14

Fluids in Motion

Plaque (fatty deposits) accumulates on the walls of arteries as cholesterol-laden blood flows by. As plaque grows, blood flows past at higher speed. If the blood is moving fast enough, it can dislodge deposits, which may then become lodged downstream and stop blood flow. If this stoppage occurs in the heart, it can cause a heart attack. In this chapter, we will learn why blood flows faster through an artery clogged with plaque and why the fast-moving stream of blood tends to pull the plaque off the artery wall.

- **How does blood flow dislodge plaque from an artery?**
- **Why can a strong wind cause the roof to blow off a house?**
- **Why do people snore?**

IN THE PREVIOUS CHAPTER, we investigated the behavior of static fluids. What happens when a gas or liquid moves across a surface—for example, when air moves across the roof of a house or when blood moves through a blood vessel? In this chapter, we will investigate and explain phenomena involving moving fluids—*fluid dynamics*.

BE SURE YOU KNOW HOW TO:

- Draw work-energy bar charts (Section 7.2).
- Apply the concept of pressure to explain the behavior of liquids (Section 13.2).
- Draw force diagrams and apply Newton's second law (Section 3.5).

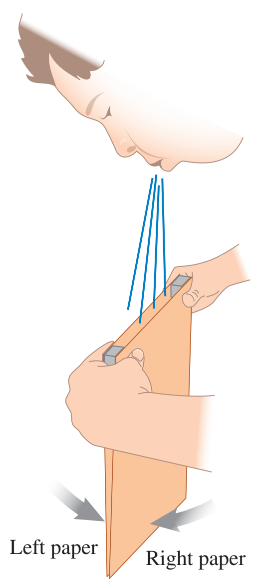
14.1 Fluids moving across surfaces—qualitative analysis

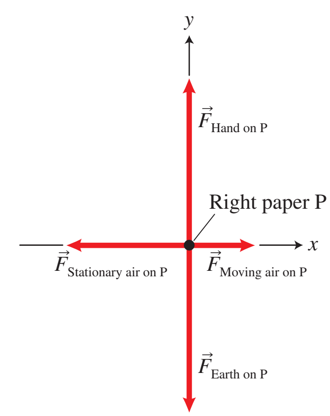
You learned in Chapters 12 and 13 that a key property of a static fluid is its pressure. What happens to that pressure when a fluid moves? To investigate this, we will analyze the simple experiment described in Observational Experiment **Table 14.1**.

OBSERVATIONAL EXPERIMENT TABLE 14.1 Pressure inside a moving fluid

VIDEO OET 14.1

Observational experiment	Analysis
Hold two pieces of paper separated by plastic blocks and blow down directly between them. You see the pieces of paper coming close together, as if they are pushed toward the moving air.	We consider the piece of paper on the right to be our system and examine the moment when the paper starts moving to the left. We draw forces exerted on the paper in the horizontal direction. For the system to start moving, the sum of the horizontal forces must point to the left—from the region of stationary air to the region of moving air.





Pattern

Based on the analysis of the experiment and using the force diagram, we can infer that air moving along the surface of an object exerts a smaller force on the object. Given that force divided by the area over which it is exerted is pressure, we conclude that the air moving along a surface exerts less pressure on the object than stationary air.

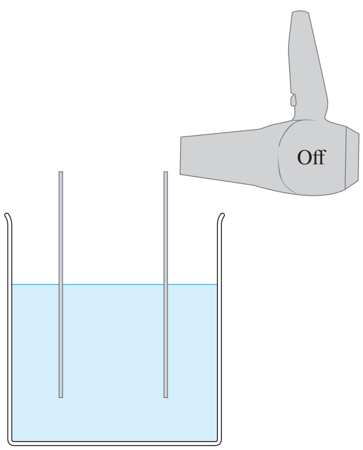
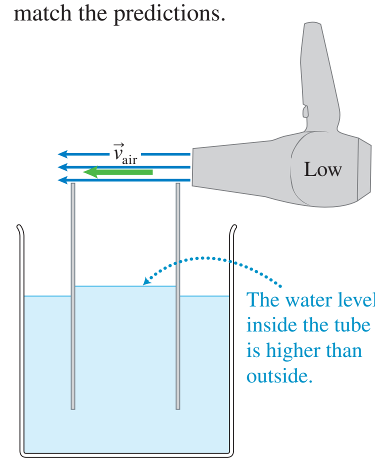
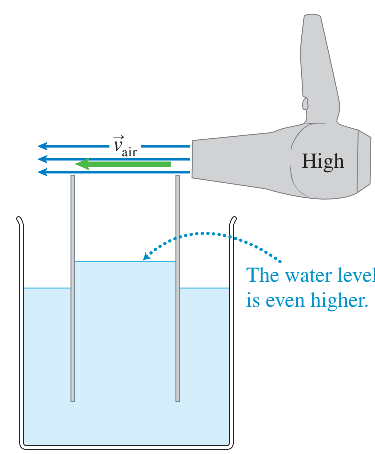
Bernoulli's effect

Extrapolating from the pattern in Table 14.1, could it be that for any fluid, the speed with which fluid is moving along the object and the pressure exerted by this fluid on the object are related: the greater the speed, the smaller the pressure? Let's test this hypothesis experimentally.

It is important to note here that in all of the experiments, the movement of the fluid was parallel to the surface of interest. When the fluid is moving perpendicular to the surface of interest, it exerts a force in the direction of motion (you can think of the momentum of fluid particles changing as the particles hit the surface). You can clearly observe this effect if you repeat the experiment in Testing Experiment **Table 14.2** with the hairdryer blowing directly downward into the tube—the water level inside the tube lowers.

TESTING EXPERIMENT TABLE 14.2 How are the speed of a fluid and its pressure related?

VIDEO TET 14.2

Testing experiment	Prediction	Outcome
We cut the top and bottom off a plastic bottle to make a plastic tube and put it into a container of water. The level of the water in the tube is the same as in the container. Then using a hair dryer set to cold, we blow cold air parallel to the surface of the water right above the tube, first using a low-speed setting and then a high-speed setting.  	If a faster-moving fluid exerts less pressure than a stationary fluid, and we blow air parallel to the opening of the tube, then the pressure at the top of the tube (but also inside the tube) should be lower than atmospheric pressure. When the speed of the moving air increases, the pressure should decrease. The pressure outside the tube, above the surface of the water in the container is atmospheric; therefore, the sum of the forces on an element of water inside the tube should point upward and the water in the tube should rise until the sum of the forces exerted on the raised water inside the tube is zero. The height of the water inside the tube should increase when the hair dryer is switched to high.	The outcomes are shown in the figures. They match the predictions.  

Conclusion

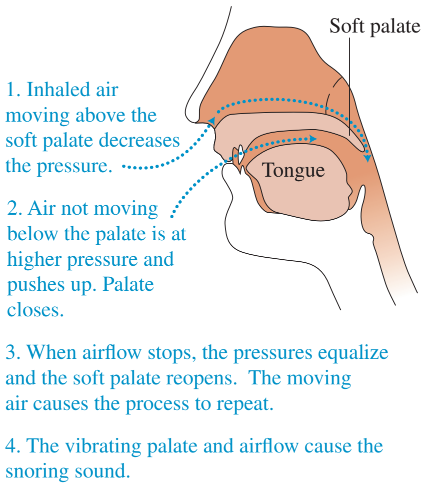
The hypothesis that the pressure exerted by a faster-moving fluid along an object is less than the pressure exerted by a slower-moving fluid was not disproved.

It is important to note here that in all of the experiments, the movement of the fluid was parallel to the surface of interest. When the fluid is moving perpendicular to the surface of interest, it exerts a force in the direction of motion (you can think of the momentum of fluid particles changing as the particles hit the surface). You can clearly observe this effect if you repeat the experiment in Table 14.2 with the hairdryer blowing directly downward into the tube—the water level inside the tube lowers.

At this point, without evidence to the contrary, we can say that as a fluid's speed parallel to a surface increases, the pressure that the moving fluid exerts on the surface decreases. This statement is a qualitative version of a principle formulated in 1738 by Daniel Bernoulli and named in his honor.

Bernoulli's effect The pressure that a fluid exerts on a surface *decreases* as the speed with which the fluid moves parallel to the surface *increases*.

