13

Static Fluids

- **How can a hot air balloon fly for hours?**
- **Why is it dangerous for scuba divers to ascend from a deep-sea dive too quickly?**
- **Why, in water, does a 15-g nail sink but a cargo ship float?**

The first hot air balloon with a person on board was launched by the Montgolfier brothers in 1783. Hot air is less dense than cold air. By carefully balancing the density of the balloon and its contents with the density of air, the pilot can control the force that the outside cold air exerts on the balloon. What do pressure, volume, mass, and temperature have to do with this force?

BE SURE YOU KNOW HOW TO:

- Draw a force diagram for a system of interest (Section 3.1).
- Apply Newton's second law in component form (Section 4.2).
- Define pressure (Section 12.2).
- Apply the ideal gas law (Sections 12.3 and 12.4).

IN THE PREVIOUS CHAPTER, we constructed the ideal gas model and used it to explain the behavior of gases. The temperature of the gas and the pressure it exerted on surfaces played an important role in the phenomena we analyzed. We ignored the effect of the gravitational force exerted by Earth on the gas particles. This simplification was reasonable, since in most of the processes we analyzed the gases had little mass and occupied a relatively small region of space, like in a piston. In this chapter, our interest expands to include phenomena in which the force exerted by Earth plays an important role. We will confine the discussion to static fluids—fluids that are not moving.

13.1 Density

We are familiar with the concept of density (from Chapter 12). To find the density of an object or a substance, determine its mass m and volume V and then calculate the ratio of the mass and volume:

$$\rho = \frac{m}{V} \tag{13.1}$$

Archimedes (Greek, 287–212 B.C.) discovered how to determine the density of an object of irregular shape. First determine its mass using a scale. Then determine its volume by submerging it in a graduated cylinder with water (**Figure 13.1**). Finally, divide the mass in kilograms by the volume in cubic meters to find the density in kilograms per cubic meter. Using this method we find that the density of an iron nail is $7860\ \mathrm{kg/m^3}$—relatively large. A gold coin has an even larger density—$19{,}300\ \mathrm{kg/m^3}$. The universe, though, contains much denser objects. For example, the rapidly spinning neutron star known as a pulsar (discussed in Chapter 9) has a density of approximately $10^{18}\ \mathrm{kg/m^3}$. **Table 13.1** lists the densities of various solids, liquids, and gases.

FIGURE 13.1 Measuring the density of an irregularly shaped object.

1. Measure mass of object.
2. Place the object in water in a graduated cylinder.
3. Measure the volume change of the water. Volume change of water = volume of object.
4. Density = $\rho = m/V$

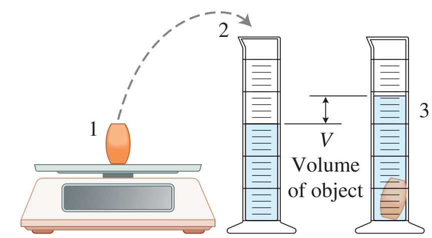


TABLE 13.1 Densities of various solids, liquids, and gases

Solids		Liquids[1]		Gases[2]	
Substance	**Density (kg/m³)**	**Substance**	**Density (kg/m³)**	**Substance**	**Density (kg/m³)**
Aluminum	2700	Acetone	791	Dry air 0 °C	1.29
Copper	8920	Ethyl alcohol	789	10 °C	1.25
Gold	19,300	Methyl alcohol	791	20 °C	1.21
Iron	7860	Gasoline	726	30 °C	1.16
Lead	11,300	Olive oil	800–920	Helium	0.178
Platinum	21,450	Mercury	13,600	Hydrogen	0.090
Silver	10,500	Milk	1028–1035	Oxygen	1.43
Bone	1700–2000	Seawater	1025	Carbon dioxide	1.98
Brick	1400–2200	Water 0 °C	999.8		
Cement	2700–3000	3.98 °C	1000.00		
Clay	1800–2600	20 °C	998.2		
Glass	2400–2800	Blood plasma	1030		
Ice	917	Blood whole	1050		
Styrofoam	25–100				
Balsa wood	120				
Oak	600–900				
Pine	500				
Planet Earth	5515				
Moon	3340				
Sun	1410				
Universe (average)	10^{-26}				
Pulsar	10^{11}–10^{18}				

[1]Densities of liquids are at 0 °C unless otherwise noted.
[2]Densities of gases are at 0 °C and 1 atm unless otherwise noted.

QUANTITATIVE EXERCISE 13.1 **Ping-pong balls with different densities**

Saturn has the lowest density of all the planets in the solar system ($M_{\text{Saturn}} = 5.7 \times 10^{26}$ kg and $V_{\text{Saturn}} = 8.3 \times 10^{23}\ \text{m}^3$). The average density of a neutron star is $10^{18}\ \text{kg/m}^3$. Compare the mass of a ping-pong ball filled with material from Saturn with that of the same ball filled with material from a neutron star. An empty ping-pong ball has a 0.037-m diameter (0.020-m radius) and a 2.7-g mass.

Represent mathematically To find the mass of a ping-pong ball filled with a particular material, we add the mass of the ball alone and the calculated mass of the material inside:

$$m_{\text{filled ball}} = m_{\text{ball}} + m_{\text{material}}$$

where $m_{\text{material}} = \rho_{\text{material}} V_{\text{ball}}$.

The density of the neutron star is given, and the density of Saturn can be found using the operational definition $\rho_{\text{Saturn}} = \dfrac{m_{\text{Saturn}}}{V_{\text{Saturn}}}$. The interior of the ping-pong ball is a sphere of volume $V_{\text{sphere}} = \frac{4}{3}\pi R^3$. Assume that the plastic shell of the ball has negligible volume. The mass of either filled ball is

$$\begin{aligned} m_{\text{filled ball}} &= m_{\text{ ball}} + m_{\text{material}} = m_{\text{ ball}} + \rho_{\text{material}} V_{\text{ball}} \\ &= m_{\text{ball}} + \rho_{\text{material}} \tfrac{4}{3}\pi R_{\text{ball}}^3 \end{aligned}$$

Solve and evaluate For the neutron-star-filled ball:

$$\begin{aligned} m_{\text{neutron star ball}} &= (0.003\ \text{kg}) + (10^{18}\ \text{kg/m}^3)\tfrac{4}{3}\pi(0.020\ \text{m})^3 \\ &= 3.4 \times 10^{13}\ \text{kg} \end{aligned}$$

For the Saturn-filled ball:

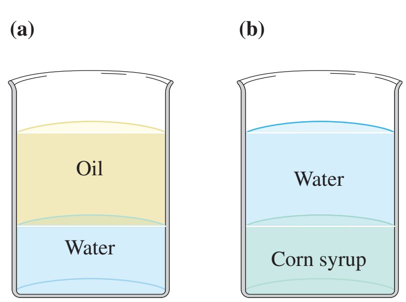
$$\begin{aligned} m_{\text{Saturn ball}} &= m_{\text{ ball}} + \left(\frac{m_{\text{Saturn}}}{V_{\text{Saturn}}}\right)\left(\tfrac{4}{3}\pi R_{\text{ball}}^3\right) \\ &= 0.003\ \text{kg} + \left(\frac{5.7 \times 10^{26}\ \text{kg}}{8.3 \times 10^{23}\ \text{m}^3}\right)\left(\tfrac{4}{3}\pi(0.020\ \text{m})^3\right) \\ &= 0.003\ \text{kg} + 0.023\ \text{kg} = 0.026\ \text{kg} \end{aligned}$$

The material from Saturn has less mass than an equal volume of water. The ball filled with the material from a neutron star has a mass of more than a billion tons!

Try it yourself The mass of the ping-pong ball filled with soil from Earth's surface is 0.050 kg. What is the density of the soil?

Answer $1400\ \text{kg/m}^3$.

FIGURE 13.2 Less dense matter floats on denser matter.



Density and floating

Understanding density allows us to pose questions about phenomena that we observe almost every day. For example, why does oil form a film on water? If you pour oil into water or water into oil, they form layers (see **Figure 13.2a**) independently of which fluid is poured first—the layer of oil is always on top of the water. The density of oil is less than the density of water. If you pour corn syrup and water into a container, the corn syrup forms a layer below the water (Figure 13.2b); the density of corn syrup is $1200\ \text{kg/m}^3$, greater than the density of water. Why, when mixed together, is the lower density substance always on top of the higher density substance?

Similar phenomena occur with gases. Helium-filled balloons accelerate upward in air, while air-filled balloons accelerate (slowly) downward. The mass of helium atoms is much smaller than the mass of any other molecules in the air. (Recall that at the same pressure and temperature, atoms and molecules of gas have the same concentration; because helium atoms have much lower mass, their density is lower.) The air-filled balloon must be denser than air. The rubber with which the skin of any balloon is made is denser than air. We can disregard the slight compression of the gas by the balloon, because even though it increases the density of the gas, the effect is the same for both the air and helium in the balloons.

Ice (solid water) floats in liquid water

The solid form of a particular substance is almost always denser than the liquid form of the substance, with one very significant exception: liquid water and solid ice. Since ice floats on liquid water, we can assume that the density of ice is less than that of water. This is in fact true: the density of water changes slightly with temperature and is the highest at 4 °C: $1000\ \text{kg/m}^3$. The density of ice is $917\ \text{kg/m}^3$. Ice has a lower density because in forming the crystal structure of ice, water molecules spread apart. The fact that water expands when it forms ice is important for life on Earth (see the second Reading Passage at the end of this chapter). Fish and plants living in lakes survive cold winters in liquid water under a shield of ice and snow at the surface. Water absorbed in the cracks of rocks freezes and expands in the winter, cracking the rock. Over the years, this process of liquid water absorption, freezing, and cracking eventually converts the rock into soil.

Why do denser forms of matter sink in less dense forms of matter? We learned (in Chapter 12) that the quantity *pressure* describes the forces that fluids exert on each other and on the solid objects they contact. Let us investigate whether pressure explains, for example, why a nail sinks in water or why a hot air balloon rises.

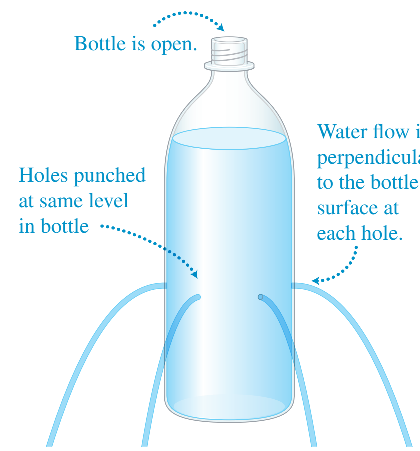
REVIEW QUESTION 13.1 How would you determine the density of an irregularly shaped object?

13.2 Pressure inside a fluid

We know that as gas particles collide with the walls of the container in which they reside, they exert pressure. In fact, if you place any object inside a gas, the gas particles exert the same pressure on the object as the gas exerts on the walls of the container. Do liquids behave in a similar way? In the last chapter we learned that the particles in a liquid are in continual random motion, somewhat similar to particles in gases.

Let's conduct a simple observational experiment. Take a plastic water bottle and poke several small holes at the same height along its perimeter. Close the holes with tacks, fill the bottle with water, open the cap, remove the tacks, and observe what happens (**Figure 13.3**). Identically shaped parabolic streams of water shoot out of the holes. The behavior of the water when the tacks are removed is analogous to a person leaning on a door that is suddenly opened from the other side—the person falls through the door. Evidently, the water inside must push out perpendicular to the wall of the bottle, just as gas pushes out perpendicular to the wall of a balloon. Due to their similar behaviors, liquids and gases are often studied together and are collectively referred to as **fluids**. In addition, since the four streams are identically shaped, the pressure at all points at the same depth in the fluid is the same.

FIGURE 13.3 Arcs of water leaving holes at the same level in a bottle.

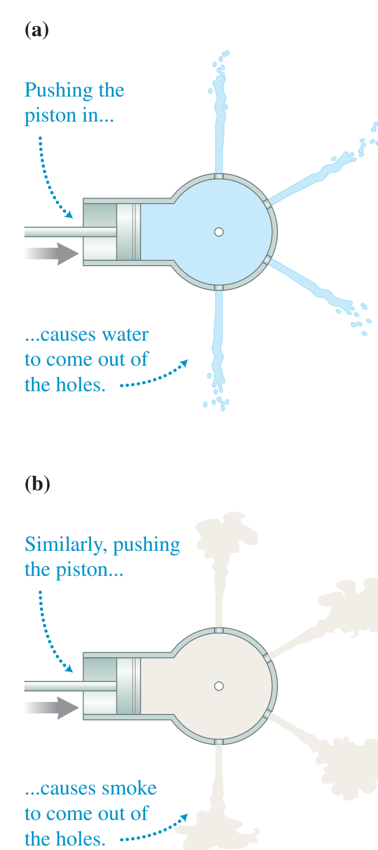


Pascal's first law

Many practical applications involve situations in which an external object (for example, a piston) exerts a force on a particular part of a fluid. What happens to the pressure at other places inside the fluid? To investigate how the pressures at different points in a fluid are related, we use a special instrument that consists of a round glass bulb with holes in it connected to a glass tube with a piston on the other side (**Figure 13.4a**). When we fill the apparatus with water and push the piston, water comes out of all of the holes, not only those that align with the piston. When we fill the apparatus with smoke and push the piston, we get the same result (Figure 13.4b). The liquid and the gas behave similarly.

How can we explain this observation? Pushing the piston in one direction caused a greater pressure in the fluid close to the piston. It seems that almost immediately the pressure throughout the fluid increased as well, as the fluid was pushed out of *all* of the holes in the bulb in the same way. This phenomenon was first discovered by French scientist Blaise Pascal in 1653 and is called Pascal's first law.

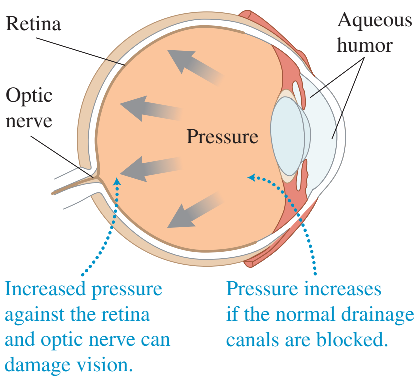
FIGURE 13.4 Pascal's first law: increasing the pressure of a fluid at one location causes a uniform pressure increase throughout the fluid.



Pascal's first law An increase in the pressure of a static, enclosed fluid at one place in the fluid causes a uniform increase in pressure throughout the fluid.

The above experiment describes Pascal's first law macroscopically. We can also explain Pascal's first law at a microscopic level using gases as an example. Gas particles inside a container move randomly in all directions. When we push harder on one of the surfaces of the container, the gas compresses near that surface. The molecules near that surface collide more frequently with their neighbors farther from the surface. They in turn collide more frequently with their neighbors. The extra pressure exerted at the one surface quickly spreads, and soon there is increased pressure throughout the gas.

FIGURE 13.5 Glaucoma is an increase in intraocular pressure, caused by blockage of the ducts that normally drain aqueous humor from the eye.



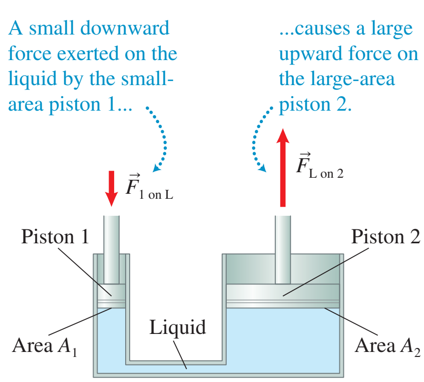
Glaucoma

Pascal's first law can help us understand a common eye problem—*glaucoma*. A clear fluid called *aqueous humor* fills two chambers in the front of the eye (**Figure 13.5**). In a healthy eye, new fluid is continually secreted into these chambers while old fluid drains from the chambers through sinus canals. A person with glaucoma has closed drainage canals. The buildup of fluid causes increased pressure throughout the eye, including at the retina and optic nerve, which can lead to blindness. Ophthalmologists diagnose glaucoma by measuring the pressure at the front of the eye. The eye pressure of a person with healthy eyes is about $1.2 \times 10^5\ \mathrm{N/m^2}$; a person with glaucoma has an elevated pressure of $1.3 \times 10^5\ \mathrm{N/m^2}$.

