks okay. What’s the deal?
42. * **Car tire dilemma** Imagine a car tire that contains 5.1 moles of air when at a gauge pressure of $2.1 \times 10^5 \text{ N/m}^2$ (the pressure above atmospheric pressure) and a temperature of 27°C . The temperature increases to 37°C , the volume decreases to 0.8 times the original volume, and the gauge pressure decreases to $1.6 \times 10^5 \text{ N/m}^2$. Can these measurements be correct if the tire did not leak? If it did leak, then how many moles of air are left in the tire?

43. * There is a limit to how much gas can pass through a pipeline, because the pipes can only tolerate so much pressure on the walls. To increase the amount of gas going through the pipeline, engineers decide to cool the gas (to reduce its pressure). Suggest how much they should lower the temperature of the gas if they want to increase the mass per unit time by 1.5 times.
44. Explain how you know that the volume of one mole of gas at standard conditions is 22.4 L.
45. * At what pressure is the density of $-50\text{ }^\circ\text{C}$ nitrogen gas (N_2) equal to 0.01 times the density of water?
46. * In the morning, the gauge pressure in your car tires is 35 psi. During the day, the air temperature increases from $20\text{ }^\circ\text{C}$ to $30\text{ }^\circ\text{C}$ and the pressure increases to 36.5 psi. By how much did the volume of one of the tires increase? What assumptions did you make?
47. **Equation Jeopardy 1** The equation below describes a process. Construct a word problem for a process that is consistent with the equations (there are many possibilities). Provide as much detailed information as possible about your proposed process.

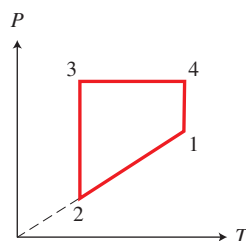
$$\frac{1.2 \times 10^5 \text{ N/m}^2}{293 \text{ K}} = \frac{2.0 \times 10^5 \text{ N/m}^2}{T}$$

48. * **Equation Jeopardy 2** The equation below describes a process. Construct a word problem for a process that is consistent with the equations (there are many possibilities). Provide as much detailed information as possible about your proposed process.

$$\Delta n = \frac{(0.67 \times 10^5 \text{ N/m}^2)(0.60 \times 10^{-6} \text{ m}^3)}{(8.3 \text{ J/mole} \cdot \text{K})(303 \text{ K})} - \frac{(1.00 \times 10^5 \text{ N/m}^2)(0.60 \times 10^{-6} \text{ m}^3)}{(8.3 \text{ J/mole} \cdot \text{K})(310 \text{ K})}$$

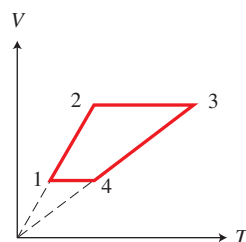
49. ** The P -versus- T graph in **Figure P12.49** describes a cyclic process comprising four hypothetical parts. (a) What happens to the pressure of the gas in each part? (b) What happens to the temperature of the gas in each part? (c) What happens to the volume of the gas in each part? (d) Explain each part microscopically. (e) Use the information from (a)–(c) to represent the same parts in P -versus- V and V -versus- T graphs. (Hint: It helps to align the P -versus- V graph beside the P -versus- T graph using the same P values on the ordinate (vertical axis) and to place the V -versus- T graph below the P -versus- T graph using the same T values on the abscissa (horizontal axis). This helps keep the same scale for the variables.)

FIGURE P12.49



50. ** The V -versus- T graph in **Figure P12.50** describes a cyclic process comprising four hypothetical parts. (a) What happens to the pressure of the gas in each part? (b) What happens to the temperature of the gas in each part? (c) What happens to the volume of the gas in each part? (d) Explain each part microscopically. (e) Use the information from (a)–(c) to represent the same process in a P -versus- T graph (below the V -versus- T graph) and a P -versus- V graph (beside the P -versus- T graph). See the hint in the previous problem about the graph alignments.
51. * **BIO Breathing** You are breathing heavily while hiking up the mountain. To inhale, you expand your diaphragm and lungs. Explain, using your knowledge of gas pressure, why this mechanical movement leads to the air flowing into your nose or mouth. Support your reasoning with diagrams if necessary.