FIGURE 14.1 Snoring occurs when the soft palate opens and closes due to the starting and stopping of airflow across it.



Bernoulli's effect has important fluid-flow implications in biological systems—for example, in the flow of blood through blood vessels. The blood pressure against the wall of a vessel depends on how fast the blood is moving—pressure is lower when the blood is moving faster. Let's look at another biological application of Bernoulli's effect.

Snoring

A snoring sound occurs when air moving through the narrow opening above the soft palate at the back of the roof of the mouth has lower pressure than nonmoving air below the palate (**Figure 14.1**). The normal air pressure below the soft palate, where the air is not moving, pushes the palate closed. When airflow stops, the pressures equalize and the passage reopens. The rhythmic opening and closing of the soft palate against the throat leads to the snoring sound.

REVIEW QUESTION 14.1 What is the empirical evidence for Bernoulli's effect?

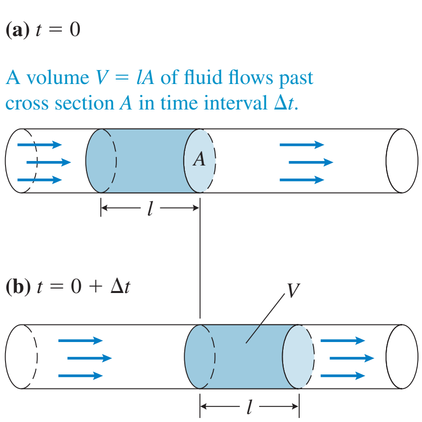
14.2 Flow rate and fluid speed

We have found qualitatively that the pressure of a fluid along the direction of flow depends on the speed of the moving fluid—the greater the speed, the lower the pressure. In this section, we will learn new physical quantities that we will need to describe the effect quantitatively. The quantitative analysis is much easier if we limit our investigations to fluids in confined regions—pipes or tubes.

Perhaps you have taken a shower in which little water flowed from the showerhead. In physics, we would say the water flow rate was low. The **flow rate Q** is an important consideration in designing showerheads. A smaller flow rate will save water, but a larger flow rate will help you rinse off faster. Flow rate is defined as the volume V of fluid that moves through a cross section of a pipe divided by the time interval Δt during which it moved (see **Figure 14.2**):

$$Q = \frac{V}{\Delta t} \tag{14.1}$$

FIGURE 14.2 Flow rate is the volume of fluid that passes a cross section of a vessel in a given time interval.



The SI unit of flow rate is m^3/s, but you may also see it as ft^3/s, ft^3/min, gallons/min, L/min, or any unit of volume divided by any unit of time interval. Notice that flow rate in m^3/s is different from the speed of the fluid v in m/s.

TIP The symbols V, t, and Q are also used in other aspects of physics. For example, a lowercase v denotes speed, the capital letter T is used for temperature, and in future chapters we will use Q for two other unrelated quantities. Because these symbols are often used to indicate different quantities, it is important when working with equations to try to visualize their meaning with concrete images (for example, the volume of water flowing out of a faucet during 1 s).

How does the flow rate relate to the speed of the moving fluid? To explore the relationship, consider Figure 14.2a. Over a certain time interval Δt the shaded volume of fluid passes a cross section of area A at some position along the pipe. Thus, after a time Δt, the back part of this fluid volume has in effect moved forward to the position shown in Figure 14.2b. The volume V of fluid in the shaded portion of the cylinder is the product of its length l and the cross-sectional area A of the pipe:

$$V = lA$$

Thus, the fluid flow rate is

$$Q = \frac{V}{\Delta t} = \frac{lA}{\Delta t} = \left(\frac{l}{\Delta t}\right)A$$

However, l is also the distance the fluid moves in a time interval Δt. Thus, $l/\Delta t$ is the average fluid speed v. Substituting $v = l/\Delta t$ into the above equation, we find that

$$Q = vA \tag{14.2}$$

The flow rate is equivalent to the average fluid speed multiplied by the cross-sectional area of the pipe.

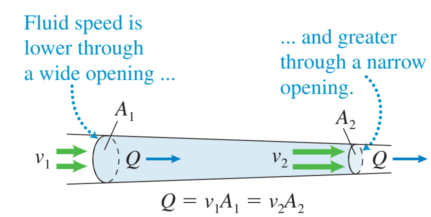
When an incompressible fluid (such as water) flows through a pipe with variable cross-sectional area (**Figure 14.3**), the amount of fluid entering at cross section A_1 must equal the amount of fluid leaving at cross section A_2. Thus the flow rate should remain constant. What must change is the speed of the fluid as it travels through the narrower part of the pipe. In the narrow section of the pipe (A_2), the speed will be greater in order to keep the flow rate constant. Thus, the flow rate past cross section A_1 equals that past cross section A_2:

$$Q_1 = v_1A_1 = v_2A_2 = Q_2 \tag{14.3}$$

where v_1 is the average speed of the fluid passing cross section A_1, and v_2 is the average speed of the fluid passing cross section A_2. Equation (14.3) is called the **continuity equation** and is used to relate the cross-sectional area and average speed of fluid flow in different parts of a rigid pipe carrying an incompressible fluid.

TIP The definition of flow rate is an operational definition, not a cause-effect relationship, because it is the speed of the fluid that depends on the flow rate and the cross-sectional area. The flow rate is determined by the source of the fluid (for example, how much you open the faucet).

FIGURE 14.3 The flow speed $v_2 > v_1$ depends on the cross-sectional area of pipe carrying the fluid.



QUANTITATIVE EXERCISE 14.1 Speed of blood flow in the aorta

The heart pumps blood at an average flow rate of $80\ \text{cm}^3/\text{s}$ into the aorta, which has a diameter of 1.5 cm. Determine the average speed of blood flow in the aorta.

Represent mathematically The flow rate can be determined by rearranging Eq. (14.2):

$$v = \frac{Q}{A}$$

where the cross-sectional area of the aorta is

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$$

Solve and evaluate Combining the above two equations, we find that the average speed of blood flow in the aorta is

$$v = \frac{Q}{A} = \frac{Q}{\pi(d/2)^2} = \frac{(80\ \text{cm}^3/\text{s})}{\pi(1.5\ \text{cm}/2)^2} = 45\ \text{cm/s}$$

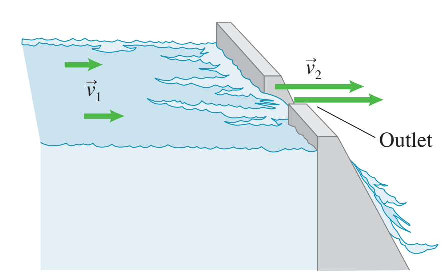
The unit is correct. The magnitude is reasonable—about half a meter each second.

Try it yourself Determine the average speed of blood flow if the diameter is reduced from 1.5 cm to 1.0 cm but the flow rate is the same.

Answer 100 cm/s.

Notice that blood speed more than doubles when the aorta diameter decreases by 33%. Vessel diameter has a very significant effect on the flow rate of fluid through a vessel, including those in biological systems. The narrower the blood vessel, the faster the blood flows, increasing the risk of dislodging plaque. Likewise, the narrower the airway from the nose to the mouth, the faster the air moves and the more likely you are to snore. These effects depend on the speed of the fluid, like blood or air in different parts of a vessel or pipe in which the diameter changes from one section to another.

FIGURE 14.4 Why is $v_2 > v_1$?



REVIEW QUESTION 14.2 Why does water in a river flow more slowly just before a dam than it does while passing through the outlet of the dam (**Figure 14.4**)?

FIGURE 14.5 Laminar flow.

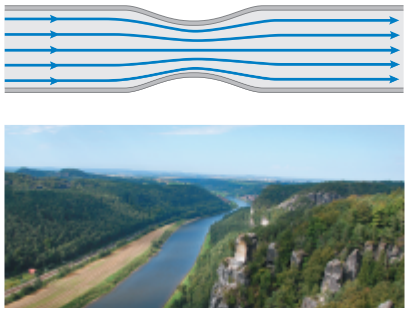


FIGURE 14.6 Turbulent flow.