Hydraulic lift

One of the technical applications of Pascal's first law is a *hydraulic lift*, a form of simple machine that converts small forces into larger forces, or vice versa. Automobile mechanics use hydraulic lifts to lift cars, and dentists and barbers use them to raise and lower their clients' chairs. The hydraulic brakes of an automobile are also a form of hydraulic lift. Most of these devices work on the simple principle illustrated in **Figure 13.6**, although the actual devices are usually more complicated in construction.

FIGURE 13.6 Schematic of a hydraulic lift.



In Figure 13.6, a downward force $\vec{F}_{1\text{ on L}}$ is exerted by piston 1 (with small area A_1) on the liquid. This piston compresses a liquid (usually oil) in the lift. The pressure in the liquid just under piston 1 is

$$P_1 = \frac{F_{1\text{ on L}}}{A_1}$$

Because the pressure changes uniformly throughout the liquid, the pressure under piston 2 is also $P = F_{1\text{ on L}}/A_1$, with a key assumption that the pistons are at the same elevation. Since piston 2 has a greater area A_2 than piston 1, the liquid exerts a greater upward force on piston 2 than the downward force on piston 1:

$$F_{\text{L on 2}} = PA_2 = \left(\frac{F_{1\text{ on L}}}{A_1}\right)A_2 = \left(\frac{A_2}{A_1}\right)F_{1\text{ on L}} \tag{13.2}$$

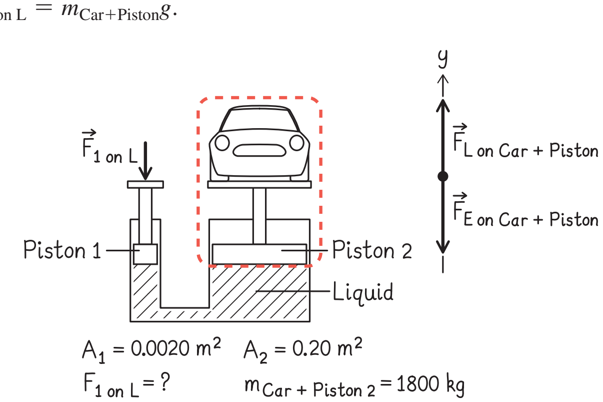
Since A_2 is greater than A_1, the lift provides a significantly greater upward force $F_{\text{L on 2}}$ on piston 2 than the downward push of the smaller piston 1 on the liquid $F_{1\text{ on L}}$.

EXAMPLE 13.2 **Lifting a car with one hand**

A hydraulic lift similar to that described above has a small piston with surface area $0.0020\ \mathrm{m^2}$ and a larger piston with surface area $0.20\ \mathrm{m^2}$. Piston 2 and the car placed on piston 2 have a combined mass of 1800 kg. What is the minimal force that piston 1 needs to exert on the fluid to slowly lift the car?

Sketch and translate The situation is similar to that shown in Figure 13.6. We need to find $F_{1\text{ on L}}$ so the fluid exerts a force great enough to support the mass of the car and piston 2. The hydraulic lift Eq. (13.2) should then allow us to determine $F_{1\text{ on L}}$.

Simplify and diagram Assume that the levels of the two pistons are the same and that the car is being lifted at constant velocity. Use the force diagram for the car and piston 2 (see diagram at right) and Newton's second law to determine $F_{\text{L on 2}}$. Note that the force that the liquid exerts on the large piston 2 $F_{\text{L on 2}}$ is equal in magnitude to the force that piston 2 and the car exert on the liquid $F_{2\text{ on L}}$, which equals the downward gravitational force that Earth exerts on the car and piston: $F_{2\text{ on L}} = m_{\text{Car+Piston}}g$.



Represent mathematically We rewrite the hydraulic lift Eq. (13.2) to determine the unknown force:

$$F_{1 \text{ on L}} = \left(\frac{A_1}{A_2}\right) F_{2 \text{ on L}} = \left(\frac{A_1}{A_2}\right) m_{\text{Car+Piston}} g$$

Solve and evaluate

$$F_{1 \text{ on L}} = \frac{(0.0020 \text{ m}^2)}{(0.20 \text{ m}^2)} [(1800 \text{ kg})(9.8 \text{N/kg})] = 180 \text{ N}$$

That is the force equal to lifting an object of mass 18 kg, which is entirely possible for a person. The units are also consistent.

Try it yourself If you needed to lift the car about 0.10 m above the ground, what distance would you have to push down on the small piston, assuming the model of the hydraulic lift applies to industrial lifts? Is the answer realistic, and what does it tell you about the operation of real hydraulic lifts?

Answer

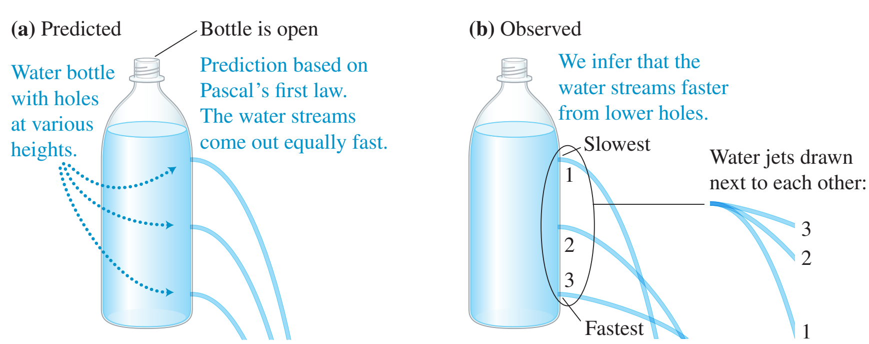
10 m. This answer is too large to be realistic. There must be some other technical details involved in the operation of industrial hydraulic lifts.

REVIEW QUESTION 13.2 If you poke many small holes in a closed toothpaste tube and squeeze it, the paste comes out equally from all holes. Why?

13.3 Pressure variation with depth

Pascal's first law states that an increase in the pressure in one part of an enclosed fluid results in an increase at all other parts of the fluid. Does that mean that the pressure is the same throughout a fluid—for example, in a vertical column of fluid? To test this hypothesis, consider another experiment with a water bottle. This time, poke vertical holes along one side of the bottle. Place tacks in the holes, and fill the bottle with water. Leave the cap off. If the pressure is the same throughout the fluid, when the tacks are removed the water should come out of each hole making an arc of the same shape, similar to projectiles thrown horizontally at the same speed, as shown in **Figure 13.7a**.

FIGURE 13.7 Water seems to be pushed harder from holes deeper in the water.

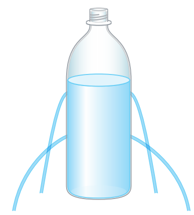
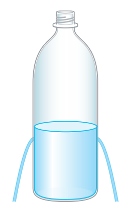
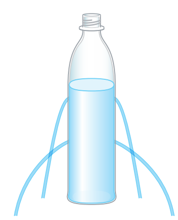


However, when the tacks are removed, we observe that the shapes of the arcs are different. The arcs produced by the water coming out from the lower holes resemble the trajectories of projectiles thrown at higher speeds (Figure 13.7b). Should we abandon Pascal's first law now because the prediction based on it did not match the outcome of the experiment? In such cases, scientists do not immediately throw out the principle but first examine the additional assumptions that were used to make the prediction. In our first experiment with the water bottle, we did not consider the impact of poking holes at different heights. Maybe this was an important factor in the experiment. Let's investigate this in Observational Experiment **Table 13.2**, on the next page.

OBSERVATIONAL EXPERIMENT TABLE 13.2

How does the location of the holes affect the streams leaving the holes?

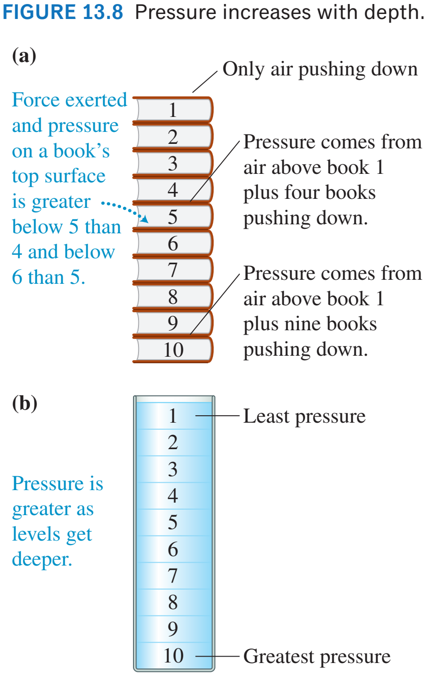
VIDEO OET 13.2

Observational experiment	Analysis
Experiment 1. Place two tacks on each side of a plastic bottle, one hole above the other, and fill the bottle with water above the top tack. Remove the tacks. Water comes out on the left and right, and the stream from the lower holes resembles a projectile thrown at a higher speed. 	There must be greater pressure inside than outside. The pressure must be greater at the bottom holes than at the top holes.
Experiment 2. Repeat Experiment 1 but this time fill the bottle with water to the same distance above the bottom tack as it was filled above the top tack in Experiment 1. Remove the tacks. The stream comes out the bottom holes with the same arc as it came out of the top holes in Experiment 1. 	The total water depth seems not to matter, just the height of the water above the hole.
Experiment 3. Repeat Experiment 1 using a thinner bottle with the water level initially the same distance above the top tack as it was in Experiment 1. Remove the tacks. The water streams are identical to those in Experiment 1. 	Because the water comes out in exactly the same arc in a bigger bottle and in a smaller bottle when the water level above the top tack is at the same height, we can conclude that the mass of the water in the bottle does not affect the pressure.

Patterns

The stream shape at a particular level:
- Depends on the depth of the water above the hole.
- Is the same in different directions at the same level.
- Does not depend on the amount of water (volume or mass) above the hole.
- Does not depend on the amount (mass or volume) or depth of the water below the hole.

FIGURE 13.8 Pressure increases with depth.



From the patterns above we reason that the pressure of the fluid at the hole depends only on the *depth* of the fluid above the hole, and not on the *mass* of the fluid above. We also see that the pressure at a given depth is the same in all directions. This is consistent with the experience you have when you dive below the surface of the water in a swimming pool, lake, or ocean. The pressure on your ears depends only on how deep you are below the surface. Pascal's first law fails to explain this pressure variation at different depths below the surface. Now we need to understand why the pressure varies with depth and to devise a rule to describe this variation quantitatively.

Why does pressure vary at different levels?

To explain the variation of pressure with depth we can use an analogy of stacking ten books on a table (**Figure 13.8a**). Imagine that each book is a layer of water in a cylindrical tube (see Figure 13.8b). Consider the pressure (force per unit area) from above on the top surface of each book. The only force exerted on the top surface of the top book is due to air pushing down from above. However, there are in effect two forces exerted on the top surface of the second book: the force that the top book exerts on it (equal in magnitude to the weight of the book) plus the force exerted by the air on the top book. The top surface of the bottom book in the stack must balance the force exerted by the nine books above it (equal in magnitude to the weight of nine books) plus the pressure force exerted by the air on the top book. So the pressure increases on the top surface of each book in the stack as we go lower in the stack.

Similar reasoning applies for the fluid-filled tube divided into a number of imaginary thin layers in Figure 13.8b. Air pushes down on the top layer. The second layer balances the weight of the top layer plus the force exerted by the air pushing down on the top layer, and so on. The pressure is lowest at the top of the fluid and greatest at the bottom.

Note that, at each layer, the pressure in a fluid is the same in all directions. If we could take a pressure sensor and place it inside the container of water, the readings of the sensor would be the same independent of the orientation of the sensor as long as its depth remains the same.

How can we quantify pressure change with depth?

We know that pressure increases with depth, and according to our model of pressure using layers, we can hypothesize that it increases with depth linearly (assuming that the density of the fluid remains constant). But what is the slope of the pressure-versus-depth graph, and what is the intercept? Consider the shaded cylinder C of water shown in **Figure 13.9a** as our system of interest. The walls on opposite sides of the cylinder push inward, exerting equal-magnitude and oppositely directed forces—the forces exerted by the sides cancel. What about the forces exerted by the water above and below? If the pressure at elevation y_2 is P_2 and the cross-sectional area of the cylinder is A, then the fluid above pushes down, exerting a force of magnitude $F_{\text{fluid above on C}} = P_2A$ (Figure 13.9b). Similarly, fluid from below the shaded section of fluid at elevation y_1 exerts on the cylinder an upward force of magnitude $F_{\text{fluid below on C}} = P_1A$. Earth exerts a third force on the shaded cylinder $\vec{F}_{\text{E on C}}$ equal in magnitude to $m_{\text{C}}g$, where m_{C} is the mass of the fluid in the cylinder. Since the fluid is not accelerating, these three forces add to zero. Choosing the y-axis pointing up, we have

$$\Sigma F_y = (-F_{\text{fluid above on C}}) + F_{\text{fluid below on C}} + (-m_{\text{C}}g) = 0$$

Substituting the earlier expressions for the forces, we have

$$(-P_2A) + P_1A + (-m_{\text{C}}g) = 0$$

The mass of the fluid in the shaded cylinder is the product of the fluid's density and the volume of the cylinder (assuming the density of the fluid is the same throughout the cylinder):

$$m_{\text{C}} = \rho_{\text{fluid}}V = \rho_{\text{fluid}}[A(y_2 - y_1)]$$

Substituting this expression for the mass in the above expression for the forces, we get

$$-P_2A + P_1A - \rho_{\text{fluid}}[A(y_2 - y_1)]g = 0$$

Divide by the common A in all of the terms and rearrange to get

$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g \tag{13.3}$$

This is Pascal's second law. As we see, pressure varies linearly with depth.

Pascal's second law—variation of pressure with depth The pressure P_1 in a static fluid at position y_1 can be determined in terms of the pressure P_2 at position y_2 as follows:

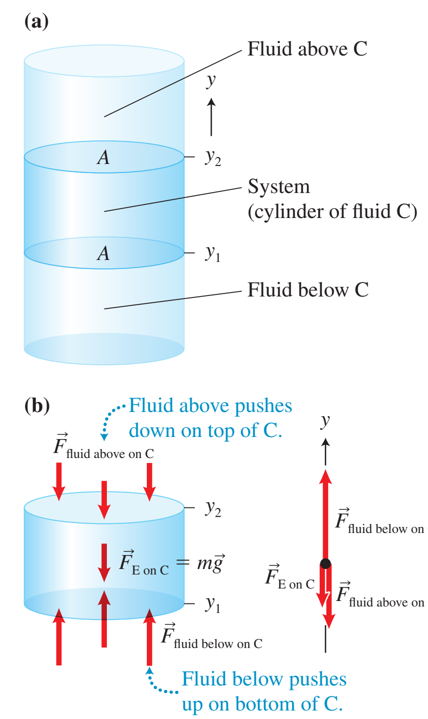
$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g \tag{13.3}$$

where ρ_{fluid} is the fluid density, assumed constant throughout the fluid, and $g = 9.8\ \text{N/kg}$. The positive y-direction is up. If y_2 is chosen to be the top of the fluid, then Eq. (13.3) can be simplified as

$$P_1 = P_{\text{atm}} + \rho_{\text{fluid}}gd \tag{13.4}$$

where P_{atm} is the atmospheric pressure and d is the depth from the top of the fluid to the level at which we want to determine the pressure.

FIGURE 13.9 Using Newton's second law to determine how fluid pressure changes with the depth in the fluid.



TIP When using Pascal's second law [Eq. (13.3)], picture the situation and be sure to include a vertical y-axis that points upward and has a defined origin, or zero point. Then choose the two points of interest and identify their vertical y-positions relative to the axis. This lets you relate the pressures at those two points.

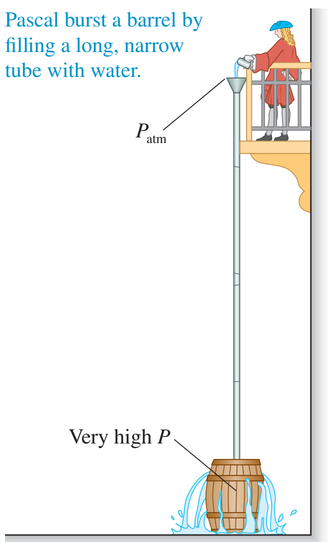


FIGURE 13.10 Pascal tests his second law.

Some history of physics books say that Blaise Pascal conducted the following testing experiment for his second law. Pascal filled a barrel with water and inserted a long, narrow vertical tube into the water from above. He then sealed the barrel (see **Figure 13.10**). He predicted that when he filled the tube with water, the barrel would burst even though the mass of water in the thin tube was small, because the pressure of the water in the barrel would depend on the depth, not the mass, of the water column. The barrel burst, matching Pascal's prediction, thus supporting the idea that the depth of the fluid above, not its mass determined the pressure.