FIGURE P12.50



53. * **BIO EST Breathing and metabolism** We need about 0.7 L of oxygen per minute to maintain our resting metabolism and about 2 L when standing and walking. Estimate the number of breaths per minute for a person to satisfy this need when resting and when standing and walking. What assumptions did you make? Remember that oxygen is about 21% of the air.
54. * **EST Sun's life expectancy** (a) Estimate the average kinetic energy of the particles in the Sun. Assume that it is made of atomic hydrogen and that its average temperature is 100,000 K. The mass of the Sun is 2×10^{30} kg. (b) For how long would the Sun shine using this energy if it radiates 4×10^{26} W/s? Is your answer reasonable? Explain.
55. ** A gas that can be described by the ideal gas model is contained in a cylinder of volume V . The temperature of the gas is T . The mass of the gas is m , and the molar mass is M . Write an expression for the total thermal energy of the gas. Now, imagine that the exact same gas has been placed in a container of volume $2V$. What happens to its pressure? What happens to its temperature? What happens to its thermal energy?
56. * **Equation Jeopardy 3** The three equations below describe a physical situation. Construct a word problem for a situation that is consistent with the equations (there are many possibilities). Provide as much detailed information as possible about the situation.

$$m = (1.3 \text{ kg/m}^3)(3.0 \text{ m} \times 5.0 \text{ m} \times 2.0 \text{ m})$$

$$N = m / [(29 \times 10^{-3} \text{ kg}) / (6.0 \times 10^{23} \text{ particles})]$$

$$U_{\text{th}} = N(3/2)(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})$$

General Problems

FIGURE P12.57

57. * **EST** A simple experimental setup for performing isobaric processes can be made with a thin glass tube filled with a gas and closed with a drop of water (**Figure P12.57**). If the pressure of the gas changes, the drop moves until the sum of the forces exerted on the drop is zero (there is no friction between the drop and the tube). The inner diameter of the glass tube is 3.0 mm and the length of the tube is 200.0 mm. At normal room temperature, a 5.0-mm-long water drop is 30.0 mm from the open end of the tube. Estimate the temperature range in which this setup can be used to show isobaric processes. (Hint: What happens to water at low temperature?)



58. * See the previous problem. Explain how the force exerted by Earth on the water drop and the orientation of the tube affects the performance of the setup.
59. * Jeff and Natalie notice that a rubber balloon, which is first in a warm room, shrinks when they take it into the garden on a cold winter day. They propose two different explanations for the observed phenomenon: (a) the balloon is slowly leaking; (b) the balloon shrinks due to decreased temperature while the pressure in the balloon remains constant (isobaric compression). In order to test their proposed explanations, Jeff and Natalie perform three consecutive experiments: they measure the volume of the balloon and the temperature of the air near the balloon (1) in the room, (2) in the garden, and (3) again in the room. Their measurements, including uncertainties, are presented in the table below.

Exp. #	Location	Temperature	Volume of the balloon
1	Room	$26.2\text{ }^\circ\text{C} \pm 0.1\text{ }^\circ\text{C}$	$7500\text{ cm}^3 \pm 400\text{ cm}^3$
2	Garden	$-15.3\text{ }^\circ\text{C} \pm 0.1\text{ }^\circ\text{C}$	$6400\text{ cm}^3 \pm 400\text{ cm}^3$
3	Room	$26.2\text{ }^\circ\text{C} \pm 0.1\text{ }^\circ\text{C}$	$7300\text{ cm}^3 \pm 400\text{ cm}^3$

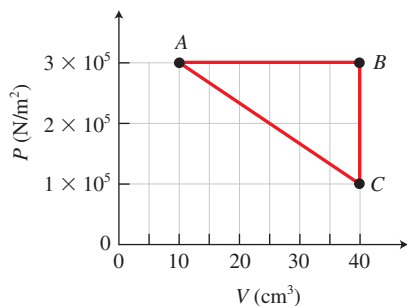
Based on the data, can Jeff and Natalie reject any of their hypotheses? Explain. Make sure you include uncertainties in your answer.

12.8 Thermal energy, the Sun, and diffusion

52. * The temperature of the Sun's atmosphere near the surface is about 6000 K, and the concentration of atoms is about 10^{15} particles/ m^3 . What are the average pressure and density of its atmosphere? What assumptions did you make to solve the problem?

60. * **Different planet compositions** Explain why planets closer to the Sun have low concentrations of light elements, but light elements are relatively abundant in giant planets such as Jupiter, Uranus, and Saturn, which are far from the Sun.
61. ** **EST Density of our galaxy** Estimate the average density of particles in our galaxy, assuming that the most abundant element is atomic hydrogen. There are about 10^{11} stars in the galaxy and the size of the galaxy is about 10^5 light-years. A light-year is the distance light travels in 1 year moving at a speed of 3×10^8 m/s. What do you need to assume about the stars in order to answer this question?
62. * **BIO Breathing** During each breath you probably inhale about 0.50 L of air. How many oxygen molecules do you inhale if you are at sea level?
63. * **Car engine** During a compression stroke of a cylinder in a diesel engine, the air pressure in the cylinder increases from 1.0×10^5 N/m² to 50×10^5 N/m², and the temperature increases from 26 °C to 517 °C. A typical compression ratio (the ratio of the largest and the smallest volume of a cylinder) for a diesel engine is 20:1. Using this information, how would you convince your friends that knowledge about ideal gases can help explain how hot gases burned in the car engine affect the motion of the car?
64. * **EST** How can the pressure of air in your house stay constant during the day if the temperature rises? Estimate the volume of your house and the number of moles of air that leave the house during the daytime. Assume that nighttime temperature and daytime temperature differ by about 10 °C. List all other assumptions that are necessary to answer the question.
65. * **Tell-all problem** Tell everything you can about a process that was performed with 2×10^{-3} moles of a gas and that is described by the pressure-versus-volume graph shown in **Figure P12.65**.