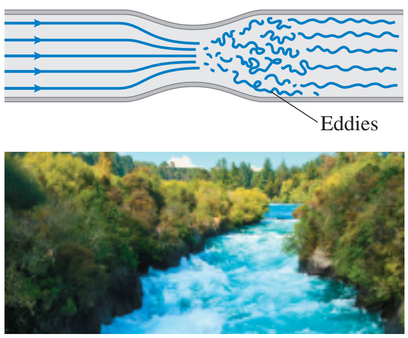
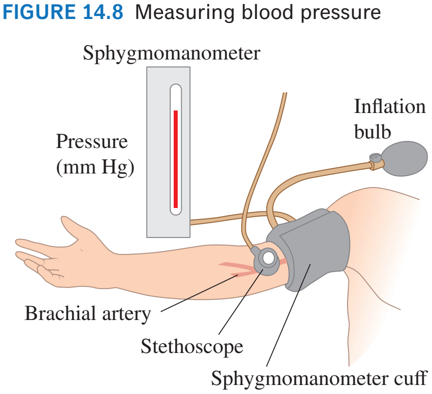


FIGURE 14.7 Streamlining a truck.

FIGURE 14.8 Measuring blood pressure



14.3 Types of fluid flow

Before we can proceed to describing Bernoulli's effect quantitatively, we need to learn more about fluid flow.

What kinds of flow can occur? There is the smooth flow that we see in a wide river and the whitewater flow that we see as water rushes and swirls through a narrow channel. Studies of fluid flow in wind tunnels indicate that there are two primary kinds of flow: laminar flow and turbulent flow. In **laminar flow**, every particle of fluid that passes a particular point follows the same path as the particles that preceded it. These paths are called **streamlines**. This is the smooth flow that we see in a wide river (see the fluid flowing smoothly in the wide tube in **Figure 14.5**). **Turbulent flow**, on the other hand, is characterized by agitated, disorderly motion. Instead of following a given path, the fluid forms whirlpool patterns called eddies, which come and go randomly, or sometimes become semi-stable (see **Figure 14.6**). Turbulent flow occurs when a fluid moves around objects and through pipes at high speed.

The force exerted by the fluid on objects is called the **drag force**. This is the force you "feel" when running against the wind. Due to the drag force, more kinetic energy is converted into thermal energy in turbulent flow than in laminar flow. Designing a car so that air moves over it with laminar flow reduces the drag force that the air exerts on the car and improves gasoline mileage. Placing a curved dome above the cab of a truck deflects air up and over the trailer, reducing turbulent flow and increasing gas mileage by more than 10% (see **Figure 14.7**).

Measuring blood pressure

To measure a person's blood pressure, a nurse uses a device called a *sphygmomanometer* (see **Figure 14.8**). The nurse places a cuff around the upper arm of a patient at about the level of the heart and places a stethoscope on the inside of the elbow above the brachial artery in the arm. The nurse then increases the gauge pressure in the cuff to about 180 mm Hg by pumping air into the cuff. The expanded cuff pushes on the brachial artery and stops blood flow in the arm. Then the nurse slowly releases the air from the cuff, decreasing the pressure of the air in it. When the pressure in the cuff is equal to the *systolic* pressure (120 mm Hg if the systolic blood pressure is normal), blood starts to squeeze through the artery past the cuff. The flow is intermittent and turbulent and causes a sound heard with the stethoscope. This turbulent sound continues until the cuff pressure decreases below the *diastolic* pressure (80 mm Hg for normal diastolic blood pressure). At that point the artery is continually open and blood flow is laminar and makes no sound. The systolic and diastolic pressure numbers together make up the blood pressure measurement.

REVIEW QUESTION 14.3 Is it easier for the heart to pump blood if the flow of the blood through the blood vessels is laminar or if it is turbulent? Explain.

14.4 Bernoulli's equation

Earlier in this chapter we investigated Bernoulli's effect qualitatively. In this section we will learn how to describe it quantitatively.

To achieve this goal, we use the case of a fluid flowing through a pipe, as shown in **Figure 14.9a**. We assume that (1) the fluid is incompressible, (2) there are no resistive forces exerted on the flowing fluid, and (3) the flow is laminar. We can apply the work-energy principle to describe the behavior of the fluid as it moves a short distance along the pipe. Consider the system to be composed of the shaded volume of fluid pictured in Figures 14.9a and b and Earth.

FIGURE 14.9 Applying the work-energy principle to fluid flow.

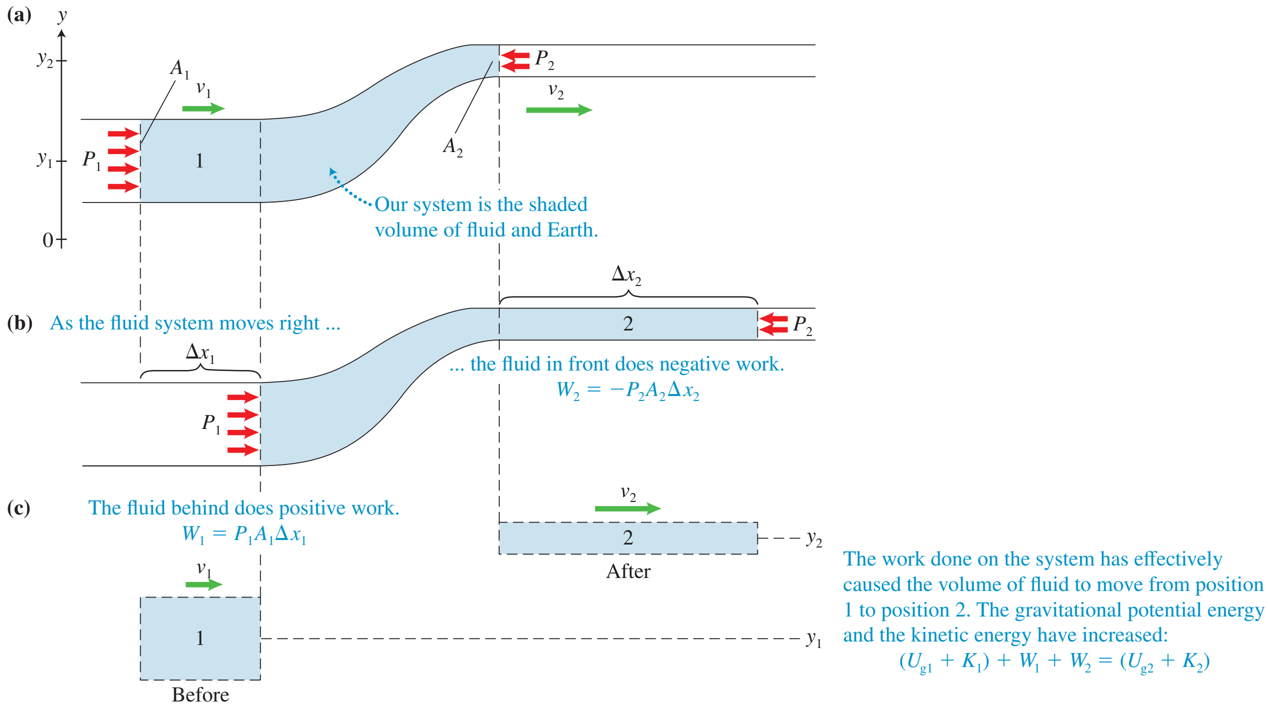


Figure 14.9a shows us the initial state of the system. As the volume of fluid flows to the right, the fluid behind it at position 1 exerts a force of magnitude $F_1 = P_1A_1$ to the right, where P_1 is the fluid pressure against the left side of the volume and A_1 is the cross-sectional area of the left side of the volume. Simultaneously, the fluid ahead of the system at position 2 exerts a force in the opposite direction of magnitude $F_2 = P_2A_2$, where P_2 is the fluid pressure against the right side of the volume and A_2 is the cross-sectional area of the right side of the volume.