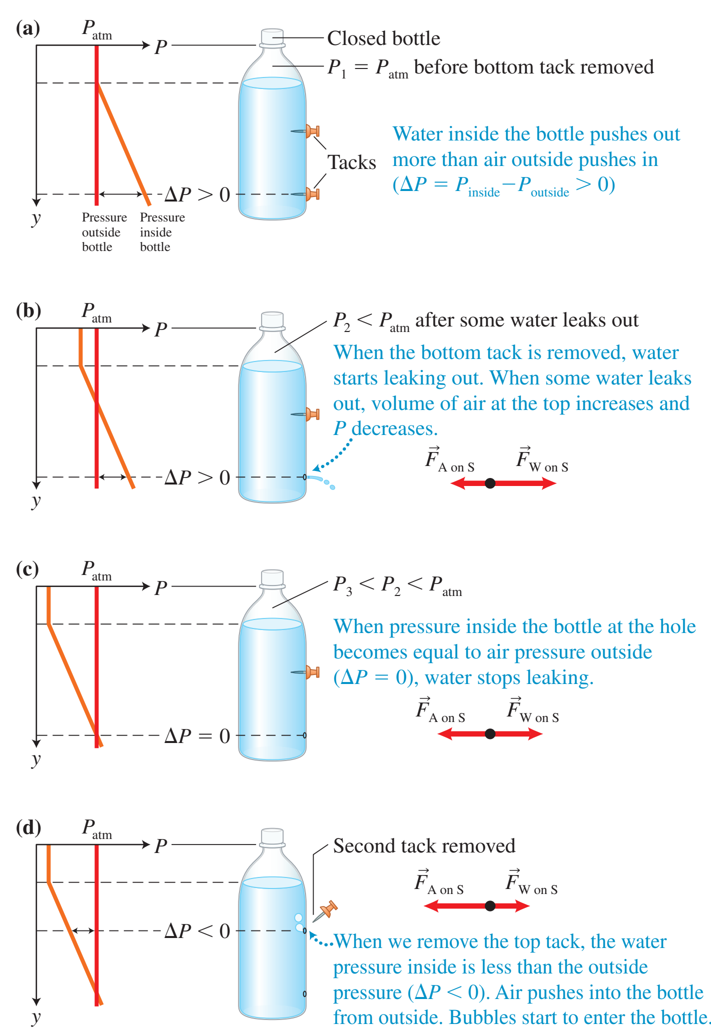
We derived Eqs. (13.3) and (13.4) using liquid as an example. The density of liquids does not change with depth because they are incompressible. Gases are compressible, and thus their density changes with depth. However, in all our examples, the changes in depth will be small enough so that we can neglect the density change in gases. Applying Eq. (13.3) to atmospheric air, we can explain why the pressure at the top of a mountain is less than at the bottom.

CONCEPTUAL EXERCISE 13.3 Closed water bottle with tacks

You have a closed plastic water bottle with two holes in the side closed with tacks, one tack near the bottom of the bottle and the other in the middle. You remove the bottom tack, and a few drops of water come out of the hole but then the leaking stops. Then you remove the top tack (with the bottom hole still open), and an air bubble enters the bottle through the top hole. Draw depth-versus-pressure graphs and force diagrams to explain this phenomenon.

Sketch and translate We sketch the bottle with two tacks in the holes. Let's choose the cap of the bottle to be the origin of the y-axis that points down; the horizontal axis will represent the pressure (Figure a).

Simplify and diagram We consider the pressure above the water level to be atmospheric at the beginning of the experiment (Figure a). Because only a few drops of water leave the bottle, we will neglect the changes in water level on the sketches and graphs. Figures b–d show the removal of the tacks and the changes in pressure on the graphs. We draw force diagrams for a tiny portion of water at the hole as our system of interest. There are two horizontal forces exerted on that system element of water: the force exerted by the outside air pushing inward $\vec{F}_{A \text{ on } S}$ and the force of the inside water pushing outward $\vec{F}_{W \text{ on } S}$. If one of the forces is smaller, then the tiny system of water will either accelerate outward and we will see the bottle leaking, or it will accelerate inward and we will see bubbles of air coming in. When a little bit of water leaks out, the volume of air above the water in the bottle increases, and its pressure decreases. This decrease leads to a decrease of the pressure everywhere inside the bottle.



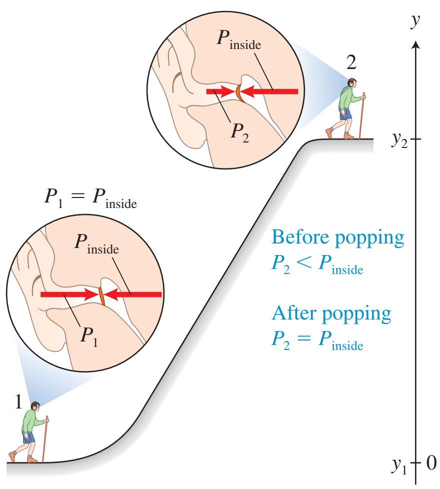
Try it yourself Predict what will happen after an air bubble enters the bottle through the top hole.

Answer

As new air enters the bottle, the air pressure above the water surface increases and so does the pressure elsewhere in the bottle. This results in more water leaking through the lower hole, which again decreases the pressure and causes more bubbles to enter the bottle, and so on. The process repeats until the water level drops to the upper hole. From then on, the water leaks out continuously until it reaches the level of the bottom hole.

QUANTITATIVE EXERCISE 13.4 **Pop your ears**

If your ears did not pop, then what would be the net force exerted by the inside and outside air on your eardrum at the top of a 1000-m-high mountain? You start your hike from sea level. The area of your eardrum is 0.50 cm^2. The density of air at sea level at standard conditions is 1.3 kg/m^3. Assume the air density remains constant during the hike. The situation at the start at $y_1 = 0$ and at the end of the hike at $y_2 = 1000$ m is sketched below.



Represent mathematically We use an upward-pointing vertical y-axis with the origin at sea level. Assume that the air pressure inside the eardrum remains constant at its sea level value $P_{inside} = P_1$. The air pressure difference between the top of the mountain and sea level is

$$P_2 - P_1 = \rho(y_1 - y_2)g$$

When at 1000 m above sea level, the outside air exerts a lower pressure P_2 on the eardrum. So the net force exerted by the air on the drum is the pressure difference of air pushing out $P_{inside} = P_1$ and air pushing in P_2 times the area of the eardrum: $F_{\text{net air on drum}} = (P_1 - P_2)A$. This pressure difference can be determined using Pascal's second law.

Solve and evaluate

$$\begin{aligned} P_1 - P_2 &= \rho(y_2 - y_1)g \\ &= (1.3\ \text{kg/m}^3)(1000\ \text{m} - 0)(9.8\ \text{N/kg}) \\ &= +0.13 \times 10^5\ \text{N/m}^2 \end{aligned}$$

This is 0.13 atm!

$$\begin{aligned} F_{\text{net air on drum}} &= (P_1 - P_2)A \\ &= (0.13 \times 10^5\ \text{N/m}^2)(0.50\ \text{cm}^2) \times (1\ \text{m}/100\ \text{cm})^2 \\ &= +0.60\ \text{N} \end{aligned}$$

The net force is exerted outward and is about half the gravitational force that Earth exerts on an apple. No wonder it can hurt until you get some air out of that middle ear!

Try it yourself Determine the difference in water pressure on your ear when you are 1.0 m underwater compared to when you are at the surface. The density of water is 1000 kg/m^3.

Answer $+9800\ \text{N/m}^2$ greater pressure when under the water, or 0.1 atm.

REVIEW QUESTION 13.3 Pascal's first law says that an increase in pressure in one part of an enclosed liquid results in an increase in pressure throughout all parts of that fluid. Why then does the pressure differ at different heights?

13.4 Measuring atmospheric pressure

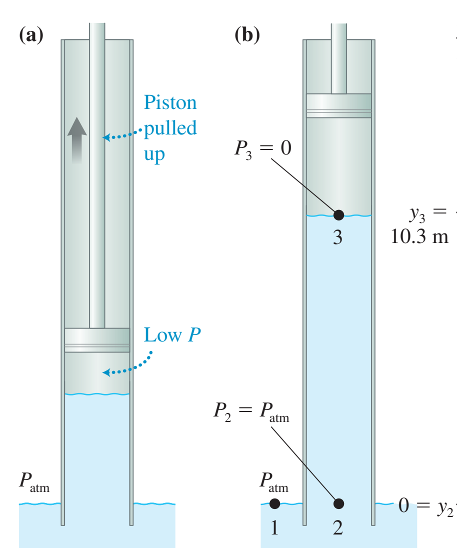
We can now use Pascal's second law to develop a method for measuring atmospheric air pressure.

Torricelli's experiments

In the 1600s, suction pumps were used to lift drinking water from wells and to remove water from flooded mines. The suction pump was like a long syringe. The pump consisted of a piston in a long cylinder that pulled up water (**Figure 13.11a**). Such pumps could lift water a maximum of 10.3 m. Why 10.3 m?

Evangelista Torricelli (1608–1647), one of Galileo's students, hypothesized that the pressure of the air in the atmosphere could explain the limit to how far water could be lifted. Torricelli did not know of Pascal's second law, which was published in the year that Torricelli died. However, it is possible that Torricelli's work influenced Pascal. Let's analyze the situation shown in Figure 13.11b.

FIGURE 13.11 A piston pulled up a cylinder causes water to rise to a maximum height of 10.3 m.



Consider the pressure at three places: point 1, at the water surface in the pool outside the cylinder; point 2, at the same elevation but inside the cylinder; and point 3, in the cylinder 10.3 m above the pool water level. The pressure at point 1 is atmospheric pressure. The pressure at point 2, according to Pascal's second law, is also atmospheric pressure, since it is at the same level as point 1. To get the water to the 10.3-m maximum height, the region above the water surface inside the cylinder and under the piston must be at the least possible pressure—essentially a vacuum. Thus, we assume that the pressure at point 3 is zero.

Now we will use Eq. (13.3) with $y_3 - y_2 = 10.3$ m to predict the pressure P_2:

$$\begin{aligned} P_2 = P_3 + \rho_{\text{water}}(y_3 - y_2)g &= 0 + (1.0 \times 10^3\ \text{kg/m}^3)(10.3\ \text{m} - 0)(9.8\ \text{N/kg}) \\ &= 1.01 \times 10^5\ \text{N/m}^2 \end{aligned}$$

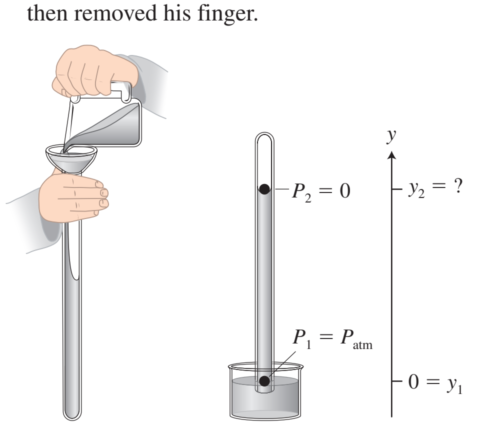
This number is exactly the value of the atmospheric pressure that we encountered in our discussion of gases (in Chapter 12). The atmospheric pressure pushing down on the water outside the tube can push water up the tube a maximum of 10.3 m if there is a vacuum (absence of any matter) above the water in the tube.

At Torricelli's time, the value of normal atmospheric pressure was unknown, so the huge number that came out of this analysis surprised Torricelli. Not believing the result, he tested it using a different liquid—mercury. Mercury is 14 times denser than water ($\rho_{\text{Hg}} = 13{,}600\ \text{kg/m}^3$); hence, the column of mercury should rise only 1/14 times as high in an evacuated tube. However, instead of using a piston to lift mercury, Torricelli devised a method that guaranteed that the pressure at the top of the column was about zero. Consider Testing Experiment **Table 13.3**.

TESTING EXPERIMENT TABLE 13.3 Testing Torricelli's hypothesis using mercury

Testing experiment

- Torricelli filled a long glass tube closed at one end with mercury.
- He put his finger over the open end and placed it upside down in a dish filled with mercury. He then removed his finger.



Predict what he observed based on the hypothesis that atmospheric pressure limits the height of the liquid in a suction pump.

Prediction

Mercury should start leaking from the tube into the dish. When it leaks, it leaves an empty evacuated space at the top of the tube. It will leak until the height of the mercury column left in the tube produces the same pressure as the atmosphere at the bottom of the column at position 1. The height of the mercury in the tube should be

$$\begin{aligned} y_2 - y_1 &= \frac{P_1 - P_2}{\rho_{\text{mercury}}g} \\ &= \frac{(1.01 \times 10^5\ \text{N/m}^2 - 0)}{(13.6 \times 10^3\ \text{kg/m}^3)\ (9.8\ \text{N/kg})} \\ &= 0.76\ \text{m} \end{aligned}$$

Outcome

Torricelli observed some mercury leaking from the tube and then the process stopped. He measured the height of the remaining mercury to be 0.76 m = 760 mm, in agreement with the prediction.

Conclusion

The outcome of the experiment was consistent with the prediction based on Torricelli's hypothesis that atmospheric pressure limits the height of the liquid being lifted in a suction pump. Thus the hypothesis is supported by evidence.

Torricelli also used his understanding of pressure and fluids to predict that in the mountains, where the atmospheric pressure is lower, the height of the mercury column should be lower. Experiments have shown that the mercury level indeed decreases at higher elevation. These experiments supported the explanation that atmospheric air pushes the liquids upward into the tubes.

Torricelli's apparatus with the mercury tube became a useful device for measuring atmospheric pressure—called a barometer. However, since mercury is toxic, the Torricelli device has since been replaced by the aneroid barometer (described in Chapter 12).

Eugenia, one of the book's authors, stands under a 17th-century diving bell displayed in the Vasa Museum in Stockholm, Sweden.

TIP We now understand why pressure is often measured and reported in mm Hg and why atmospheric pressure is 760 mm Hg. The atmospheric pressure ($101{,}000\ \text{N/m}^2$) can push mercury (density $13{,}600\ \text{kg/m}^3$) 760 mm up a column.

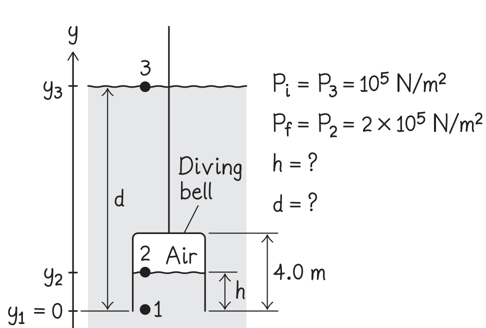
Diving bell

Our understanding of atmospheric pressure allows us to explain many simple experiments that lead to important practical applications. For example, have you ever submerged a transparent container upside down under water? If you have, you have seen that at first little water enters the container; then as the inverted container is pushed deeper into the water, more water enters the container. One practical application of this phenomenon is a diving bell—a large, bottomless chamber lowered under water with people and equipment inside. Divers use the diving bell to take a break and refill on oxygen. In the past, diving bells were used for underwater construction, such as building bridge foundations.

EXAMPLE 13.5 Diving bell

The bottom of a 4.0-m-tall cylindrical diving bell is at an unknown depth underwater. The pressure of the air inside the bell is $2.0 \times 10^5\ \text{N/m}^2$ (having been about $1.0 \times 10^5\ \text{N/m}^2$ before the bell entered the water). The density of ocean water is slightly higher than fresh water: $\rho_{\text{ocean water}} = 1027\ \text{kg/m}^3$. How high is the water inside the bell, and how deep is the bottom of the bell under the water?

Sketch and translate A labeled sketch shows the situation. We want to find the height h of the water in the bell and the depth d that the bottom of the bell is under the water.



Simplify and diagram Assume that the temperature is constant so we can apply our knowledge of an isothermal process (PV is constant for a constant temperature gas) to the air inside the bell and use it to relate the state of the air inside the bell before it is submerged to its state after it is submerged. This will let us find the ratio of the air volumes before and after, and from that we can determine h. Once we have that result, we can use Pascal's second law to determine d.