FIGURE P12.65



66. ** Two massless, frictionless pistons are inside a horizontal tube opened at both ends. A 10-cm-long thread connects the pistons. The cross-sectional area of the tube is 20 cm². The pressure and temperature of gas between the pistons and the outside air are the same and are equal to $P = 1.0 \times 10^5$ N/m² and $T = 24$ °C. At what temperature of the gas will the thread break if it breaks when the tension reaches 30 N?
67. * A closed cylindrical container is divided into two parts by a light, movable, frictionless piston. The container's total length is 100 cm. Where is the piston located when one side is filled with nitrogen (N₂) and the other side with the same mass of hydrogen (H₂) at the same temperature?
68. How many molecules are there in 1 g of air at normal conditions? If these molecules were distributed uniformly on Earth's surface, *estimate* the number that would be under your feet right now. The radius of Earth is about 6400 km.
69. ** The speed of sound in an ideal gas is given by the relationship

$$v = \sqrt{\frac{\gamma RT}{M}}$$

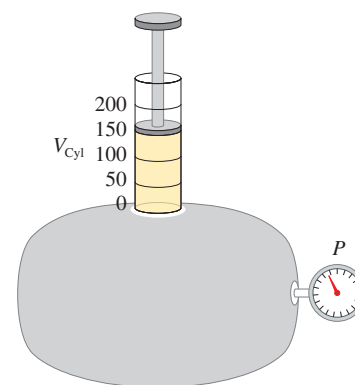
where R = the universal gas constant = 8.314 J/mol · K; T = the absolute temperature; M = the molar mass of the gas in kg/mol; and γ is a characteristic of the specific gas. For air, $\gamma = 1.4$ and the average molar mass for dry air is 28.95 g/mol. (a) Show that the equation gives you correct units. (b) Give reasons why the temperature of the gas is in the numerator and the molar mass of the gas is in the denominator.

70. * Using the information from problem 12.69, calculate the speed of sound in the air. What assumptions are you making?
71. * **Speed of sound in summer and winter** Using the information in problem 12.69, estimate how much faster sound travels in summer than in winter. Explain how you arrived at your answer and the assumptions that you made.

72. ** A chamber of unknown volume is filled with an unknown amount of ideal gas. A cylinder with an open bottom and calibrated volume is connected to the chamber and is closed with a movable piston. A gas pressure sensor is also mounted on the chamber (**Figure P12.72**). The whole experimental setup is kept at a constant temperature of 300 K. Doris moves the piston from mark 0 to mark 200 mL in steps of 50 mL and records the corresponding pressure in the chamber. She moves the piston very slowly and waits several minutes before recording the pressure values. Use Doris's data in the table to determine the volume of the chamber and the number of moles of gas. (Note: This is a problem that requires linearization of data.)

V_{Cyl} (mL)	P (10^5 N/m ²)
0	4.35
50	3.85
100	3.50
150	3.15
200	2.90

FIGURE P12.72



Reading Passage Problems

BIO Vascular wall tension and aortic blowout The walls of blood vessels contain varying amounts of elastic fibers that allow the vessels to expand and contract as the pressure and amount of fluid inside vary (these fibers are more prevalent in the aorta and large arteries than in the small arterioles and capillaries). These fibers in the cylindrical walls produce a wall tension T , defined as

$$T = \frac{F}{L}$$

where L is the length of an imaginary cut parallel to the axis of the vessel and F is the magnitude of the force that each side of the cut must exert on the other side to hold the two sides together.

Three forces are exerted on a short section of wall fiber—the system.