In Figure 14.9b, the shaded volume of fluid has moved to the right. Because the pipe is narrower at 2, the right side of the fluid at position 2 moves a greater distance than the left side of the same volume of fluid in the wider part of the pipe at position 1. The net effect of the movement of the fluid a short distance to the right is summarized in Figure 14.9c. The volume of fluid initially at position 1 moving at speed v_1 has now been transferred, in effect, to position 2 where it moves at speed v_2. The volume of fluid stays constant, since we assume the fluid is incompressible. The fluid is now moving faster through the narrow tube at position 2 than it was earlier when moving through the wider tube at position 1, thus we have an increase in kinetic energy. The fluid at position 2 is at a higher elevation than when at position 1, thus the gravitational potential energy of the system increases. The energies changed as a result of the work done by the forces exerted by the fluid behind and ahead of the shaded volume. We can represent this quantitatively using the generalized work-energy equation [Eq. (7.3)].

$$(K_1 + U_{g1}) + W = (K_2 + U_{g2})$$

If we move the terms with the subscript 1 to the right side of the equation, we have

$$W = (K_2 - K_1) + (U_{g2} - U_{g1})$$

or

$$W = \Delta K + \Delta U_g \tag{14.4}$$

Let us now write expressions for each of the terms in the above equation.

Work done Two forces are doing work on the system. The fluid behind the system exerts a force F_1 to the right on the left side of the system over a distance Δx_1. The fluid ahead of the system exerts a force F_2 to the left on the right side of the system over a distance Δx_2. (Figures 14.9a and b show fluid pressures; the corresponding forces have magnitudes $F_1 = P_1A_1$ and $F_2 = P_2A_2$.) The force F_1 does positive work since it points in the direction of the motion of the system. The force F_2 does negative work since it points in the direction opposite the motion of the system. The total work done on the system is

$$\begin{aligned} W &= F_1\,\Delta x_1 \cos 0^\circ + F_2\,\Delta x_2 \cos 180^\circ \\ &= P_1A_1\,\Delta x_1 - P_2A_2\,\Delta x_2 \end{aligned}$$

The volume of fluid V that has moved from the left to the right is $V = A_1\,\Delta x_1 = A_2\,\Delta x_2$ since the fluid is incompressible. The preceding expression for work becomes

$$W = P_1V - P_2V = (P_1 - P_2)V$$

Change in kinetic energy The mass m of the system (the moving volume of fluid) is related to its density ρ and volume V:

$$m = \rho V$$

As the system moves from the initial to the final state, the speed of the element of fluid that effectively moved changes from v_1 to v_2. Thus, the kinetic energy change of the mass m of fluid shown in Figure 14.9c is

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}\rho V v_2^2 - \tfrac{1}{2}\rho V v_1^2$$

Change in gravitational potential energy The gravitational potential energy of the system has also changed because part of the system has moved from elevation y_1 to elevation y_2. The change in gravitational potential energy is then

$$\Delta U_g = mg(y_2 - y_1) = \rho V g(y_2 - y_1)$$

We can now substitute the above three expressions into Eq. (14.4) to get

$$(P_1 - P_2)V = \left(\tfrac{1}{2}\rho V v_2^2 - \tfrac{1}{2}\rho V v_1^2\right) + \rho V g(y_2 - y_1)$$

If the common V is canceled from each term, we find that

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

By dividing by V in that last step we have changed the units of each term in the equation from energy (measured in joules) to **energy density** (measured in joules per cubic meter). Energy density is the same as energy per unit volume of fluid and appears on the right side of the above equation. The left-hand side represents the amount of work done on the fluid per unit volume of fluid.

Bernoulli's equation relates the pressures, speeds, and elevations at two points on the same streamline in laminar flow in a fluid:

$$P_1 - P_2 = \tfrac{1}{2}\rho\,(v_2^2 - v_1^2) + \rho g(y_2 - y_1) \qquad (14.5)$$

The equation can be rearranged into an alternate form:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad (14.6)$$

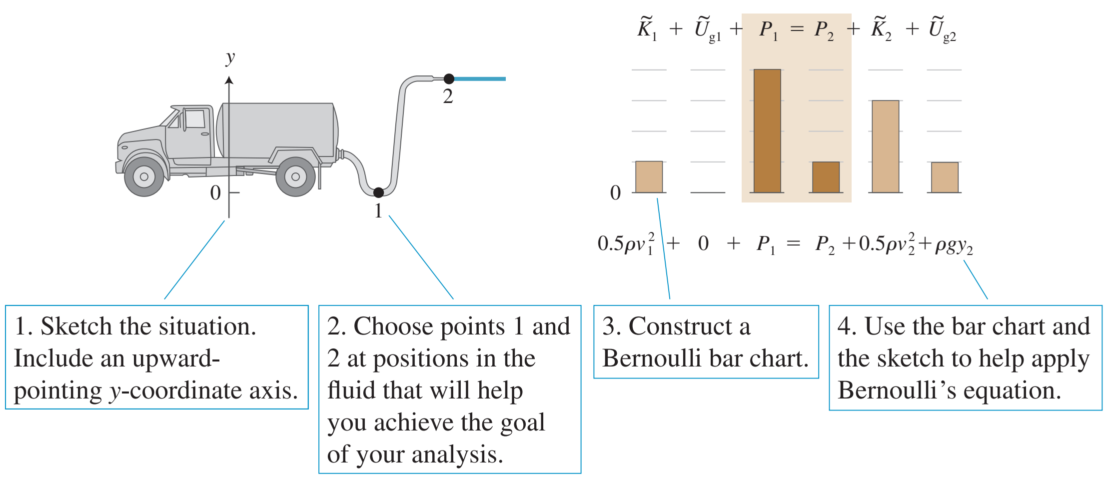
The sum of the kinetic and gravitational potential energy densities and the pressure at position 1 equals the sum of the same three quantities at position 2.

It is important to remember the assumptions that we used to derive Bernoulli's equation. It describes quantitatively the flow of a frictionless, nonturbulent, incompressible fluid.

Using Bernoulli bar charts to understand fluid flow

Bernoulli's equation looks fairly complex and might be difficult to use for visualizing fluid dynamics processes. However, since Bernoulli's equation is based on the work-energy principle, we can represent such processes using energy bar charts similar to the ones used in Chapters 7 and 13 (here the bars represent pressures and energy densities; we use a ~ over the energy symbols to emphasize that they are energy densities, not energies). Physics Tool Box 14.1 describes how to construct a fluid dynamics bar chart for the following process. A fire truck pumps water through a big hose up to a smaller hose on the ledge of a building. Water sprays out of the smaller hose onto a fire. Compare the pressure in the hose at the lowest point of the larger hose to the pressure at the exit of the smaller hose.

PHYSICS TOOL BOX 14.1 Constructing a bar chart for a moving fluid



To start, first draw a sketch of the process. Choose positions 1 and 2 at appropriate locations in order to help answer the question. One of the positions might be a place where you want to determine the pressure in the fluid and the other position a place where the pressure is known. For the water pump-hose process, it is useful to choose position 1 (the location of the unknown pressure) and position 2 at the exit of the water from the small hose (at known atmospheric pressure).

Represent this process by placing bars of appropriate relative lengths on the chart (the absolute lengths are not known). It is often easiest to start by analyzing the gravitational potential energy density. Use a vertical y-axis with a well-defined origin to keep track of the gravitational potential energy densities. For the fire hose process, choose position 1 as the origin of the vertical coordinate system. The gravitational potential energy density at position 1 is then zero. The exit of the water from the small hose is at higher elevation; thus, there is a positive gravitational potential energy density bar for position 2 (in the bar chart we arbitrarily assign it one positive unit of energy density).

Next, consider the kinetic energy. The water flows from a wider hose at position 1 to a narrower hose at position 2. Thus the kinetic energy density at 2 is greater than at 1—thus the longer bar at position 2. We arbitrarily assume that the kinetic energy density bar for position 1 is one unit and that for position 2 is three units.

Notice now that the total length of the bars on the right side of the chart is much higher than on the left side (the difference is three units). To account for the difference we need to consider the change in pressure. Since the fluid pressures at 1 and 2 are analogous to the work done on the system in an ordinary work-energy bar chart, P_1 and P_2 appear in the shaded box in the center of the bar chart, where work is represented. The difference in the pressure heights should account for the total difference in the energy densities. Thus, we draw the bar for P_1 three units higher than for P_2. The bar chart is now complete. We can use it to write a mathematical description for the process and solve for any unknown quantity.