Represent mathematically Apply the mathematical expression for an isothermal process to the process of submerging the bell, where the initial state is just before the bell starts to be submerged in the water and the final state is when submerged to some unknown depth. We have

$$P_3 V_i = P_2 V_f \quad \Rightarrow \quad V_f = \frac{P_3}{P_2} V_i$$

Next, use Pascal's second law to determine d. The bottom of the bell is at $y_1 = 0$; the water surface is at $y_3 = d$. Compare the pressure at the ocean surface P_3 to the pressure at the water surface inside the bell P_2:

$$P_2 = P_3 + \rho(y_3 - y_2)g$$

Solve and evaluate $V_f = \dfrac{P_3}{P_2} V_i = \dfrac{1.0 \times 10^5\ \text{N/m}^2}{2.0 \times 10^5\ \text{N/m}^2} V_i = \dfrac{1}{2} V_i$

Thus, the air volume inside the diving bell is half of what it was before entering the water. The bell is therefore half full of water, which means the height of the water level inside is

$$h = \frac{1}{2}(4.0\ \text{m}) = 2.0\ \text{m} = y_2$$

(CONTINUED)

Rearrange Pascal's second law to solve for the position y_2 of the water level inside the bell:

$$y_2 = 2.0\text{ m} = y_3 - \frac{P_2 - P_3}{\rho g} = d - \frac{(2.0 \times 10^5\text{ Pa} - 1.0 \times 10^5\text{ Pa})}{(1027\text{ kg/m}^3)(9.8\text{ N/kg})}$$

$$= d - 10\text{ m}$$

Thus, $d = 2.0\text{ m} + 10\text{ m} = 12\text{ m}$. The position of the bottom of the bell is 12 m below the ocean's surface.

Try it yourself Suppose the air pressure in the bell is 3.0 atm. In this case, how high is the water in the bell, and how deep is the bottom of the bell?

Answer The water is 2.7 m high in the bell (there is 1.3 m of air at the top), and the bottom of the bell is 23 m below the ocean's surface.

REVIEW QUESTION 13.4 What does it mean if atmospheric pressure is 760 mm Hg?

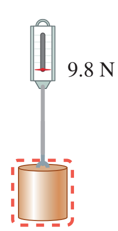
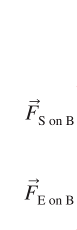
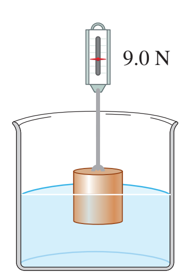
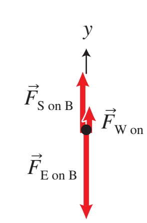
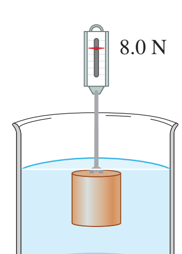
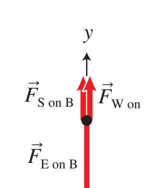
13.5 Buoyant force

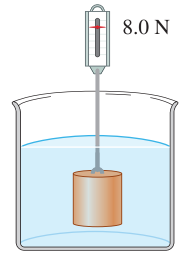
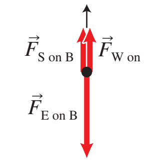
Pascal's first law tells us that pressure changes in one part of a fluid result in pressure changes in other parts. Pascal's second law describes how the pressure in a fluid varies depending on the depth in the fluid. Do these laws explain why some objects float and others sink? Consider Observational Experiment **Table 13.4**.

OBSERVATIONAL EXPERIMENT TABLE 13.4

Effect of depth of submersion on a steel block suspended in water

VIDEO OET 13.4

Observational experiment	Analysis
Experiment 1. Hang a 1.0-kg block from a spring scale. The force that the scale exerts on the block balances the downward force that Earth exerts on the block $(mg = (1.0\text{ kg})(9.8\text{ N/kg}) = 9.8\text{ N})$. 	Force diagram for the block B: 
Experiment 2. Lower the block into a container of water, so it is partially submerged. The water level rises. The reading of the scale decreases. 	We explain the decreased reading on the scale by the water pushing upward a little on the block. 
Experiment 3. Lower the same block into the container of water to the point where the block is completely submerged. As the water level rises, the reading of the scale decreases. 	The upward force exerted by the water increases. 

Observational experiment	Analysis
Experiment 4. Lower the block into the container of water so that the block is completely submerged near the bottom. The water level and the reading of the scale do not change. 	The upward force exerted by the water does not change once it is completely submerged. 

Patterns

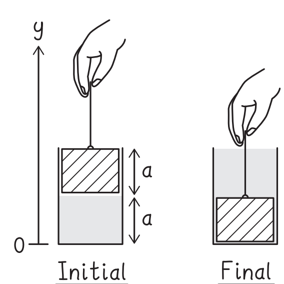
We notice two effects:

1. The level of the water in the container rises as more of the block is submerged in the water.
2. The scale reading decreases as more of the block is submerged. The water exerts an upward force on the block. The magnitude of this force depends on how much of the block is submerged. After it is totally submerged, the force does not change, even though the depth of submersion changes.

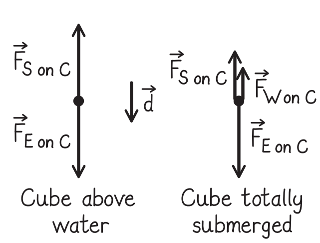
CONCEPTUAL EXERCISE 13.6 Qualitative force and energy analysis for an object in a fluid

You hang a solid metal cube (with side a) by a string over a rectangular container filled with water up to height a. The cube barely fits inside the container. Starting when the cube is just above the water in the container, you very slowly lower the cube into the water until it is totally submerged. Represent this process by drawing force diagrams for the cube when it is above the water and when it is totally submerged. Also draw an energy bar chart for the cube, Earth, and the water as the system for those states.

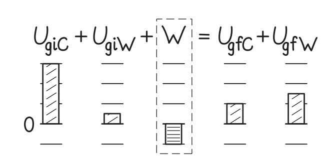
Sketch and translate We draw a sketch of the situation; the system and the initial and final states are specified in the problem statement. The submerged cube displaces water that is now above the cube. The volumes of the cube and water are the same, but the water has less mass due to its smaller density.



Simplify and diagram Because you are moving the cube very slowly, we can neglect the kinetic energy of the cube and the water in this process. We also assume that the cube has no acceleration during this motion and all friction effects are negligible. On the force diagrams we show the three forces exerted by the three objects with which the cube interacts: Earth, the water, and the string. Because there is no acceleration, the sum of the forces is zero.

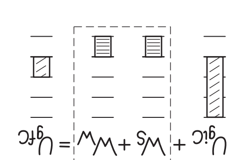


The string does negative work on the cube because the force it exerts on the cube points opposite to the cube's displacement. Given that both water and the cube interact with Earth, we consider the gravitational potential energies of these interactions separately. We take zero gravitational potential energy to be at the bottom of the container. In the initial state, the system has initial gravitational potential energy due to the location of the cube and water. The gravitational potential energy of cube-Earth interaction decreases as the cube moves down; the gravitational potential energy of water-Earth interaction increases as some water moves up (being replaced by the cube). However, the total gravitational potential energy of the system must decrease due to the negative work done by the string.



Try it yourself Draw the bar charts for the two states, choosing the cube and Earth as a system.

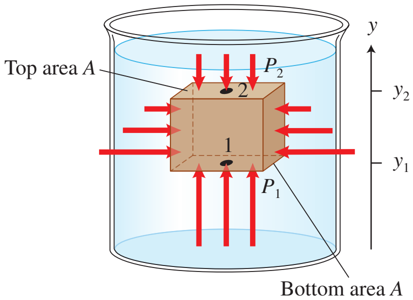
Answer



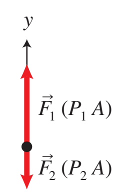
In Table 13.4, after the block is completely submerged, the scale reads 8.0 N instead of 9.8 N. Evidently, the water exerts a 1.8-N upward force on the block. What is the mechanism responsible for this force?

FIGURE 13.12 A fluid exerts an upward buoyant force on the block.

(a) The force exerted on the block by the fluid below is greater than the force exerted by the fluid above.



(b) The upward force of the fluid on the bottom surface is greater than the downward force of the fluid on the top surface.



The magnitude of the force the fluid exerts on a submerged object

Consider *only* the fluid forces exerted on the block shown in **Figure 13.12a**. The fluid pushes inward on the block from all sides, including the top and the bottom. The forces exerted by the fluid on the vertical sides of the block cancel, since the pressure at a specific depth is the same magnitude in all directions.

What about the fluid pushing down on the top and up on the bottom of the block? The pressure is greater at elevation y_1 at the bottom of the block than at elevation y_2 at the top surface of the block. Consequently, the force exerted by the fluid pushing up on the bottom of the block is greater than the force exerted by the fluid pushing down on the top of the block. Arrows in Figure 13.12b represent the forces that the fluid exerts on the top and bottom of the block. The vector sum of these two fluid forces always points up and is called a buoyant force $\vec{F}_{\text{F on B}}$ (fluid on block).

To calculate the magnitude of the upward buoyant force $F_{\text{F on B}}$ exerted by the fluid on the block, we use Eq. (13.3) to determine the pressure P_1 of the fluid on the bottom surface of the block compared to the pressure P_2 of the fluid on the top surface (see Figure 13.12):

$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g$$

The magnitudes of the forces exerted by the fluid on the top and on the bottom of the block are the products of the pressure P and the area A of the top and bottom surfaces of the block:

$$P_1 A = P_2 A + \rho_{\text{fluid}}(y_2 - y_1)Ag$$

or

$$F_1 = F_2 + \rho_{\text{fluid}}(y_2 - y_1)Ag$$

The volume of the block is

$$V_{\text{B}} = A(y_2 - y_1)$$

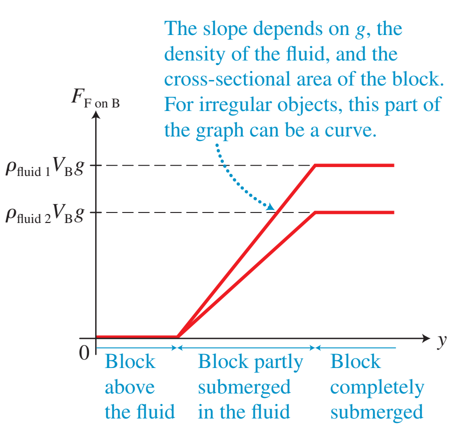
where A is the cross-sectional area of the block and $(y_2 - y_1)$ is its height. Substitute this volume into the above force equation and rearrange it to get an expression for the magnitude of the total upward **buoyant force** $F_{\text{F on B}}$ that the fluid exerts on the block (the object of interest):

$$F_{\text{F on B}} = F_1 - F_2 = \rho_{\text{fluid}}\, gV$$

Note that for a totally submerged block, V is the volume of the block. However, when it is partially submerged, V is the volume of the submerged part.

We can now understand the results of the experiments in Table 13.4. When submerging the block further into the water, the scale reading decreased because the buoyant force was increasing. The scale reading stopped changing after the block was completely under the water. Once completely underwater, the submerged volume did not change; the upward buoyant force exerted by the fluid on the block remained constant. Fluids of different densities exert different upward forces on the same object submerged to the same depth (**Figure 13.13**).

FIGURE 13.13 Buoyant force dependence on the depth of the object and the density of the fluid.



Archimedes' principle—the buoyant force A stationary fluid exerts an upward buoyant force on an object that is totally or partially submerged in the fluid. The magnitude of the force is the product of the fluid density ρ_{fluid}, the volume $V_{\text{displaced}}$ of the fluid that is displaced by the object, and the gravitational constant g:

$$F_{\text{F on O}} = \rho_{\text{fluid}} V_{\text{displaced}} g = \rho_{\text{fluid}} V_{\text{subm}} g \tag{13.5}$$

TIP The derivation of Eq. (13.5) was for a solid cube, but the result applies to objects of any shape, though calculus is needed to establish that.

For simplicity, we will always use the volume of the submerged part of the object in our calculations and label it V_{subm}. Because $\rho_{\text{fluid}} V_{\text{displaced}} = m_{\text{displaced fluid}}$, we see that **the buoyant force is equal in magnitude to the force that Earth exerts on the amount of fluid displaced by the object.** However, the natures of these two forces are different. Earth exerts a gravitational force at a distance; the buoyant force is a contact force.

REVIEW QUESTION 13.5 Why does a fluid exert an upward force on an object submerged in it?

13.6 Skills for analyzing static fluid problems

In this section we adapt our problem-solving strategy to analyze processes involving static fluids.

PROBLEM-SOLVING STRATEGY 13.1 Analyzing situations involving static fluids

EXAMPLE 13.7 Buoyant force exerted by air on a human

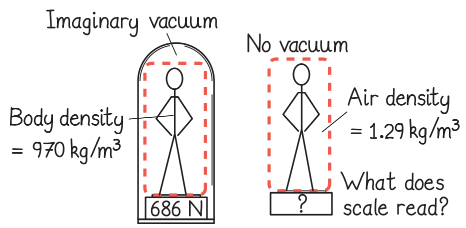
Suppose your mass is 70.0 kg and your density is 970 kg/m^3. If you could stand on a scale in a vacuum chamber on Earth's surface, the reading of the scale would be $mg = (70.0\text{ kg})(9.80\text{ N/kg}) = 686\text{ N}$. What will the scale read when you are completely submerged in air of density 1.29 kg/m^3?

Sketch and translate

- Make a labeled sketch of the situation and choose the system of interest. If applicable, decide on the initial and final states.
- Include all known information in the sketch and indicate the unknown(s) you wish to determine.

You are the system.

The scale reads 686 N when in a vacuum. Your density is 970 kg/m^3 and the density of air is 1.29 kg/m^3. What does it read when you are submerged in air?

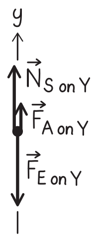


Simplify and diagram

- Indicate any assumptions you are making.
- Identify objects outside the system that interact with it.
- Construct a force diagram for the system, including a vertical coordinate axis. The buoyant force is just one of the forces included in the diagram.
- Construct a bar chart or any other graphical representation that might help solve the problem.

Assume that the air density is uniform.

Three objects exert forces on you. Earth exerts a downward gravitational force $F_{\text{E on Y}} = mg = 686\text{ N}$. The air exerts an upward buoyant force $F_{\text{A on Y}} = \rho_{\text{air}} g V_{\text{Y}}$. The scale exerts an unknown upward normal force of magnitude $N_{\text{S on Y}}$.



(CONTINUED)

Represent mathematically

- Use the force diagram to help apply Newton's second law in component form.
- Use the energy bar chart to calculate work and energy if needed.
- Use the expression for the buoyant force and the definitions of pressure and density if needed; sometimes you might need the ideal gas law.

The y-component form of Newton's second law for your body with zero acceleration is (assuming the upward direction as positive)

$$0 = N_{\text{S on Y}} + F_{\text{A on Y}} + (-F_{\text{E on Y}})$$

or

$$N_{\text{S on Y}} = +F_{\text{E on Y}} - F_{\text{A on Y}}$$

The buoyant force that the air exerts on your body has magnitude $F_{\text{A on Y}} = \rho_{\text{air}} V_{\text{Y}} g$. The volume of your body is $V_{\text{Y}} = (m/\rho_{\text{body}})$. The magnitude of the buoyant force that the air exerts on you is

$$F_{\text{A on Y}} = \rho_{\text{air}}\left(\frac{m}{\rho_{\text{body}}}\right)g = mg\left(\frac{\rho_{\text{air}}}{\rho_{\text{body}}}\right)$$

Thus the reading of the scale should be:

$$N_{\text{S on Y}} = mg - mg\left(\frac{\rho_{\text{air}}}{\rho_{\text{body}}}\right)$$

Solve and evaluate

- Insert the known information and solve for the desired unknown.
- Evaluate the final result in terms of units, reasonable magnitude, and whether the answer makes sense in limiting cases.

$$N_{\text{S on Y}} = +686\text{ N} - (686\text{ N})\frac{1.29\text{ kg/m}^3}{970\text{ kg/m}^3} = 685\text{ N}$$

According to Newton's third law, the force that you exert on the scale $N_{\text{Y on S}}$ is equal in magnitude to the force the scale exerts on you.

The reading of the scale is actually 0.1% less when you step on the scale in air—not a big deal. We can usually neglect air's buoyant force. Notice that overall the atmospheric air pushes up on objects, not down.

Try it yourself What will the scale read if you weigh yourself in a swimming pool with your body completely submerged?

Answer 0 N. Because you are less dense than water, the buoyant force exerted by the water on you will completely support you, and the scale will not push upward on you at all.