(1) The fluid inside pushes outward, due to fluid pressure from inside $P_{\text{inside fluid pushing out}} = P_{\text{out}}$; (2) fluid outside the vessel pushes inward, due to fluid pressure from outside $P_{\text{outside fluid pushing in}} = P_{\text{in}}$; and (3) the wall next to the system exerts wall tension T on each side of that wall. The pressure difference across the wall $\Delta P = P_{\text{out}} - P_{\text{in}}$, the wall tension T , and the radius R of the cylindrical vessel are related by Laplace's law:

$$\Delta P = \frac{T}{R} \quad \text{or} \quad T = \Delta P \cdot R$$

The inward gauge pressure P_{in} of tissue surrounding the vessels is approximately zero. Thus, the pressure difference $\Delta P = P_{\text{out}} - P_{\text{in}} = P_{\text{vessel}} - 0 = P_{\text{vessel}}$ is the gauge pressure in the blood vessel. We can now estimate the wall tension for different types of vessels. The tension in the aorta is approximately

$$T = \Delta P \cdot R \approx (100 \text{ mm Hg}) \left(\frac{133 \text{ N/m}^2}{1 \text{ mm Hg}} \right) (1.3 \times 10^{-2} \text{ m}) = 170 \text{ N/m}$$

Using similar reasoning, the wall tension in the low-pressure, very small radius capillaries is about 0.016 N/m—about 0.0003 times the tension needed to tear a facial tissue. Because such little tension is needed to hold a capillary together, its wall can be very thin, allowing easy diffusion of various molecules across the wall.

The walls of a healthy aorta can easily provide the tension needed to support the increased blood pressure when it fills with blood from the heart during each heartbeat. However, aging and various medical conditions may weaken the aortic wall in a short section, and increased blood pressure can cause it to stretch. The weakened wall bulges outward; this is called an aortic aneurism. The increased radius causes increased tension, which can increase bulging. This cycle can result in a rupture to the aorta: an aortic blowout.

73. Why is the wall tension in capillaries so small?
- There are so many capillaries.
 - Their radii are so small.
 - The outward pressure of the blood inside is so small.
 - b and c
 - a, b, and c
74. Which answer below is closest to the wall tension in a typical arteriole of radius 0.15 mm and 60 mm Hg blood pressure?
- 0.001 N/m
 - 0.01 N/m
 - 0.1 N/m
 - 1 N/m
 - 10 N/m
75. According to Laplace's law, elevated blood pressure in an artery should cause the wall tension in the artery to do what?
- Increase
 - Remain unchanged
 - Decrease
 - Impossible to decide
76. As a person ages, the fibers in arteries become less elastic and the wall tension increases. According to Laplace's law, this will cause the blood pressure to do what?
- Increase
 - Remain unchanged
 - Decrease
 - Impossible to decide
77. Aortic blowout occurs when part of the wall of the aorta becomes weakened. What does this cause?
- A bulge and increased radius of the aorta when the blood pressure inside increases
 - An increased radius of the aorta, which causes increased tension in the wall
 - An increased tension in the aorta, which causes the radius to increase
 - a and b
 - a, b, and c

BIO Portable hyperbaric chamber In 1997, a hiking expedition was stranded for 38 days on the Tibetan side of Mount Everest at altitudes between 5200 m and 8000 m. While at those altitudes, 10 climbers suffered acute mountain sickness. A 37-year-old climber with acute pulmonary edema (buildup of fluid in the lungs) was treated with a portable Gamow bag (see **Figure 12.20**), named after its inventor, Igor Gamow. The Gamow bag is a windowed cylindrical portable hyperbaric chamber constructed of nonpermeable nylon that requires constant pressurization with a foot pump attached to the bag. The climber enters the bag, and a person outside pumps air into the bag so that the air pressure inside is somewhat higher than the outside pressure.

FIGURE 12.20 A Gamow bag, used to help climbers with acute altitude sickness.



The bag and pump have a 6.76-kg mass. The volume of the inflated bag is 0.476 m^3 . The maximal bag pressure is $0.15 \times 10^5 \text{ N/m}^2$ above the air pressure at the site where it is used. In the 1997 climb, with the temperature at -20°C , the bag was filled in about 2 min with 10–20 pumps per minute. This raised the pressure in the bag to $0.58 \times 10^5 \text{ N/m}^2$ (equivalent to an elevation of 4400 m) instead of the actual outside pressure of $0.43 \times 10^5 \text{ N/m}^2$ at the 6450-m elevation at which the climber was treated. The treatment lasted for 2 h, with the climber inhaling about 15 times/min at about 0.5 L/inhalation, and was successful—the pulmonary edema disappeared.

78. What is closest to the volume of the Gamow bag?
- 50 L
 - 100 L
 - 200 L
 - 500 L
 - 1000 L
79. What is closest to the temperature at the 6450-m elevation on the day described in the problem?
- 37 K
 - 253 K
 - 20 K
 - -20 K
 - 273 K
80. What is closest to the number n of moles of air in the filled bag when at 4400 m?
- 3 moles
 - 10 moles
 - 13 moles
 - 110 moles
 - 170 moles
81. What is closest to the number n of moles of air in the bag if its pressure is at the 6450-m level?
- 3 moles
 - 10 moles
 - 13 moles
 - 110 moles
 - 170 moles
82. Estimate the fraction of the air in the Gamow bag that an occupant would inhale in 1 h, assuming no replacement of the air.
- 0.01
 - 0.1
 - 0.3
 - 0.5
 - 1.0