REVIEW QUESTION 14.4 Compare and contrast work-energy bar charts and Bernoulli bar charts.

14.5 Skills for analyzing processes using Bernoulli's equation

In this section, we will adapt our problem-solving strategy to analyze processes involving moving fluids. In this case, we describe and illustrate a strategy for finding the speed of water as it leaves a bottle. The general strategy is on the left side of the table and the specific process is on the right.

PROBLEM-SOLVING STRATEGY 14.1 Applying Bernoulli's equation

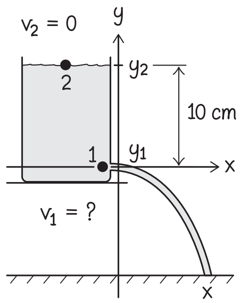
EXAMPLE 14.2 Removing a tack from a water bottle

What is the speed with which water flows from a hole punched in the side of an open plastic bottle? The hole is 10 cm below the water surface.

Sketch and translate

- Sketch the situation. Include an upward-pointing y-coordinate axis. Choose an origin and positive direction for the coordinate axis.
- Choose points 1 and 2 at positions in the fluid where you know the pressure/speed/position or that involve the quantity you are trying to determine.
- Choose a system.

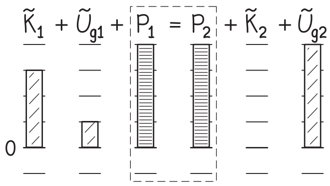
- Choose the origin of the vertical y-axis to be the location of the hole.
- Choose position 1 to be the place where the water leaves the hole and position 2 to be a place where the pressure, elevation, and water speed are known—at the water surface $y_2 = 0.10$ m. The pressure in Bernoulli's equation at both positions 1 and 2 is atmospheric pressure, since both positions are exposed to the atmosphere ($P_1 = P_2 = P_{atm}$).
- Choose Earth and the water as the system.



Simplify and diagram

- Identify any assumptions you are making. For example, can we assume that there are no resistive forces exerted on the flowing fluid?
- Construct a Bernoulli bar chart.

- Assume that no resistive forces are exerted on the flowing fluid.
- Assume that y_2 and y_1 stay constant during the process, since the elevation of the surface decreases slowly compared to the speed of the water as it leaves the tiny hole.



- Because the water at the surface is moving very slowly relative to the hole, assume that $v_2 = 0$.
- Draw a bar chart that represents the process.

Represent mathematically

- Use the sketch and bar chart to help apply Bernoulli's equation.
- You may need to combine Bernoulli's equation with other equations, such as the equation of continuity $Q = v_1A_1 = v_2A_2$ and the definition of pressure $P = F/A$.

- We see from the sketch and the bar chart that the speed of the fluid at position 2 is zero (zero kinetic energy density) and that the elevation is zero at position 1 (zero gravitational potential energy density). Also, the pressure is atmospheric at both 1 and 2. Thus

$$\tfrac{1}{2}\rho(0)^2 + \rho g y_2 + P_{atm} = P_{atm} + \tfrac{1}{2}\rho v_1^2 + \rho g(0)$$

$$\Rightarrow \rho g y_2 = \tfrac{1}{2}\rho v_1^2$$

Solve and evaluate

- Solve the equations for an unknown quantity.
- Evaluate the results to see if they are reasonable (the magnitude of the answer, its unit, how the answer changes in limiting cases, and so forth).

- Solve for v_1:

$$v_1 = \sqrt{2gy_2}$$

Substituting for g and y_2, we find that

$$v_1 = \sqrt{2(9.8\ \text{m/s}^2)(0.10\ \text{m})} = 1.4\ \text{m/s}$$

- The unit m/s is the correct unit for speed. The magnitude seems reasonable for water streaming from a bottle (if we obtained 120 m/s it would be unreasonably high).

Try it yourself In the above situation the water streams out of the bottle onto the floor a certain horizontal distance away from the bottle. The floor is 1.0 m below the hole. Predict this horizontal distance using your knowledge of projectile motion. (Hint: Use Eqs. (4.7) and (4.8).)

Answer The equations yield a result of 0.63 m. However, if we were to actually perform this experiment with a tack-sized hole, the water would land short of our prediction because of resistive forces exerted by the hole on the water. In order to make the water land closer to 0.63 m from the bottle, we must increase the diameter of the hole. We discuss the effect of resistive forces on fluid flow later in the chapter.

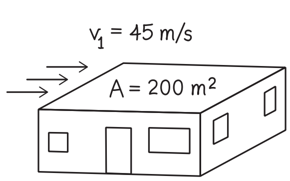
Blowing the roof off a house

You've no doubt seen images of roofs being blown from houses during tornadoes or hurricanes. How does that happen? On a windy day, the air inside the house is not moving, whereas the air outside is moving very rapidly. The air pressure inside the house is therefore greater than the air pressure outside, creating a net pressure that pushes outward against the roof and windows. If the net pressure becomes great enough, the roof and/or the windows will blow outward off of the house. In the following example, we do a quantitative estimate of the net force exerted by the inside and outside air on a roof.

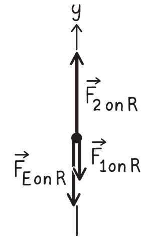
EXAMPLE 14.3 Effect of high-speed air moving across the roof of a house

During a storm, air moves at speed 45 m/s (100 mi/h) across the top of the 200-m² flat roof of a house. Estimate the net force exerted by the air pushing up on the inside of the roof and the outside air pushing down on the outside of the roof. Indicate any assumptions made in your estimate.

Sketch and translate The situation is shown at right. We need to determine the pressure just above and below the roof.



Simplify and diagram A force diagram for the roof is shown here. The air above the house exerts a downward force on the roof $F_{1\text{ on R}} = P_1A$, where P_1 is the air pressure above the house and A is the area of the roof. The air inside the house pushes up, exerting a force on the roof $F_{2\text{ on R}} = P_{\text{atm}}A$, where P_{atm} is the assumed atmospheric pressure of the stationary air inside the house. We assume that the air is incompressible and flows without friction or turbulence and that the roof is fairly thin so that the air has approximately the same gravitational potential energy density at points 1 and 2.



Represent mathematically With the y-axis oriented upward, the net force exerted by the air on the roof is

$$F_{\text{net Air}} = F_{2\text{ on R}} - F_{1\text{ on R}} = P_{\text{atm}}A - P_1A$$
$$= (P_{\text{atm}} - P_1)A$$

We use Bernoulli's equation to find this pressure difference.

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = \tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1$$
$$\Rightarrow P_{\text{atm}} + 0 + \rho g y_2 = \tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1$$
$$\Rightarrow P_{\text{atm}} - P_1 = \tfrac{1}{2}\rho v_1^2 + (\rho g y_1 - \rho g y_2) = \tfrac{1}{2}\rho v_1^2 + 0$$

We can now determine the net force exerted by the air on the roof.

$$F_{\text{net Air}} = (P_{\text{atm}} - P_1)A = \tfrac{1}{2}\rho v_1^2 A$$

(CONTINUED)

Solve and evaluate

$$F_{\text{net Air}} = \tfrac{1}{2}\rho v_1^2 A$$
$$= \tfrac{1}{2}(1.3\ \text{kg/m}^3)(45\ \text{m/s})^2(200\ \text{m}^2) = 2.6 \times 10^5\ \text{N}$$

The result is an upward net force that is enough to lift more than ten cars of combined mass 27,000 kg.

We were a little lax in applying Bernoulli's equation in this example. The equation relates the properties of a fluid at two points along the same streamline. A streamline does not flow between just below the roof and just above the roof. We could, with some more complex reasoning, use the equation correctly by considering two streamlines that start far from the house at the same pressure. One ends up in the house under its roof with the air barely moving. The other passes just above the roof with the air moving fast. We would get the same result in a somewhat more cumbersome manner.

Try it yourself A 2.0 m $\times$ 2.0 m canvas covers a trailer. The trailer moves at 29 m/s (65 mi/h). Determine the net force exerted on the canvas by the air above and below it.

Answer An upward force of 2200 N (about 500 lb). No wonder the canvas covering a truck trailer moving on a highway balloons outward.