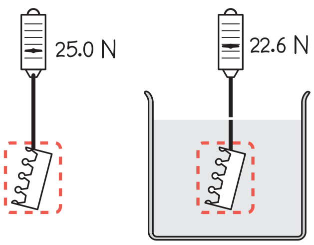
We will use these strategies to analyze several more situations. In the next example, it might not be obvious how to arrive at the answer to the question with the information provided. Following the suggested problem-solving routine will help you arrive at a solution.

EXAMPLE 13.8 Is the crown made of gold?

You need to determine if a crown is made from pure gold or some less valuable metal. From Table 13.1 you know that the density of gold is $19{,}300\text{ kg/m}^3$. You find that the force that a string attached to a spring scale exerts on the crown is 25.0 N when the crown hangs in air and 22.6 N when the crown hangs completely submerged in water.

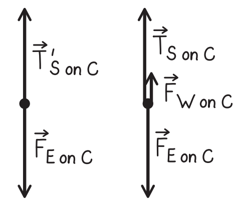
You can determine the mass of the crown easily from the measurement of the scale when the crown hangs in air. But how can you determine the volume from the given information? Crowns have irregular shapes, and it would be difficult to determine its volume by simple measurements and calculations.

Sketch and translate We draw a sketch of the situation and label the givens. If you could measure the mass m_C of the crown and its volume V_C, you could calculate the density $\rho_C = m_C/V_C$ of the crown—it should be $19{,}300\text{ kg/m}^3$.



Simplify and diagram Let's follow the recommended strategy and see what happens. First, we draw force diagrams for the crown hanging in air and again when hanging in water. When the crown is in air, the upward force exerted by the string attached to the spring scale balances the downward force exerted by Earth. We ignore the buoyant force that air exerts on the crown when hanging in air, since it will be very small in magnitude compared with the other forces exerted on the crown.

When the crown is in water, the upward force exerted by the string (the force measured by the scale) and the upward buoyant force that the water exerts on the crown combine to balance the downward gravitational force that Earth exerts on the crown.



Represent mathematically Since the crown is in equilibrium, the forces exerted on it must add to zero in both cases. When the crown is hanging in air, the vertical component form of Newton's second law is

$$0 = \Sigma F_y = T'_{\text{S on C}} + (-F_{\text{E on C}})$$

where $T'_{\text{S on C}}$ is the 25.0-N string tension force exerted on the crown when it is suspended in air. Thus,

$$F_{\text{E on C}} = m_{\text{C}} g = 25.0 \text{ N}$$

or

$$m_{\text{C}} = \frac{25.0 \text{ N}}{9.8 \text{ N/kg}} = 2.55 \text{ kg}$$

The vertical component form of Newton's second law when the crown hangs in water becomes (the upward direction is positive):

$$\Sigma F_y = T_{\text{S on C}} + F_{\text{W on C}} + (-F_{\text{E on C}}) = 0$$

where $T_{\text{S on C}} = 22.6$ N is the magnitude of the string tension force, and the buoyant force that the water exerts on the crown is $F_{\text{W on C}} = \rho_{\text{W}} V_{\text{C}}\, g$. Substituting in the above, we get

$$T_{\text{S on C}} + \rho_{\text{W}} V_{\text{C}} g - m_{\text{C}} g = 0$$

Solve and evaluate We see now that the last equation can be used to determine the volume of the crown:

$$\begin{aligned} V_{\text{C}} &= \frac{m_{\text{C}} g - T_{\text{S on C}}}{\rho_{\text{W}} g} \\ &= \frac{25.0 \text{ N} - 22.6 \text{ N}}{(1000 \text{ kg/m}^3)(9.8 \text{ N/kg})} = 0.000245 \text{ m}^3 \end{aligned}$$

We now know the crown mass and volume and can calculate its density:

$$\rho = \frac{m}{V} = \frac{2.55 \text{ kg}}{0.000245 \text{ m}^3} = 10{,}400 \text{ kg/m}^3$$

Oops! Since 10,400 kg/m^3 is much less than the 19,300 kg/m^3 density of gold, the crown is not made of pure gold. The goldsmith must have combined the gold with some less expensive metal.

Try it yourself What is the density of the crown if the scale reads 0 when submerged in water?

Answer 1000 kg/m^3.

REVIEW QUESTION 13.6 Two objects have the same volume, but one is heavier than the other. When they are completely submerged in oil, on which one does the oil exert a greater buoyant force?

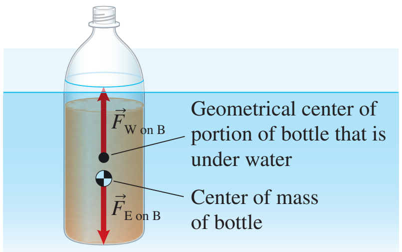
13.7 Ships, balloons, climbing, and diving

As we learned in Section 13.1, whether an object floats or sinks depends on its density relative to the density of the fluid. The reason for this lies in the interactions of the object with the fluid and Earth. Specifically, the magnitude of the buoyant force is $F_{\text{F on O}} = \rho_{\text{fluid}} V_{\text{subm}}\, g$, and the magnitude of the force exerted by Earth is $F_{\text{E on O}} = \rho_{\text{object}} V_{\text{object}}\, g$. These forces are exerted in the opposite directions. The relative densities of the fluid and the object and consequently the relative magnitudes of the forces determine what happens to the object when placed in the fluid.

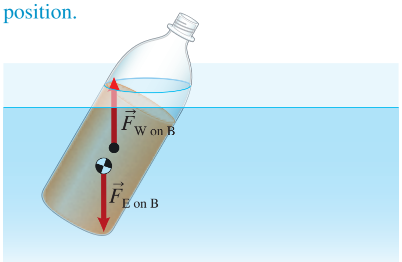
- If the object's density is less than that of the fluid $\rho_{\text{object}} < \rho_{\text{fluid}}$, then $\rho_{\text{object}} V_{\text{object}}\, g < \rho_{\text{fluid}} V_{\text{object}}\, g$; the object floats *partially* submerged since the buoyant force can balance the gravitational force with less than the entire object below the surface of the fluid.
- If the densities are the same $\rho_{\text{object}} = \rho_{\text{fluid}}$, then $\rho_{\text{object}} V_{\text{object}}\, g = \rho_{\text{fluid}} V_{\text{object}}\, g$; the sum of the forces exerted on the object is zero and it remains wherever it is placed totally submerged at any depth in the fluid.
- If the object is denser than the fluid $\rho_{\text{object}} > \rho_{\text{fluid}}$, then $\rho_{\text{object}} V_{\text{object}}\, g > \rho_{\text{fluid}} V_{\text{object}}\, g$; the magnitude of the gravitational force is always greater than the magnitude of the buoyant force. The object sinks until it reaches the bottom of the container.

FIGURE 13.14 Making a bottle float with stable equilibrium.

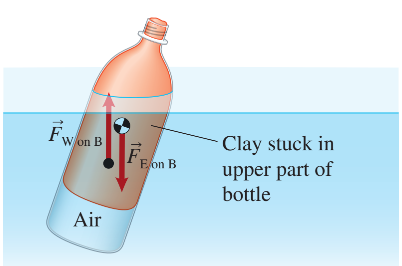
(a) Bottle partially filled with sand floats upright if the center of mass is below the geometrical center of bottle.



(b) If the bottle is tipped, torques due to forces exerted on the bottle return it to the upright position.



(c) If this bottle tips slightly, torques due to forces exerted on the bottle cause it to overturn.



These cases show that by changing the average density of an object relative to the density of the fluid, the object can be made to float or sink in the same fluid. In this section we investigate this phenomenon and its many practical applications.

Building a stable ship

For years ships were made of wood. In the middle of the 17th century, people decided to try building metal ships. Many thought that this idea was absurd: iron is denser than water and an iron boat would certainly sink. In 1787 British engineer John Wilkinson succeeded in building the first iron ship that did not sink. Since the middle of the 19th century, large ships have been made primarily of steel, which is less dense than iron but much denser than water. These ships can float because part of the volume of a ship is filled with air, which reduces the average density of the ship to a density lower than that of water.

Making a ship or a raft float is only part of the challenge of building watercraft. Another problem is to maintain stable equilibrium for the ship, allowing it to right itself if it tilts to one side due to wind or rough seas. Refresh your knowledge of stable equilibrium (Section 8.6) before you read on.

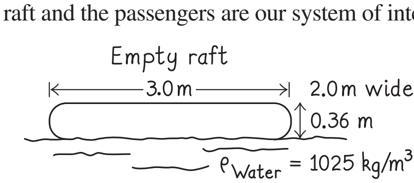
Consider a floating bottle partially filled with sand. Earth exerts a gravitational force at the center of mass of the bottle (**Figure 13.14a**). The buoyant force exerted by the water on the bottle is effectively exerted at the geometrical center of the part of the bottle that is underwater, which equals the center of mass of the displaced water. If this point is above the center of mass of the bottle, then any slight tipping causes these forces to produce a torque that attempts to return the bottle to an upright position (see Figure 13.14b). However, if the geometrical center of the part of the bottle that is underwater is below the center of mass of the bottle, slight tipping causes the gravitational force to produce a torque that enhances the tipping—unstable equilibrium (Figure 13.14c).

Although ships are more complicated than water bottles filled with sand, it is important to load a ship in such a way that when it begins to heel and one side of the hull begins to rise from the water, the center of mass of the displaced water is above the center of mass of the ship. This is why ships have their cargo stored at the bottom.

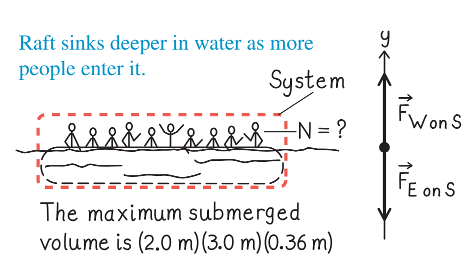
EXAMPLE 13.9 **Should we take this trip?**

The top of an empty life raft of cross-sectional area $2.0\text{ m} \times 3.0\text{ m}$ is 0.36 m above the waterline. How many 75-kg passengers can the raft hold before water starts to flow over its top? The raft is in seawater of density 1025 kg/m^3.

Sketch and translate We make a sketch of the unloaded raft. As people get on the raft, it sinks deeper into the water, and the upward buoyant force increases until the raft reaches a maximum submerged volume, when the maximum number of people are on board. The maximum submerged volume is $V_{\text{subm}} = 2.0\text{ m} \times 3.0\text{ m} \times 0.36\text{ m} = 2.16\text{ m}^3$. We need to determine the maximum buoyant force the seawater can exert on the raft and then decide how to convert this into the number of passengers the raft can hold. The raft and the passengers are our system of interest.



Simplify and diagram We have no information about the mass of the raft; thus we will assume it is negligible. We draw a sketch of the filled raft and a force diagram for the raft with passengers (the system). The vertical axis points up. There are two forces exerted on the system: the upward force exerted by the water $\vec{F}_{\text{W on S}}$ of magnitude $\rho_{\text{water}} g V_{\text{subm}}$ and the downward force exerted by Earth $\vec{F}_{\text{E on S}}$. The magnitude of the force Earth exerts on N people is $N_{\text{people}} m_{\text{person}} g$. As the system is in equilibrium, the net force exerted on it is zero.



Represent mathematically Using the upward direction as positive, apply the vertical component form of Newton's second law:

$$\Sigma F_y = F_{\text{W on S}} + (-F_{\text{E on S}}) = 0$$
$$\rho_{\text{water}} g V_{\text{subm}} - N_{\text{people}} m_{\text{person}}\, g = 0$$

Assuming that all people have the same mass, we find the number of people:

$$N_{\text{people}} = \frac{\rho_{\text{water}} V_{\text{subm}} g}{m_{\text{person}} g} = \frac{\rho_{\text{water}} V_{\text{subm}}}{m_{\text{person}}}$$

Solve and evaluate

$$N_{\text{people}} = \frac{\rho_{\text{water}} V_{\text{subm}}}{m_{\text{person}}}$$
$$= \frac{(1025\ \text{kg/m}^3)(0.36\ \text{m}\ \times 2.0\ \text{m}\ \times 3.0\ \text{m})}{75\ \text{kg}} = 29.5$$

The raft can precariously hold 29 passengers, which is a reasonable number. The number is inversely proportional to the mass of a person. This makes sense—the heavier the people, the fewer of them the raft should hold. The units, dimensionless, also make sense. We assumed that the raft has negligible mass. If we take the mass into account, the number of people will be smaller.

Try it yourself Suppose that 10 people of average mass 80 kg entered the raft. Now how far would the water line be from the top of the raft?

Answer

The raft would sink 0.13 m into the water, and the water line would be 0.23 m below the top of the raft.

Ballooning

Balloons used for transportation are filled with hot air. Why hot air? The density of 100 °C air is 0.73 times the density of 0 °C air. Thus, balloonists can adjust the average density of the balloon (the balloon's material, people, equipment, etc.) to match the density of air so that the balloon can float at any location in the atmosphere (up to certain limits). A burner under the opening of the balloon regulates the temperature of the air inside the balloon and hence its volume and density. This allows control over the buoyant force that the outside cold air exerts on the balloon. The same approach that we used in Example 13.9 allows us to predict that a balloon with radius of 5.0 m and a mass of 20.0 kg filled with air at the temperature of 100 °C can carry about 160 kg (three slim 53-kg people or two medium-mass 80-kg people). This is not a heavy load.

At one time, hydrogen was used in closed balloons instead of air. The density of hydrogen is 1/14 times the density of air. Unfortunately, hydrogen can burn explosively in the presence of oxygen. The hydrogen-filled Hindenburg, a German airship, caught fire and exploded in 1937, killing 36 people (see **Figure 13.15**). Balloonists then turned to helium—an inert gas that does not interact readily with other types of atoms.

FIGURE 13.15 The Hindenburg explosion.

Effects of altitude on humans

Table 13.5 on the next page shows that atmospheric pressure decreases with altitude. Therefore, climbers and balloonists have to guard against altitude sickness, caused by the low pressure and lack of oxygen. Below 3000 m, altitude has little effect on performance. Between 3000 m and 4600 m, climbers experience compensated hypoxia—increased heart and breathing rates. Between 4600 m and 6000 m, manifest hypoxia sets in. Heart and breathing rates increase dramatically, and cognitive and sensory function and muscle control decline. Climbers may feel lethargy and euphoria and even experience hallucinations. Between 6000 m and 8000 m, climbers undergo critical hypoxia, characterized by rapid loss of muscular control, loss of consciousness, and possibly death.

These symptoms were exhibited clearly on April 15, 1875 by three French balloon pioneers attempting to set an altitude record. They carried bags of oxygen with them, but as their elevation increased, slowly lost the mental awareness needed to use the bags. Instruments indicate that the balloon reached a maximum elevation of 8600 m twice. During the second time, two of the balloonists died. The third lost consciousness but survived.

TABLE 13.5 Pressure of air and pressure due to oxygen in the air (called partial pressure) at different elevations

Location	Elevation (m)	P_{air} (atm)	P_{oxygen} (atm)
Sea level	0	1.0	0.21
Mount Washington	1917	0.93	0.18
Pikes Peak	4301	0.59	0.12
Mount McKinley	6190	0.47	0.10
Mount Everest	8848	0.34	0.07
Jet travel	12,000	0.23	0.05

Scuba diving

The sport of scuba diving depends on an understanding of fluid pressure and buoyant force to avoid cases of excess internal pressure, oxygen overload, and decompression sickness.

Assuming that the surface area of your body is about $2\ m^2$, the air exerts a 200,000-N force (about 20 tons) on the surface of your body. Fortunately, fluids inside the body push outward and balance the force exerted by the outside air. For example, the pressure inside your lungs is approximately atmospheric pressure. What would happen if the fluid pressure on the inside remained constant while the pressure on the outside doubled or tripled? Would you be crushed, the way a can or barrel is crushed by outside air pressure when the air pressure inside the can is much lower than the pressure outside (**Figure 13.16**)? Scuba divers face this problem.

FIGURE 13.16 External air pressure collapses an evacuated can.

When air is pumped out of a can, outside air pressure causes it to collapse.

We know that atmospheric pressure (1 atm) is equivalent to the pressure of a 10-m column of water. Therefore, at depth $d = 10$ m, the water pressure is 2 atm. At 40 m below the water surface, the pressure is about 5 atm. This would surely be a problem for a scuba diver if the internal pressure were only 1 atm!