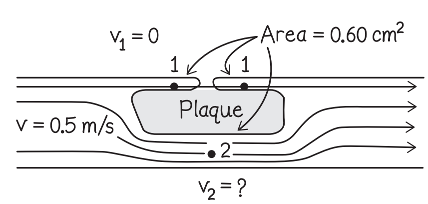
Dislodging plaque

The physical principles of a roof being lifted from a house also explain how plaque can become dislodged from the inner wall of an artery. Plaque may block a considerable portion of the area where blood normally flows. Suppose the radius of the vessel opening is one-third its normal value because of the plaque. Then the area available for blood flow, proportional to r^2, is about one-ninth the normal value. The speed of flow in the narrowed portion of the artery will be about nine times greater than in the unblocked part of the vessel. The kinetic energy density term in Bernoulli's equation is proportional to v^2 and therefore is 81 times greater in the constricted area than in the open part of the vessel.

Notice that in Bernoulli's equation, the sum of the gravitational potential energy density, the kinetic energy density, and the pressure at one location should equal the sum of the same three terms at some other location along a streamline in the blood. As blood speeds by the plaque, its kinetic energy density is 81 times greater, and consequently its pressure is much less than the pressure in the open vessel just before and just after the plaque. This pressure differential could cause the plaque to be pushed off the wall and tumble downstream, causing a blood clot (a process called thrombosis). Let's estimate the net force that the blood exerts on the plaque.

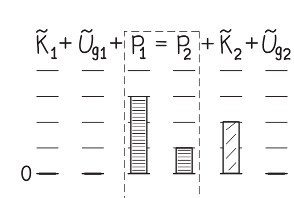
EXAMPLE 14.4 **A clogged artery**

Blood flows through the unobstructed part of a blood vessel at a speed of 0.5 m/s. The blood then flows past a plaque that constricts the cross-sectional area to one-ninth the normal value. The surface area of the plaque parallel to the direction of blood flow is about $0.60\ \text{cm}^2 = 6.0 \times 10^{-5}\ \text{m}^2$. Estimate the net force that the fluid exerts on the plaque.



Sketch and translate A simplified two-dimensional sketch of three-dimensional the situation is shown above right. The third dimension is perpendicular to the page of the book. The plaque is as large in this dimension as in the x-dimension, but the area where it is attached to the top wall is very small. Point 1 is above the plaque in stationary blood pooled in a channel where the plaque attaches to the artery wall. Point 2 is in the bloodstream below the plaque, where blood flows rapidly past it. The net force depends on the differences in pressure at points 1 and 2. Thus, we need to first find the pressure difference. Since points 1 and 2 are not on the same streamline, we cannot automatically use Bernoulli's equation. But since the streamlines that do go through points 1 and 2 were side by side before they reached the plaque, each streamline will have the same $P + (1/2)\rho v^2 + \rho g y$ value. This means we can equate the $P + (1/2)\rho v^2 + \rho g y$ values at points 1 and 2.

Simplify and diagram Assume for simplicity that the blood is nonviscous and flows with laminar flow without turbulence. Assume also that the vertical distance between points 1 and 2 is small ($y_1 - y_2 \approx 0$) and that the area of the stationary blood above the plaque is the same as the area where the blood moves below the plaque. A bar chart represents the process. The blood pressure at position 2 is less than at position 1 because the blood flows at high speed through the constricted artery, whereas it sits at rest in the channels at position $1 (v_1 = 0)$.



Represent mathematically Compare the two points using Bernoulli's equation:

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$
$$= \tfrac{1}{2}\rho(v_2^2 - 0) + 0 = \tfrac{1}{2}\rho v_2^2$$

Since the cross-sectional area of the vessel at the location of the plaque is one-ninth its normal area (the radius is one-third the normal value), the blood must be flowing at nine times its normal speed. The net force exerted by the blood on the plaque downward and perpendicular to the direction the blood flows will be

$$F_{\text{net blood on P } y} = F_{\text{blood 1 on P}} + (-F_{\text{blood 2 on P}})$$
$$= P_1 A - P_2 A = (P_1 - P_2)A$$
$$\Rightarrow F_{\text{net blood on P } y} = \tfrac{1}{2}\rho v_2^2 A$$

Solve and evaluate

$$F_{\text{net blood on P } y} = \tfrac{1}{2}(1050\ \text{kg/m}^3)\,(9 \times 0.5\ \text{m/s})^2(6.0 \times 10^{-5}\ \text{m}^2)$$
$$= 0.64\ \text{N} \approx 0.6\ \text{N}$$

This is about the weight of one-half of an apple pulling on this tiny plaque. In addition, an "impact" force caused by blood hitting the plaque's upstream side contributes to the risk of breaking the plaque off the side wall of the vessel. The loose plaque can then tumble downstream and block blood flow in a smaller vessel in the heart (causing a heart attack) or in the brain (causing a stroke).

Try it yourself Air of density $1.3\ \text{kg/m}^3$ moves at speed $10\ \text{m/s}$ across the top surface of a clarinet reed that has an area of $3\ \text{cm}^2$. The air below the reed is not moving and is at atmospheric pressure. Determine the net force exerted on the reed by the air above and below it.

Answer 0.02 N upward, toward the inside of the mouthpiece.

Using Bernoulli's equation to explain how airplanes fly

You may have heard that airplanes can fly because of the special shape of their wings, which causes the air above the wing to move faster than the air below the wing ($\vec{v}_{\text{above}} > \vec{v}_{\text{below}}$, shown in **Figure 14.10a**). Let us apply Bernoulli's equation to an airplane wing to evaluate this claim. A Boeing 747-8's maximum takeoff mass is 442 metric tons, its takeoff speed is 330 km/h, and its wing surface area is $554\ \text{m}^2$. We assume that the speed of air above the wing is equal to the takeoff speed and the speed of air below the wing is zero (this is a rather unrealistic assumption, but it allows us to estimate the largest possible force due to Bernoulli's effect). In this case we find

$$\Delta p = \tfrac{1}{2}\rho_{\text{air}} v_{\text{takeoff}}^2 = 0.5 \times (1.3\ \text{kg/m}^3) \times (92\ \text{m/s})^2 = 5501\ \text{N/m}^2$$

and

$$F_{\text{air on airplane}} = \Delta p \cdot A = 3.1 \times 10^6\ \text{N}$$

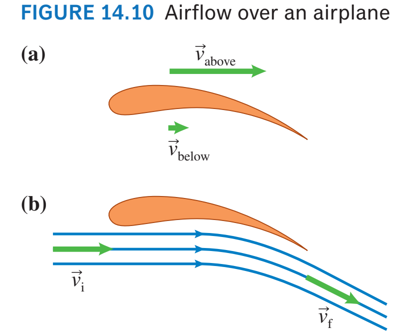
The force exerted by Earth on the airplane is

$$F_{\text{E on airplane}} = m_{\text{airplane}}\, g = 4.3 \times 10^6\ \text{N}$$

This force is much larger than the largest possible lifting force exerted on the plane due to Bernoulli's effect.

In other words, Bernoulli's effect is at work, but it does not provide a complete explanation of how airplanes fly. The important missing force comes from the change of direction of the air's motion. Due to the shape and the tilt of the wing, the air passing the wing changes its direction of motion from horizontal ($\vec{v}_{\text{i}}$) to slightly downward ($\vec{v}_{\text{f}}$) (Figure 14.10b). For this to happen, the wing must exert a downward force on the air; therefore (remember Newton's third law), the air exerts an equal and opposite force upward on the wing. It is this force that provides the main lift and (together with the force due to Bernoulli's effect) makes the airplane fly. Note that the origin of this force is the change of the direction of motion of air and has nothing to do with Bernoulli's effect.

FIGURE 14.10 Airflow over an airplane wing.



REVIEW QUESTION 14.5 In Example 14.2 we said that the pressure was the same at two levels when we drew the bar chart. Doesn't the pressure in a fluid increase with depth?

14.6 Viscous fluid flow

In our previous discussions and examples in this chapter, we assumed that there were no resistive forces exerted on the moving fluid or that fluids flow without friction. That is, we assumed no interaction either between the fluid and the walls of the pipes it flows in, or between the layers of the fluid. However, in Example 14.2 we found that this assumption was not reasonable. In fact, for many processes, such as the transport of blood in the small vessels in our bodies, fluid friction is very important. When we cannot neglect this friction inside the fluid, we call the fluid **viscous**.