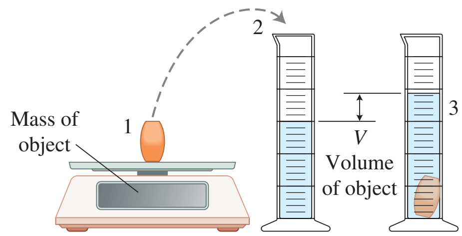
To avoid this problem, divers breathe compressed air. While moving slowly downward, a diver adjusts the pressure outlet from the compressed air tank in order to accumulate gas from the cylinder into her lungs and subsequently into other body parts, increasing the internal pressure to balance the increasing external pressure. If a diver returns to the surface too quickly, the great gas pressure in the lungs can force bubbles of gas into the bloodstream. These bubbles can behave like blood clots, blocking blood flow to the brain and possibly causing death. Blood vessels can rupture if the pressure difference between the inside and outside of the vessel is too great. Thus, a diver rises to the surface slowly so that pressure changes gradually and bubbles of gas do not form. This gradual process is called *decompression*.

When humans travel to dangerous environments (mountaintops, the deep sea, or outer space), physics intersects with human physiology. A careful understanding of gases, liquids, and the effects of changing pressures on the human body is needed to allow humans to survive in these places. As we explore the universe, we will need to learn to adapt to ever more challenging environments.

REVIEW QUESTION 13.7 A ship's waterline marks the maximum safe depth of the ship in the water when it has a full cargo. An empty ship is at the dock with its waterline somewhat above the water level. How could you estimate its maximum cargo?

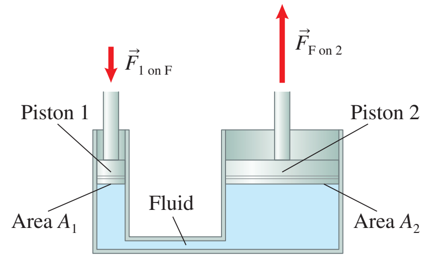
Summary

Density ρ The ratio of the mass m of a substance divided by the volume of that substance. (Section 13.1)



$$\rho = \frac{m}{V}$$ Eq. (13.1)

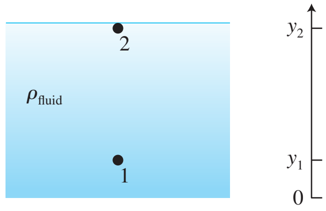
Pascal's first law—hydraulic lift An increase in the pressure in one part of an enclosed fluid increases the pressure throughout the fluid. In a hydraulic lift, a small force F_1 exerted on a small piston of area A_1 can cause a large force F_2 to be exerted on a large piston of area A_2. (Section 13.2)



For a hydraulic lift:

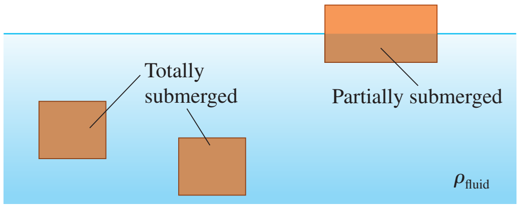
$$F_{\text{F on 2}} = PA_2 = (A_2/A_1)\,F_{\text{1 on F}}$$ Eq. (13.2)

Pascal's second law—variation of pressure with depth On a vertical upward-pointing y-axis, the pressure of a fluid P_1 at position y_1 depends on the pressure P_2 at position y_2 and on the density of the fluid. (Section 13.3)



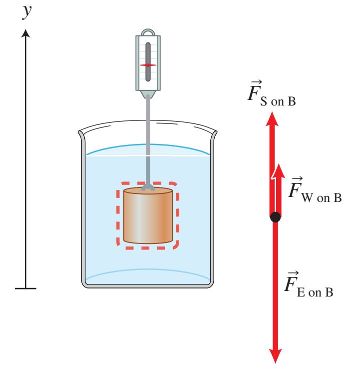
$$P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g$$ Eq. (13.3)

Buoyant force A fluid exerts an upward-pointing buoyant force on an object totally or partially immersed in the fluid. The force depends on the density of the fluid and on the volume of the fluid displaced. Once the object is totally submerged, the buoyant force does not change. (Section 13.5)



$$F_{\text{F on O}} = \rho_{\text{fluid}} V_{\text{displaced}}\, g$$ Eq. (13.5)

Newton's second law Use the standard problem-solving strategies with this law (sketches, force diagrams, math descriptions) to find some unknown quantity. The problems often involve the buoyant force, pressure, and density. (Section 13.6)



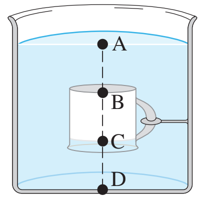
$$a_y = \frac{\Sigma F_y}{m}$$ Eq. (3.7y)

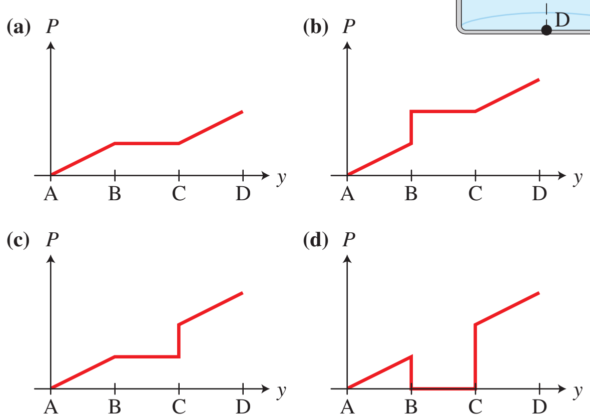
Questions

Multiple Choice Questions

1. Rank in increasing order the pressure that the *italicized objects* exert on the surface.
   I. A person standing with *bare feet* on the floor
   II. A person in *skis* standing on snow
   III. A person in *Rollerblades* standing on a road
   IV. A person in *ice skates* standing on ice
   (a) I, II, III, IV (b) IV, III, I, II (c) IV, III, II, I
   (d) III, II, IV, I (e) II, I, III, IV
2. Choose a device that reduces the pressure caused by a force.
   (a) Scissors (b) Knife (c) Snowshoes
   (d) Nail (e) Syringe
3. What does it mean if the density of a material equals 2000 kg/m^3?
   (a) The mass of the material is 2000 kg.
   (b) The volume of the material is 1 m^3.
   (c) The ratio of the mass of any amount of this material to the volume is equal to 2000 kg/m^3.
4. An upside-down mug with some air trapped in it is fixed under water, as shown in **Figure Q13.4**. Which qualitative pressure-versus-position graph correctly shows how the gauge pressure changes along the dashed line through the mug from A to D?

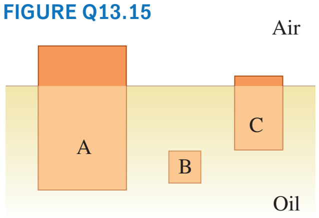
FIGURE Q13.4





5. If you hold a cylinder vertically, what is the net force exerted by the atmospheric pressure on it?
   (a) Downward (b) Upward (c) Zero
6. How do we know that a fluid exerts an upward force on an object submerged in the fluid?
   (a) Fluid pushes on the object in all directions.
   (b) The reading of a scale supporting the object when submerged in the fluid is less than when not in the fluid.
   (c) The fluid pressure on the bottom of the object is greater than the pressure on the top.
   (d) Both b and c are correct.
7. When you suspend an object from a spring scale, it reads 15 N. Then you place the same object and scale under a vacuum jar and pump out the air. What happens to the reading of the scale?
   (a) It increases slightly.
   (b) It decreases slightly.
   (c) It says the same.
   (d) Don't have enough information to answer
8. Why can't a suction pump lift water higher than 10.3 m?
   (a) Because it does not have the strength to pull up higher
   (b) Because the atmospheric pressure is equal to the pressure created by a 10.3-m-high column of water
   (c) Because suction pumps are outdated lifting devices
   (d) Because most suction cups have an opening to the bulb that is too narrow
9. If Torricelli had a wider tube in his mercury barometer, what would the height of the mercury column in the tube do?
   (a) Decrease (b) Increase (c) Stay the same
10. A wooden cube is floating in a fish tank that is filled with water. Imagine that you take this setup to a space station on the Moon. Air pressure and temperature inside the station are similar to conditions on Earth. After bringing the setup from Earth to the space station, you will observe that (choose all correct statements)
    (a) the amount of water displaced by the cube increases.
    (b) the amount of water displaced by the cube decreases.
    (c) the amount of water displaced by the cube stays the same.
    (d) the buoyant force exerted on the cube increases.
    (e) the buoyant force exerted on the cube decreases.
    (f) the buoyant force exerted on the cube stays the same.
    (g) the pressure at the bottom of the fish tank is about 1/6 of the value on Earth.
    (h) the pressure at the bottom of the fish tank is significantly less than 1/6 of the value on Earth.
    (i) the pressure at the bottom of the fish tank is slightly less than the value on Earth.
    (j) the pressure at the bottom of the fish tank is the same as on Earth.
11. Two identical beakers with the same amount of water sit on the arms of an equal arm balance. A wooden block floats in one of them. What does the scale indicate?
    (a) The beaker with the block is heavier.
    (b) The beaker without the block is heavier.
    (c) The beakers weigh the same.
12. A piece of steel and a bag of feathers are suspended from two spring scales in a vacuum. Each scale reads 100 N. What happens when you repeat the experiment outside under normal conditions?
    (a) The scale with feathers reads more than the scale with steel.
    (b) The scale with feathers reads less that the scale with steel.
    (c) The scales have the same reading, but the reading is less than the reading in a vacuum.
13. A metal boat floats in a pool. What happens to the level of the water in the pool if the boat sinks?
    (a) It rises.
    (b) It falls.
    (c) It stays the same.
14. When a boat sails from seawater to fresh water, the buoyant force exerted on the boat
    (a) decreases. (b) increases. (c) stays the same.
15. Three blocks are floating in oil as shown in **Figure Q13.15**. Which block has the highest density?
    (a) A (b) B (c) C
    (d) All blocks have the same density.
16. Three blocks are floating in oil as shown in Figure Q13.15. On which block does the oil exert the greatest buoyant force?
    (a) A (b) B (c) C
    (d) The oil exerts the same force on all of them.

FIGURE Q13.15

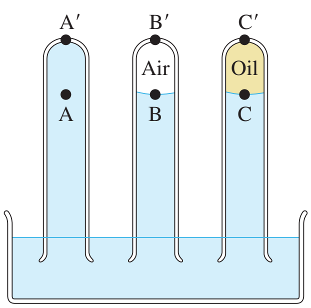


Conceptual Questions

17. Describe a method to measure the density of a liquid.
18. How can you determine the density of air?
19. Design an experiment to determine whether air has mass.
20. Does air exert a net upward force or a net downward force on an object submerged in the air? How can you test your answer experimentally?
21. What causes the pressure that air exerts on a surface that is in the air?
22. Why, when you fill a teapot with water, is the water always at the same level in the teapot and in the spout?
23. What experimental evidence supports Pascal's first law?

24. Fill a plastic cup to the very top with water. Put a piece of paper on top of the cup so that the paper covers the cup at the edges and is not much bigger than the surface of the cup. Turn the cup and paper upside down (practice over the sink first) and hold the bottom of the cup (now on the top). Why doesn't the water fall out of the cup?
25. Why does a fluid exert a net upward force on an object submerged in the fluid?
26. Describe how you could predict whether an object will float or sink in a particular liquid without putting it into the liquid.
27. Why can you lift objects while in water that are too heavy to lift when in the air?
28. When placed in a lake, a solid object either floats on the surface or sinks. It does not float at some intermediate location between the surface and the bottom of the lake. However, a weather balloon floats at some intermediate distance between Earth's surface and the top of its atmosphere. Explain.
29. A flat piece of aluminum foil sinks when placed under water. Take the same piece and shape it so that it floats in the water. Explain why the method worked.
30. Ice floats in water in a beaker. Will the level of the water in the beaker change when the ice melts? Explain.
31. The density of ice at 0 °C is less than the density of water at 0 °C. How is this related to the existence of life on Earth?
32. How would you determine the density of an irregular-shaped unknown object if (a) it sinks in water and (b) it floats in water? List all the steps and explain the reasoning behind them.
33. Why do people sink in fresh water and in most seawater (if they do not make an effort to stay afloat) but do not sink in the Dead Sea?
34. A bucket filled to the top with water has a piece of ice floating in it. Will the pressure on the bottom of the bucket change when the ice melts? Explain.
35. Marjory thinks that the mass of a fluid above a certain level should affect the pressure at this level. Describe how you will test her idea.
36. You are holding a brick that is completely submerged in water. Draw a force diagram for the brick. Why does it feel lighter in water than when you hold it in the air?
37. A bucket filled with water has a piece of ice floating in it. Will the level of water rise when the ice melts? Justify your answer.
38. Explain qualitatively and quantitatively how we drink through a straw. Make sure you can account for the water going up the length of the straw.
39. Three test tubes are inverted in a Petri dish as shown in **Figure Q13.39**. The first is completely filled with water, the second has an air bubble at the top, and the third has oil at the same level as the air bubble. (a) Rank the pressures at points A, B, and C from largest to smallest. (b) Rank the pressures at points A′, B′, and C′ from largest to smallest.

FIGURE Q13.39



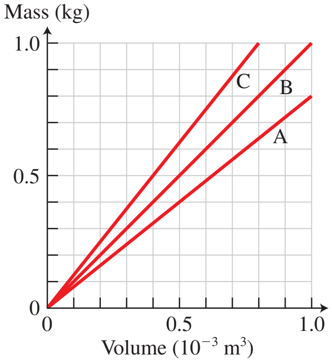
Problems

Below, **BIO** indicates a problem with a biological or medical focus. Problems labeled **EST** ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

13.1 Density

1. Determine the average density of Earth. What data did you use? What assumptions did you make?
2. * **EST** **Height of atmosphere** Use data for the normal pressure and the density of air near Earth's surface to estimate the height of the atmosphere, assuming it has *uniform* density. Indicate any additional assumptions you made. Are you on the low or high side of the real number?
3. **EST** A single-level home has a floor area of 200 m^2 with ceilings that are 2.6 m high. Estimate the mass of the air in the house.
4. * **BIO** A diet decreases a person's mass by 5%. Exercise creates muscle and reduces fat, thus increasing the person's density by 2%. Determine the percent change in the person's volume.
5. **Pulsar density** A pulsar, an extremely dense rotating star made of neutrons, has a density of $10^{18}\ kg/m^3$. Determine the mass of a pulsar contained in a volume the size of your fist (about 200 cm^3).
6. Use the graph lines in **Figure P13.6** to determine the densities of the three liquids, A, B, and C, in SI units. If you place them in one container, how will they position themselves? How does the density of each liquid change as its volume increases? As its mass decreases? Compare the masses of the three liquids when they occupy the same volume. Compare the volumes of the three liquids when they have the same mass.

FIGURE P13.6



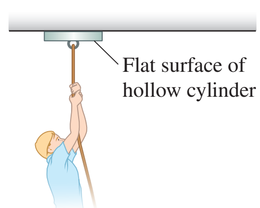
7. Imagine that you have gelatin cut into three cubes: the side of cube A is *a* cm long, the side of cube B is double the side of A, and the side of cube C is three times the side of A. Compare the following properties of the cubes: (a) density, (b) volume, (c) surface area, (d) cross-sectional area, and (e) mass.
8. An object made of material A has a mass of 90 kg and a volume of 0.45 m^3. If you cut the object in half, what would be the density of each half? If you cut the object into three pieces, what would be the density of each piece? What assumptions did you make?
9. You have a steel ball that has a mass of 6.0 kg and a volume of $3.0 \times 10^{-3}\ m^3$. How can this be?
10. * A material is made of molecules of mass 2.0×10^{-26} kg. There are 2.3×10^{29} of these molecules in a 2.0-m^3 volume. What is the density of the material?
11. You compress all the molecules described in Problem 13.10 into 1.0 m^3. Now what is the density of the material? What type of material could possibly behave this way?
12. * Bowling balls are heavy. However, some bowling balls float in water. Use available resources to find the dimensions of a bowling ball and explain why some balls float while others do not.
13. * **EST** Estimate the average density of a glass full of water and then the glass when the water is poured out (do not forget the air that now fills the glass instead of water).

13.2 Pressure inside a fluid

14. * Anita holds her physics textbook and complains that it is too heavy. Andrew says that her hand should exert no force on the book because the atmosphere pushes up on it and balances the downward pull of Earth on the book (the book's weight). Jim disagrees. He says that the atmosphere presses down on things and that is why they feel heavy. Who is correct? Approximately how large is the force that the atmosphere exerts on the bottom of the book? Why does this force not balance the force exerted by Earth on the book?

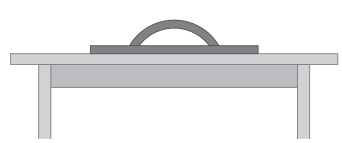
15. * The air pressure in the tires of a 980-kg car is $3.0 \times 10^5\ \mathrm{N/m^2}$. Determine the average area of contact of each tire with the road. Indicate any assumptions that you made.
16. * EST Estimate the pressure that you exert on the floor while wearing hiking boots. Now estimate the pressure under each heel if you change into high-heeled shoes. Indicate any assumptions you made.
17. **Hydraulic car lift** You are designing a hydraulic lift for a machine shop. The average mass of a car it needs to lift is about 1500 kg. What should be the specifications on the dimension of the pistons if you wish to exert a force on a smaller piston of not more than 500 N? How far down will you need to push the piston in order to lift the car 30 cm?
18. EST **Force of air on forehead** Estimate the force that air exerts on your forehead. Describe the assumptions you made.
19. * EST A 30-cm-diameter cylindrical iron plunger is held against the ceiling, and the air is pumped from inside it. A 72-kg person hangs by a rope from the plunger (Figure P13.19). List the quantities that you can estimate about the situation and estimate them. Make assumptions if necessary.

FIGURE P13.19



20. You have a rubber pad with a handle attached to it (Figure P13.20). If you press the pad firmly on a smooth table, it is impossible to lift it off the table. Why? What force would you need to exert on the handle to lift it? The surface area of the pad is $0.023\ \mathrm{m^2}$.

FIGURE P13.20

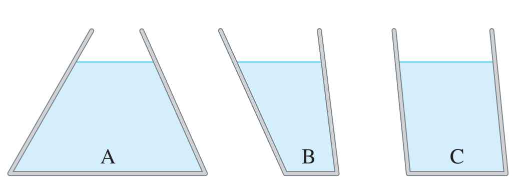


21. * EST **Toy bow and arrow** A child's toy arrow has a suction cup on one end. When the arrow hits the wall, it sticks. Draw a force diagram for the arrow stuck on the wall and estimate the magnitudes of the forces exerted on it when it is in equilibrium. The mass of the arrow is about 10 g. Why are the words "suction cup" not appropriate?

13.3 Pressure variation with depth

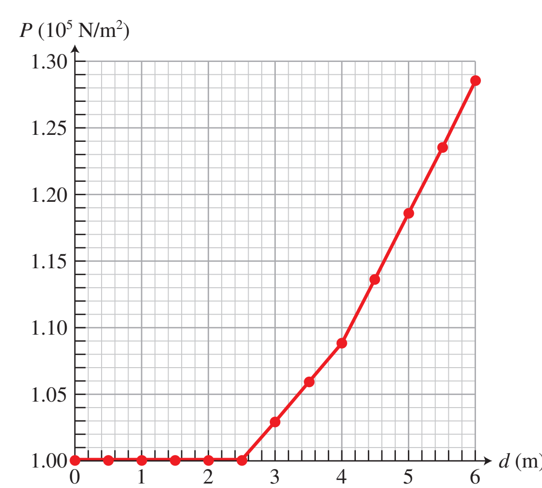
22. * **Pressure on the Titanic** The Titanic rests 4 km (2.5 miles) below the surface of the ocean. What physical quantities can you determine using this information?
23. You have three reservoirs (Figure P13.23). Rank the pressures at the bottom of each and explain your rankings. Then rank the net force that the water exerts on the bottom of each reservoir. Explain your rankings.

FIGURE P13.23



24. **Water reservoir and faucet** The pressure at the top of the water in a city's gravity-fed reservoir is $1.0 \times 10^5\ \mathrm{N/m^2}$. Determine the pressure at the faucet of a home 42 m below the reservoir.
25. **Dutch boy saves Holland** An old story tells of a Dutch boy who used his fist to plug a 2.0-cm-diameter hole in a dike that was 3.0 m below sea level, thus preventing the flooding of part of Holland. What physical quantities can you determine from this information? Determine them.
26. BIO EST **Blood pressure** Estimate the pressure of the blood in your brain and in your feet when standing, relative to the average pressure of the blood in your heart of $1.3 \times 10^4\ \mathrm{N/m^2}$ above atmospheric pressure.
27. * BIO **Intravenous feeding** A glucose solution of density $1050\ \mathrm{kg/m^3}$ is transferred from a collapsible bag through a tube and syringe into the vein of a person's arm. The blood pressure in the arm exceeds the atmospheric pressure by $1400\ \mathrm{N/m^2}$. How high above the arm must the top of the liquid in the bottle be so that the pressure in the glucose solution at the needle exceeds the pressure of the blood in the arm?
28. * **Mountain climbing** Determine the change in air pressure as you climb from elevation of 1650 m at the timberline of Mount Rainier to its 4392-m summit, assuming an average air density of $0.82\ \mathrm{kg/m^3}$. Will the real change be more or less than the one you calculated? Explain.
29. BIO EST **Giraffe raises head** Estimate the pressure change of the blood in the brain of a giraffe when it lifts its head from the grass to eat a leaf on an overhead tree. Without special valves in its circulatory system, the giraffe could easily faint when lifting its head.
30. * A truck transporting chemicals has crashed, and some dangerous liquid has spilled onto the ground and possibly entered a water well. An inspector fixes a pressure sensor to the end of a long string and lets the sensor slowly descend from the top of the well to the bottom. Using this device, he obtains the graph in Figure P13.30 that shows how the pressure P in the well changes with distance d measured from the top of the well. (a) Explain what features of the graph support the idea that there is another liquid in the well in addition to water. (b) Determine the density of the unknown liquid. Is the liquid above the water or below the water? (c) Determine the depth of the water, the depth of the unknown liquid, and the depth of the well.

FIGURE P13.30

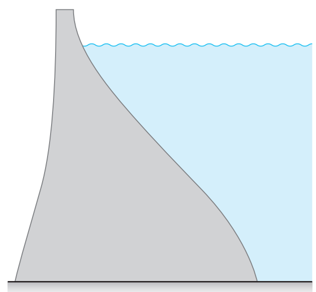


31. **Drinking through a straw** You are drinking water through a straw in an open glass. Select a small volume of water in the straw as a system and draw a force diagram for the water inside this volume that explains why the water goes up the straw.
32. * **More straw drinking** While you are drinking through the straw, the pressure in your mouth is 30 mm Hg below atmospheric pressure. What is the maximum length of a straw in an open glass that you can use to drink a fruit drink of density $1200\ \mathrm{kg/m^3}$?
33. * Your office has a $0.020\ \mathrm{m^3}$ cylindrical container of drinking water. The radius of the container is about 14 cm. When the container is full, what is the gauge pressure that the water exerts on the sides of the container halfway down from the top? All the way down?
34. * BIO EST **Eardrum** Estimate the net force on your 0.5-cm^2 eardrum that air exerts on the inside and the outside after you drive from Denver, Colorado (elevation 1609 m) to the top of Pikes Peak (elevation 4301 m). Assume that the air pressure inside and out are balanced when you leave Denver and that the average density of the air is $0.80\ \mathrm{kg/m^3}$. What other assumptions did you make?
35. BIO **Eardrum again** You now go snorkeling. What is the net force exerted on your eardrum when you are 2.4 m under the water, assuming the pressure was equalized before the dive?

36. Water and oil are poured into opposite sides of an open U-shaped tube. The oil and water meet at the exact center of the U at the bottom of the tube. If the column of oil of density 900 kg/m^3 is 16 cm high on one side, how high is the water on the other side?

37. * Examine the vertical cross section of the Hoover Dam shown in **Figure P13.37**. Explain why the dam is thicker at the bottom than at the top.

FIGURE P13.37

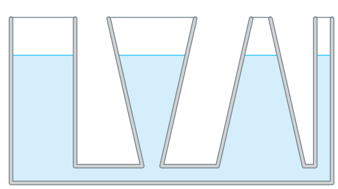


38. * A test tube of length L and cross-sectional area A is submerged in water with the open end down so that the edge of the tube is a distance h below the surface. The water goes up into the tube so its height inside the tube is l. Describe how you can use this information to decide whether the air that was initially in the tube obeys the mathematical description for an isothermal process (Boyle's law). List your assumptions.

39. Half of a 20-cm-tall beaker is filled with glycerol (density 1259 kg/m^3) and the other half with olive oil (density 800 kg/m^3). (a) Draw a graph that shows how the density of the liquids in the beaker changes with the distance from the surface to the bottom of beaker. (b) Draw a graph that shows how the pressure in the liquids changes with the distance from the surface to the bottom of the beaker.

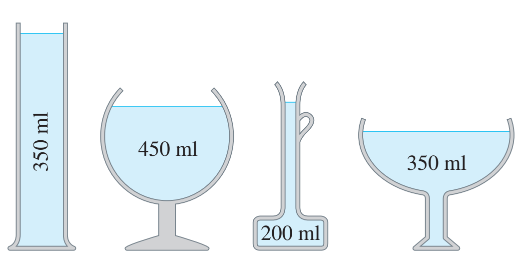
40. Blaise Pascal found a seemingly paradoxical situation when he poured water into the apparatus shown in **Figure P13.40**. The water level was the same in all four parts of the apparatus despite differences in the shapes of the parts and the masses of water in each part. Explain qualitatively the outcome of Pascal's experiment.

FIGURE P13.40



41. Four containers are filled with different volumes of water as shown in **Figure P13.41**. Rank the containers in order of decreasing pressure that the water exerts on the bottom of the containers.

FIGURE P13.41



42. **Venus pressure and underwater pressure** Atmospheric pressure on Venus is $9.0 \times 10^6\ N/m^2$. How deep underwater on Earth would you have to go to feel the same pressure?

13.4 Measuring atmospheric pressure

43. The reading of a barometer in your room is 780 mm Hg. What does this mean? What is the pressure in pascals?

44. How long would Torricelli's barometer have had to be if he had used oil of density 950 kg/m^3 instead of mercury?

45. Sometimes gas pressure is measured with a device called a *liquid manometer* (**Figure P13.45**). Explain how this instrument can be used to measure the pressure of gas in a bulb attached to one of the tubes.

FIGURE P13.45

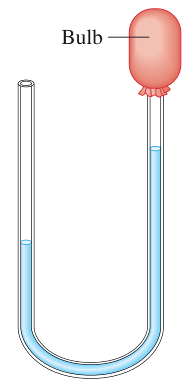
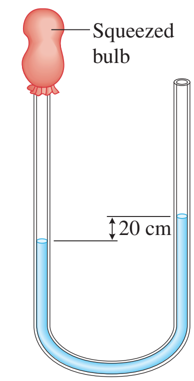


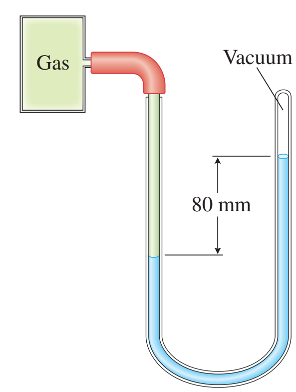
FIGURE P13.46



46. You use a liquid manometer with water to measure the pressure inside a rubber bulb. Before you squeeze the bulb, the water is at the same level in both legs of the tube. After you squeeze the bulb, the water in the opposite leg rises 20 cm with respect to the leg connected to the bulb (**Figure P13.46**). What is the pressure in the bulb? What assumptions did you make? How will the answer change if the assumptions are not valid?

47. * In a mercury-filled manometer (**Figure P13.47**), the open end is inserted into a container of gas and the closed end of the tube is evacuated. The difference in the height of the mercury is 80 mm. The radius of the connecting tube is 0.50 cm. (a) Determine the pressure inside the container in newtons per square meter. (b) An identical manometer has a connecting tube that is twice as wide. If the difference in the height of the mercury is the same, then what is the pressure in the container?

FIGURE P13.47



48. Examine the reading of the manometer that you use to measure the pressure inside car tires. What are the units? Does the manometer measure the absolute pressure of the air inside the tires or gauge pressure? How do you know?

13.5 Buoyant force

49. Draw a force diagram for an object that is floating at the surface of a liquid. Is the force exerted by air on the object included in your diagram? Explain.

50. Draw a cubic object that is completely submerged in a fluid but not resting on the bottom of the container. Then draw arrows to represent the forces exerted by the fluid on the top, sides, and bottom of the object. Make the arrows the correct relative lengths. What is the direction of the total force exerted by the fluid on the object?

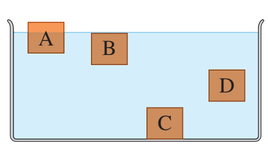
51. Three people are holding three identical sealed metal containers, ready to release them. Container A is filled with air at atmospheric pressure and room temperature, container B is filled with helium at atmospheric pressure and room temperature, and container C is evacuated (all air has been pumped out). Draw force diagrams for the containers after they are released.

52. * Four cubes of the same volume are made of different materials: lead (density 11,300 kg/m^3), aluminum (density 2700 kg/m^3), wood (density 800 kg/m^3), and Styrofoam (density 50 kg/m^3). You place the cubes in a large container filled with water. Rank the buoyant forces that the water exerts on the cubes from largest to smallest.

53. * You place four identical cubes made of oak (density 900 kg/m^3) in water, olive oil (density 880 kg/m^3), alcohol (density 790 kg/m^3), and mercury (density 13,600 kg/m^3). Rank the buoyant forces that the liquids exert on the cubes from largest to smallest.

54. ** You fill a 20-cm-tall container with glycerol so that the glycerol (density 1260 kg/m^3) reaches the 10-cm mark. You place an oak cube (density 900 kg/m^3) into the container. Each side of the cube is 10 cm. (a) What is the distance x from the upper face of the cube to the glycerol surface assuming the cube is in an upright position? (b) Make a prediction using qualitative reasoning (without mathematics) about what will happen to the cube in (a) if you now add olive oil (density 800 kg/m^3) to the container until it is completely full. Will the distance x decrease, increase, or stay the same? (c) Determine the new distance x using mathematics (physics laws) and compare your result with the prediction in (b).
55. * A 30-g ball with volume 37.5 cm^3 is attached to the bottom of a glass beaker with a light string. When the beaker is filled with water, the ball floats fully submerged under the water surface. Draw a force diagram for the ball and determine the force exerted on the ball by the string.
56. ** You have a ball (volume V, average density ρ_B) and a glass beaker. The ball is attached to the bottom of the beaker with a light spring (coefficient k). When you fill the beaker with a liquid of density ρ_L, the ball floats fully submerged under the surface with the spring extended by x. Draw a force diagram for the ball and derive the expression for x in terms V, ρ_B, ρ_L, and k. Evaluate the expression using unit analysis and limiting case analysis.
57. * This textbook says that the upward force that a fluid exerts on a submerged object is equal in magnitude to the product of the density of the fluid, the gravitational constant g, and the volume of the submerged part of the object. Where did this equation come from?
58. * **Design** This textbook says that the upward force that a fluid exerts on a submerged object is equal in magnitude to the product of the density of the fluid, the gravitational constant g, and the volume of the submerged part of the object. Design an experiment to test this expression, including a prediction about the outcome of the experiment.
59. * You have four objects at rest, each of the same volume. Object A is partially submerged, and objects B, C, and D are totally submerged in the same container of liquid, as shown in **Figure P13.59**. Draw a force diagram for each object. Rank the densities of the objects from least to greatest and indicate whether any objects have the same density.

FIGURE P13.59

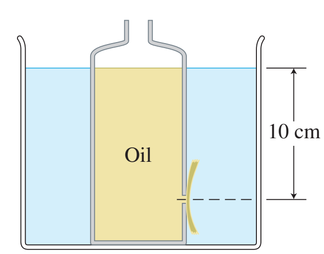


60. * **Does air affect what a scale reads?** A 60-kg woman with a density of 980 kg/m^3 stands on a bathroom scale. Determine the reduction of the scale reading due to air.
61. * When analyzing a sample of ore, a geologist finds that it weighs 2.00 N in air and 1.13 N when immersed in water. What is the density of the ore? What assumptions did you make to answer the question? If the assumptions are not correct, how would the answer be different?
62. * A pin through a hole in the middle supports a meter stick. Two identical blocks hang from strings at an equal distance from the center so the stick is balanced. What happens to the stick if one block is submerged in water of density 1000 kg/m^3 and the other block in kerosene of density 850 kg/m^3?
63. * A meter stick is supported by a pin through a hole in the middle. (a) Two blocks made of the same material but different sizes hang from strings at different positions in such a way that the stick balances. What happens when the blocks hang entirely submerged in beakers of water? (b) Next you hang two blocks of different masses but the same volume at different positions so the stick balances. What happens when these blocks hang completely submerged in beakers of water? Support your answer for each part using force diagrams with arrows drawn with the correct relative lengths.