Consider the following situation. You have an object that can slide on a frictionless horizontal surface, say, a puck on smooth ice. You push the puck abruptly and then let go. What happens to the puck? Once in motion, the puck will continue to slide at constant speed with respect to the ice even if nothing else pushes it. However, if there is friction between the contacting surfaces (there is a little sand in the ice), then the puck starts slowing down; for it to continue moving at constant speed, someone or something has to push it forward to balance the opposing friction force.

By analogy, if a fluid flows through a horizontal tube without friction (nonviscous fluid), we would expect it to continue to flow at a constant rate with no additional forward pressure. But if friction is present (the fluid is viscous), there must be greater pressure at the back of the fluid than at the front of the fluid to maintain a constant flow rate. If this is the case, the force exerted on any volume of the fluid due to the forward pressure is greater than the force exerted on the same volume of the fluid due to the pressure in the opposite direction.

Factors that affect fluid flow rate

What factors affect the flow rate in the vessel with friction? What is the functional dependence of those factors? Let's think about the physical properties of the fluid and the vessel that can affect the flow rate. The following quantities might be important.

Pressure difference The flow rate should depend on how hard the fluid is pushed forward, that is, on the difference between the fluid pressure pushing forward from behind and the fluid pressure pushing back from in front of the fluid, or $(P_1 - P_2)$.

Radius of the tube The radius r of the tube carrying the fluid should affect the flow rate. From everyday experience we know that it is more difficult to push (a greater pressure difference) fluid through a tube of tiny radius than through a tube with a large radius.

Length of the tube The length l of the tube might also affect the ease of fluid flow. A long tube offers more resistance to flow than a shorter tube.

Fluid type Water flows much more easily than molasses does. Thus some property of a fluid that characterizes its "thickness" or "stickiness" should affect the flow.

Let's design an experiment to investigate exactly how the first three of these four factors $(P, r, \text{and } l)$ affect the fluid flow rate Q. As shown in **Figure 14.11**, a pump that produces an adjustable pressure P_1 causes fluid to flow through tubes of different radii r and lengths l. We collect the fluid exiting the tube and measure the flow rate Q, which is the volume V of fluid leaving the tube in a certain time interval Δt divided by that time interval. The results of the experiments are reported in **Table 14.3**.

How is the flow rate affected by each of the three factors?

Pressure difference Looking at the first three rows of the table, we notice that the flow rate is proportional to the pressure difference $(Q \propto P_1 - P_2)$.

FIGURE 14.11 How do $P_1 - P_2$, r, and l affect the flow rate Q?

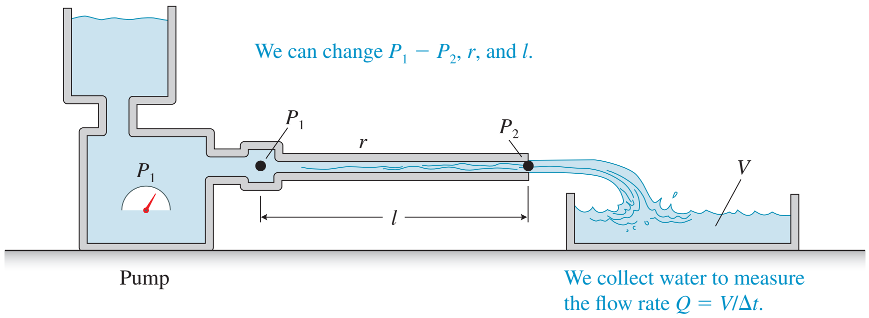


TABLE 14.3 Different quantities affect the flow rate *Q* of fluid through a tube (data are reported in relative units)

$P_1 - P_2$ (Pressure difference)	r (Radius)	l (Length)	Q (Flow rate)
1	1	1	1
2	1	1	2
3	1	1	3
1	2	1	16
1	3	1	81
1	1	2	0.5
1	1	3	0.33

Radius of the tube Looking at rows 1, 4, and 5, we notice that the flow rate increases rapidly as the radius increases. Doubling the radius causes the flow rate to increase by a factor of 16 (2^4). Tripling the radius causes the flow rate to increase by a factor of 81 (3^4). It seems that the flow rate is proportional to the fourth power of the radius of the tube ($Q \propto r^4$).

Length of the tube Looking at rows 1, 6, and 7, we notice that the flow rate decreases as the length of the tube increases. It seems that the flow rate is proportional to the inverse of the length ($Q \propto 1/l$).

These three relationships can be combined in a single equation:

$$Q \propto \frac{r^4(P_1 - P_2)}{l}$$

The equation has been confirmed by numerous experiments.

Viscosity and Poiseuille's law

In this experiment we did not investigate the fourth factor: the type of fluid. Under the same conditions, water flows faster than oil, which flows faster than molasses. If we use the same pressure difference to push different fluids through the same tube, we find that the fluids have different flow rates. The quantity by which we measure this effect on flow rate is called the **viscosity** η of the fluid. The flow rate is inversely proportional to viscosity:

$$Q \propto \frac{1}{\eta}$$

In 1840, using an experiment similar to that described above, French physician and physiologist Jean Louis Marie Poiseuille established a relationship between these physical quantities. However, instead of writing the flow rate in terms of the other four quantities, he wrote an expression for the pressure difference needed to cause a particular flow rate.

Poiseuille's law The forward-backward pressure difference $P_1 - P_2$ needed to cause a fluid of viscosity η to flow at a rate Q through a vessel of radius r and length l is

$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \qquad (14.7)$$

The pressure difference term $P_1 - P_2$ on the left side of Poiseuille's law determines the net force pushing the fluid. The flow rate Q on the far right side is a consequence of this net push on the fluid. The term before Q on the right side (the $\left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}$ term) can be thought of as the resistance of the fluid to flow: the same pressure difference will produce a smaller flow rate if the resistance is greater. The resistance is greater if the fluid has greater viscosity η, is greater for a longer tube (greater l), and is far more resistive if the vessel through which the fluid flows has a smaller radius ($1/r^4$ is much greater

TIP Notice that the pressure difference needed to cause a particular flow rate is proportional to the inverse of the fourth power of the radius of the vessel. If the radius of a vessel carrying fluid is reduced by a factor of 0.5, the pressure difference needed to cause the same flow rate must increase by $(1/0.5)^4 = 16$. We need 16 times the pressure difference to cause the same flow rate.

TABLE 14.4 Viscosities of some liquids and gases

Substance	Viscosity η ($\mathrm{N \cdot s/m^2}$)
Air (30 °C)	1.9×10^{-5}
Water vapor (30 °C)	1.25×10^{-5}
Water (0 °C)	1.8×10^{-3}
Water (20 °C)	1.0×10^{-3}
Water (40 °C)	0.66×10^{-3}
Water (80 °C)	0.36×10^{-3}
Blood, whole (37 °C)	4×10^{-3}
Oil, SAE No. 10	0.20

for small r). This idea has many applications relative to the circulatory system—see the example a little later in this section.

From Poiseuille's law we can determine the unit for viscosity. To do this, we express the viscosity using other quantities in Eq. (14.7):

$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \Rightarrow \eta = \frac{(P_1 - P_2)r^4\pi}{8Ql}$$

We use the latter equation to find the units for the viscosity. Remember that the units of pressure are

$$\mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2} = \frac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot m^2}} = \frac{\mathrm{kg}}{\mathrm{s^2 \cdot m}}$$

and the units for flow rate are $\mathrm{m^3/s}$. Using these units, we get

$$\eta = \frac{(\mathrm{kg})(\mathrm{m^4})(\mathrm{s})}{(\mathrm{s^2 \cdot m})(\mathrm{m^3})(\mathrm{m})} = \frac{\mathrm{kg}}{\mathrm{s \cdot m}}$$

We can also rewrite the last combination of units as $\mathrm{N \cdot s/m^2}$. Remember that $\mathrm{N/m^2} = \mathrm{Pa}$; thus the unit for viscosity can be written $\mathrm{Pa \cdot s}$. A list of viscosities of several fluids appears in **Table 14.4** using $\mathrm{N \cdot s/m^2}$ for the units of η.