13.6 Skills for analyzing static fluid problems

64. **Goose on a lake** A 3.6-kg goose floats on a lake with 40% of its body below the 1000-kg/m^3 water level. Determine the average density of the goose.
65. ** **Floating in seawater** A person of average density ρ_1 floats in seawater of density ρ_2. What fraction of the person's body is submerged? Explain.
66. * **Floating in seawater** A person of average density 980 kg/m^3 floats in seawater of density 1025 kg/m^3. What can you determine using this information? Determine it.
67. ** (a) Determine the force that a vertical string exerts on a 0.80-kg rock of density 3300 kg/m^3 when it is fully submerged in water of density 1000 kg/m^3. (b) If the force exerted by the string supporting the rock increases by 12% when the rock is submerged in a different fluid, what is that fluid's density? (c) If the density of another rock of the same volume is 12% greater, what happens to the buoyant force the water exerts on it?
68. * **Snorkeling** A 60-kg snorkeler (including snorkel, mask, and other gear) displaces 0.058 m^3 of water when 1.2 m under the surface. Determine the magnitude of the buoyant force exerted by the 1025-kg/m^3 seawater on the person. Will the person sink or drift upward?
69. * A helium balloon of volume 0.12 m^3 has a total mass (the helium plus the balloon) of 0.12 kg. Determine the buoyant force exerted on the balloon by the air if the air has density 1.13 kg/m^3. Determine the initial acceleration of the balloon when released.
70. * BIO **Protein sinks in water** A protein molecule of mass 1.1×10^{-22} kg and density $1.3 \times 10^3 \text{ kg/m}^3$ is placed in a vertical tube of water of density 1000 kg/m^3. (a) Draw a motion diagram and a force diagram at the moment immediately after the molecule is released. (b) Determine the initial acceleration of the protein.
71. * How can you determine if a steel ball of known radius is hollow? List the equipment that you will need for the experiment, and describe the procedure and calculations. Can you determine how big the hollow part is if present in the ball?
72. ** **Crown composition** A crown is made of gold and silver. The scale reads its mass as 3.0 kg when in air and 2.75 kg when in water. Determine the masses of the gold and the silver in the crown. The density of gold is $19{,}300 \text{ kg/m}^3$ and that of silver is $10{,}500 \text{ kg/m}^3$.
73. * You place an open bottle filled with olive oil (density 880 kg/m^3) in a container filled with water so that the surfaces of both liquids are at the same level. The bottle has a hole in it 10 cm below the surface that is initially closed with adhesive tape (**Figure P13.73**). (a) Using Pascal's laws, predict what will happen when you remove the tape. Indicate any assumptions that you made. (b) Draw pressure-versus-depth graphs (similar to the graphs in Conceptual Exercise 13.3) for the states before and after you remove the tape. (c) If you predict any changes, determine their numerical values.

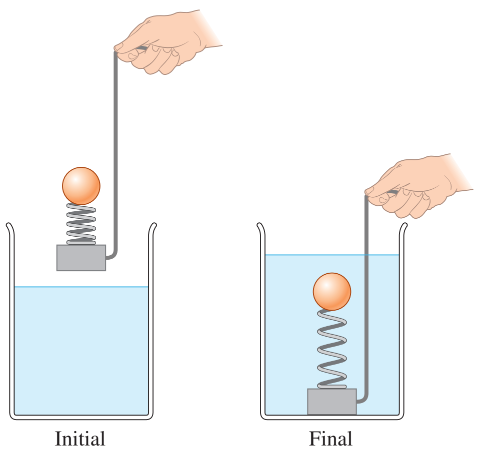
FIGURE P13.73



74. * You hold a ping-pong ball fully underwater (the initial state). When you release the ball, it jumps out of the water to a certain height above the surface. Let the final state be when the ball is just above the water surface, moving upward. Represent the process with energy bar charts (a) choosing the water, the ball, and Earth as the system, and (b) choosing the ball and Earth as the system. Indicate any assumptions that you made.
75. * You hang a steel ball on a string above a beaker that is filled to the top with water (the initial state). The beaker is sitting on a large empty tray. You slowly lower the ball until it reaches the bottom of the beaker. Some water spills over the rim of the beaker to the tray (the final state). (a) Represent the process with an energy bar chart choosing the water, the ball, and Earth as the system. (b) Repeat the analysis for a similar process with the same ball and the same amount of water, but a beaker tall enough so that no water spills over into the tray. Indicate any assumptions that you made.

76. * One end of a light spring is attached to a ping-pong ball, the other end to a heavy metal block that is fixed to a thin-wire holder (see **Figure P13.76**). You hold this setup so that the metal block is above the surface of water in a beaker (the initial state). You then slowly lower the setup until the metal block touches the bottom of the beaker (the final state). Represent the process with an energy bar chart, choosing the water, the ball, the spring, the block, and Earth as the system. Assume that the mass of the ball and the mass of the spring are much smaller than the mass of the block and so can be ignored. Indicate any other assumptions that you made.

FIGURE P13.76



13.7 Ships, balloons, climbing, and diving

77. * **Wood raft** Logs of density $600\ \text{kg/m}^3$ are used to build a raft. What is the maximum mass of the load that can be supported by a raft built from 300 kg of logs?
78. * EST **Standing on a log** A floating log is L long and d in diameter. What is the mass of a person who can stand on the log without getting her feet wet?
79. * **Ferryboat** A ferryboat is 12 m long and 8 m wide. Two cars, each of mass 1600 kg, ride on the boat for transport across the lake. How much farther does the boat sink into the water?
80. EST **Iceberg** Icebergs are large pieces of freshwater ice. Estimate the percentage of the volume of an iceberg that is underwater. Indicate any assumptions that you made.
81. * **Life preserver** A life preserver is manufactured to support a 70-kg person with 20% of his volume out of the water. If the density of the life preserver is $100\ \text{kg/m}^3$ and it is completely submerged, what must its volume be? List your assumptions.
82. ** To increase the effect of the buoyant force on a submarine, the crew replaces seawater in the ballast tanks with air. To push the seawater out of the tanks, they use compressed air from air flasks. The submarine is located 10 m below the surface in seawater with density $1030\ \text{kg/m}^3$ and temperature 6 °C. How many kilograms of air should the crew let out from the flasks to increase the difference between the gravitational force and the buoyant force exerted on the submarine by 5×10^6 N? Assume the temperature of air in the flasks is the same as the seawater temperature.

General Problems

83. * Compare the density of water at 0 °C to the density of ice at 0 °C. Suggest possible explanations in terms of the molecular arrangements inside the liquid and solid forms of water that would account for the difference. If necessary, use extra resources to help answer the question.
84. * **Collapsing star** The radius of a collapsing star destined to become a pulsar decreases by 10% while at the same time 12% of its mass escapes. Determine the percent change in its density.
85. * **Deep dive** The *Trieste* research submarine traveled 10.9 km below the ocean surface while exploring the Mariana Trench in the South Pacific, the deepest place in the ocean. Determine the force needed to prevent a 0.10-m-diameter window on the side of the submarine from imploding. Assume that the pressure inside the submarine is 1 atm and the density of the water is $1025\ \text{kg/m}^3$.
86. * EST **Bursting a wine barrel** Pascal placed a long 0.20-cm-radius tube in a wine barrel of radius 0.24 m. He sealed the barrel where the tube entered it. When he added wine of density $1050\ \text{kg/m}^3$ to the tube so the column of wine was 8.0 m high, the cover of the barrel burst off the top of the barrel. Estimate the net force that caused the cover to come off.
87. * BIO **Lowest pressure in lungs** Experimentally determine the maximum distance you can suck water up a straw. Use this number to determine the pressure in your lungs above or below atmospheric pressure while you are sucking. Be sure to indicate any assumptions you made and show clearly how you reached your conclusion.
88. ** **Measuring ocean depth with a soda bottle** You can use an empty soda bottle with a cap to measure the depth to which you dive in the ocean. Dive to a certain depth, turn the closed bottle upside down (with the cap toward the sea bottom), and open the cap. A certain volume of seawater will enter the bottle, compressing the air that was trapped in it. Keep the bottle in an upside-down position and carefully close the bottle. Then swim up to the surface and measure the volume V_{water} of water in the bottle and the total volume V_0 of the bottle. (a) Derive the expression for the depth to which you dived in terms of V_{water}, V_0, and other relevant parameters. Indicate any assumptions that you made. Evaluate the expression using unit analysis and limiting case analysis.
89. * Justin is observing pearl-like strings of bubbles that move upward in his father's glass of champagne. He notices that as the bubbles rise, their volume increases (see **Figure P13.89**). He proposes the following explanation: "The pressure inside the bubble is equal to the pressure in the liquid surrounding the bubble. Because the pressure in the liquid decreases toward the surface of the liquid, the pressure in the ascending bubble decreases and therefore the volume of the bubble increases." Justin estimates that the size of a bubble near the surface is approximately twice that of a bubble 10 cm below the surface. Can Justin reject his explanation based on these data? Explain. If you answered yes, propose a different explanation that is consistent with the data and describe how you could test it. (Hint: Champagne contains dissolved carbon dioxide.)

FIGURE P13.89

90. ** You have an empty water bottle. Predict how much mass you need to add to it to make it float half-submerged. Then add the calculated mass and explain any discrepancy that you found. How did you make your prediction?
91. ** BIO **Flexible bladder helps fish sink or rise** A 1.0-kg fish of density $1025\ \text{kg/m}^3$ is in water of the same density. The fish's bladder contains $10\ \text{cm}^3$ of air. The bladder compresses to $4\ \text{cm}^3$. Now what is the density of the fish? Will it sink or rise? Explain.
92. * **Plane lands on Nimitz aircraft carrier** When a 27,000-kg fighter airplane lands on the deck of the aircraft carrier Nimitz, the carrier sinks 0.25 cm deeper into the water. Determine the cross-sectional area of the carrier.
93. Derive an equation for determining the unknown density of a liquid by measuring the magnitude of a force $T_{\text{S on O}}$ that a string needs to exert on a hanging object of unknown mass m and density ρ to support it when the object is submerged in the liquid.

Reading Passage Problems

BIO Free diving So-called "no-limits" free divers slide to deep water on a weighted sled that moves from a boat down a vinyl-coated steel cable to the bottom of a dive site. The diver reaches depths where a soda can would implode. After reaching the target depth, the diver releases the sled and an air bag opens and brings the diver quickly back to the surface. The divers have no external oxygen supply—just lungs full of air at the start of the dive. In August 2002, Tanya Streeter of the Cayman Islands held the women's no-limits free dive record at 160 m. In 2005 Patrick Musimu set the men's record with a 209.6-m free dive in the Red Sea just off the Egyptian coast (the record was later broken by Herbert Nitsch of Austria).

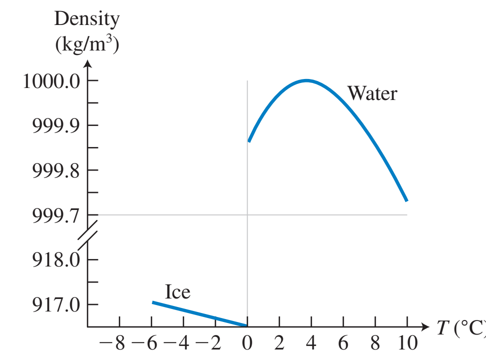
Musimu's 2005 dive took 3 minutes 28 seconds. He began the dive with his 9-L lungs full of air. By the time he passed the 200-m mark, Musimu's lungs had contracted to the size of a tennis ball. His body transferred blood from his limbs to essential organs such as the heart, lungs, and brain. This "blood shift" occurs when mammals submerge in water. Blood plasma fills the chest cavity, especially the lungs. Without this adaptation, the lungs would shrink and press against the chest walls, causing permanent damage. When he reached his target, Musimu released the weighted segment of the specialized sled that had taken him down and opened an airbag, which began his return to the surface at an average speed of 3–4 m/s.

94. Assuming Musimu weighs 670 N (150 lb) and is 1.6 m tall, 0.30 m wide, and 0.15 m thick, which answer below is closest to the magnitude of the force that the deep water exerted on one side of his body?
    (a) 0 (b) 670 N (130 lb)
    (c) 15,000 N (3000 lb) (d) 10^5 N (20,000 lb)
    (e) 10^6 N (200,000 lb)
95. Musimu's training allows him to hold up to 9 L $=$ 9000 cm^3 of air when in a 1 atm environment. Which answer below is closest to the volume of that air if at pressure 22 atm?
    (a) 100 cm^3 (b) 200 cm^3 (c) 400 cm^3
    (d) 9000 cm^3 (e) 2×10^5 cm^3
96. As Musimu descends, the buoyant force that the water exerts on him
    (a) remains approximately constant.
    (b) increases a lot because the pressure is so much greater.
    (c) decreases significantly because his body is being compressed and made much smaller.
    (d) is zero for the entire dive.
    (e) There is not enough information to answer the question.
97. Why don't his lungs, heart, and chest completely collapse?
    (a) The return balloon helps counteract the external pressure.
    (b) There is no external force pushing directly on the organs.
    (c) The sled that helps him descend protects the front of his body.
    (d) Blood plasma moves from his extremities to his chest and the organs in it.
    (e) The air originally in the lungs is transferred to the vital organs.

Lakes freeze from top down We all know that ice cubes float in a glass of water. Why? Virtually every substance contracts when it solidifies—the solid is denser than the liquid. If this happened to water, ice cubes would sink to the bottom of a glass, and ice sheets would sink to the bottom of a lake. Fortunately, this doesn't happen. Liquid water expands by 9% when it freezes into solid ice at 0 °C, from a liquid density of a little less than 1000 kg/m^3 to a solid density of 917 kg/m^3.

But this is not the only special thing about water. While the density of most substances increases when they are cooled, water density shows a very peculiar temperature dependence (see **Figure 13.17**). As the temperature decreases, water density increases, but only until 4 °C, when water density reaches its maximum value, 1000.0 kg/m^3. As the temperature decreases further, water density decreases until at 0 °C water freezes and density abruptly decreases to 917 kg/m^3.

FIGURE 13.17



It is this peculiar water-density-temperature dependence that plays a vital role in the survival of animals and plants that live in water. In the winter when the water in a lake freezes, the solid ice stays at the top, forming an ice sheet. Water just below the ice sheet cools, but when it reaches 4 °C it becomes the most dense and sinks to the bottom of the lake. Since water colder than 4 °C is less dense, it stays above, keeping the bottom of the lake at a constant 4 °C. Note that if water were most dense at the freezing point, then in the winter the very cold water at the surface of lakes would sink. In this case the lake would freeze from the bottom up, and all life in it would be destroyed.

The expansion of water when it freezes has another important environmental benefit: the so-called freeze-thaw effect on sedimentary rocks. Water is absorbed into cracks in these rocks and then freezes in cold weather. The solid ice expands and cracks the rock, like a wood-cutter splitting logs. This continual process of liquid water absorption, freezing, and cracking releases mineral and nitrogen deposits into the soil and can eventually break the rock down into soil.

98. When is water densest?
    (a) When liquid at 0 °C
    (b) When solid ice at 0 °C
    (c) When liquid at 4 °C
    (d) Water density is always 1000.0 kg/m^3.
99. Why does water freeze from the top down?
    (a) The denser water at 0 °C sinks to the bottom of the lake.
    (b) The less dense ice at 0 °C rises above the liquid water at 0 °C.
    (c) The denser water at 4 °C sinks to the bottom of the lake.
    (d) Because of both a and b
    (e) Because of both b and c
100. Using Newton's second law, expressions for buoyant force and other forces, and the densities of liquid and solid water at 0 °C, find the fraction of an iceberg or an ice cube that is under liquid water.
    (a) 0.84 (b) 0.88 (c) 0.92
    (d) 0.96 (e) 1.00
101. A swimming pool at 0 °C has a very large chunk of ice floating in it—like an iceberg in the ocean. When the ice melts, what happens to the level of the water at the edge of the pool?
    (a) It rises. (b) It stays the same.
    (c) It drops. (d) It depends on the size of the chunk.
102. Which of the following is/are benefits of the temperature dependence of the density of water?
    (a) Fish and plants can survive winters without being frozen.
    (b) Over time, soil is formed from sedimentary rocks.
    (c) Water pipes when frozen in the winter do not burst.
    (d) Two of the above three
    (e) All of the first three