QUANTITATIVE EXERCISE 14.5 **Blood flow through a narrow artery**

Because of plaque buildup, the radius of an artery in a person's heart decreases by 40%. Determine the ratio of the present flow rate to the original flow rate if the pressure across the artery, its length, and the viscosity of blood are unchanged.

Represent mathematically In this exercise, we are interested in the change in the flow rate and not in the change in pressure. Consequently, we rearrange Poiseuille's law for the flow rate in terms of the other quantities:

$$Q = \left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r^4$$

If the radius decreases by 40%, the new radius is $100\% - 40\% = 60\%$ of the original. Thus the radius r of the vessel at the present time is related to the radius r_0 years earlier by the equation $r = 0.60r_0$.

Solve and evaluate The ratio of the flow rates is

$$\frac{Q}{Q_0} = \frac{\left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r^4}{\left(\frac{\pi}{8}\right)\left(\frac{\Delta P}{\eta l}\right)r_0^4} = \frac{r^4}{r_0^4} = \left(\frac{r}{r_0}\right)^4 = (0.60)^4 = 0.13$$

The flow rate is only 13% of the original flow rate! To compensate for such a dramatically reduced flow rate, the person's blood pressure will increase.

Try it yourself Determine the reduction in flow rate, assuming a constant pressure difference, if the radius of the vessel is reduced 90% (to 0.10 times its original value). This is not an unusual reduction for people with high blood pressure.

Answer $Q/Q_0 = 0.0001$, or 0.01% of its original value!

Limitations of Poiseuille's law: Reynolds number

Poiseuille's law describes the flow of a fluid accurately only when the flow is laminar. Experiments indicate that to determine when the flow is laminar or turbulent, we need to calculate what is called the pipe **Reynolds number** R_e:

$$R_e = \frac{2\bar{v}r\rho}{\eta} \tag{14.8}$$

where $\bar{v}$ is the average speed of the fluid, ρ is its density, η is the viscosity, and r is the radius of the pipe that carries the fluid. Experiments show that if the Reynolds number is less than 2000, the fluid flow is laminar; if it is more than 3000, the flow is turbulent; and between 2000 and 3000 the flow is unstable and can be either laminar or turbulent.

The Reynolds number, used as a criterion to decide whether the flow inside a pipe is laminar or turbulent, came from experiments conducted in the 1880s by Osborne Reynolds. Using a glass pipe with flowing water inside, he adjusted a valve to control the speed of the water flow. He then added colored water to the stream and observed that when the speed of the water was low, the colored layer of water could be clearly seen inside the pipe. When the speed of water flow increased beyond a certain limit, the colored part would break apart into vortices and mix with the rest of the water. Reynolds expressed the criterion for the type of flow with a unitless number (hence the name the Reynolds number), which he derived by taking the ratio of the net force pushing the fluid and the resistive force opposing it.

The transition from laminar to turbulent flow is also observed when an object and fluid are moving relative to each other. Reynolds derived a similar criterion (expressed as a unitless number) for this phenomenon. In general, the flow in pipes remains laminar up to much larger R_e numbers than does the flow of a fluid around an object.

REVIEW QUESTION 14.6 Describe some of the physics-related effects on the cardiovascular system of medication that lowers the viscosity of blood.

14.7 Drag force

So far in this chapter all of our analyses have focused on a moving fluid. In Section 14.6 we were concerned with the resistive forces as fluids move through a tube. Now we focus on solid objects moving through a fluid—for example, a swimmer moving through water, a skydiver falling through the air, or a car traveling through air. As you know from experience, the fluid in these and in other cases exerts a resistive **drag force** on the object moving through the fluid. So far we have been neglecting this force in our mechanics problems. Now we will not only learn how to calculate this force but also learn whether our assumptions were reasonable: for example, is the resistive drag force indeed insignificant when people and objects fall from small and large heights?

Laminar drag force Imagine that an object moves relatively slowly through a fluid (for example, a marble sinking in oil). In this case the fluid flows around the solid object in streamline laminar flow, with no turbulence. However, the fluid does exert a drag force on the object. For a spherical object O of radius r moving at speed v through a liquid of viscosity η, the magnitude of this nonturbulent drag force $F_{\text{D F on O}}$ exerted by fluid on the object is given by the equation

$$F_{\text{D F on O}} = 6\pi\eta r v \tag{14.9}$$

This equation is called **Stokes's law**. Notice that the drag force is proportional to the speed of the object relative to the fluid and to the radius of the sphere.

Turbulent drag force A rock sinks in oil much faster than a small marble. In this case the motion of oil past the sinking rock is turbulent, and Eq. (14.9) does not apply. A different Reynolds number, called the object Reynolds number, can be used to determine whether the flow of fluid past an object is laminar or turbulent:

$$R_e = \frac{vl\rho}{\eta} \tag{14.10}$$

where v is the object's speed with respect to the fluid, l is the characteristic dimension of the object, in most cases the diameter, and ρ and η are the density and viscosity of the fluid. When the Reynolds number is calculated using this equation, the threshold value for laminar flow is 1. If the Reynolds number is much more than 1, the flow is turbulent and we cannot use Stokes's law. In this case, a new equation for drag force applies:

$$F_{\text{D F on O}} \approx \tfrac{1}{2} C_{\text{D}} \rho A v^2 \tag{14.11}$$

where ρ is the density of fluid, A is the cross-sectional area of the object as seen along its line of motion, and C_{D} is a dimensionless number called the *drag coefficient*. The drag coefficient depends on the shape of the object (the lower the number, the smaller the drag force and the more laminar the flow past the object). For example, the drag coefficient for a sphere is 0.5 and for a dolphin it is 0.005.

Drag force exerted on a moving vehicle

Does Stokes's law apply to moving cars? At 60 mi/h (about 27 m/s), for a car about 2 m wide in air with density 1.3 kg/m^3 and viscosity 2×10^{-5} N·s/m^2, the estimated Reynolds number is

$$R_{\text{e}} = \frac{vl\rho}{\eta} = \frac{(27\ \text{m/s})(2\ \text{m})(1.3\ \text{kg/m}^3)}{(2 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2)} \approx 4 \times 10^6$$

This is much more than 1. We need to use Eq. (14.11) for the drag force.

TIP Note that if the speed of a car doubles, the drag force exerted on it quadruples. Thus, because of air drag, when you increase your driving speed, you reduce your gas mileage.

We can estimate the magnitude of the drag force that air exerts on a compact car traveling at 27 m/s (60 mi/h) assuming that the drag coefficient C_{D} is approximately 0.3 for a well-designed car, the cross-sectional area of a car is 2 m^2, and the air density is 1.3 kg/m^3.

Because the flow of air past the car is turbulent, we use Eq. (14.11) to estimate the drag force that the air exerts on the car:

$$F_{\text{D Air on Car}} = \tfrac{1}{2} C_{\text{D}} \rho A v^2 = \tfrac{1}{2}(0.3)(1.3\ \text{kg/m}^3)(2\ \text{m}^2)(27\ \text{m/s})^2 = 280\ \text{N}$$

or a force of about 60 lb. Designing cars to minimize the drag force improves fuel efficiency.

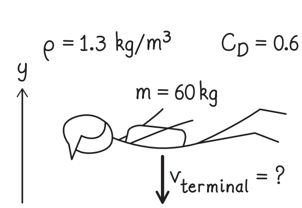
Terminal speed

As a skydiver falls through the air, her speed increases, as does the drag force that the air exerts on her. Eventually, the diver's speed becomes so great that the magnitude of the upward resistive drag force that the air exerts on the diver equals the magnitude of the downward gravitational force that Earth exerts on the diver. The sum of the forces exerted on the diver is zero, so the diver moves downward at a constant speed, known as **terminal speed**. Let's estimate the terminal speed for a skydiver.

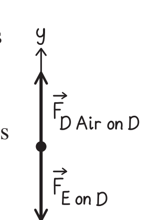
EXAMPLE 14.6 Terminal speed of skydiver

Estimate the terminal speed of a 60-kg skydiver falling through air of density 1.3 kg/m^3, assuming a drag coefficient $C_{\text{D}} = 0.6$.

Sketch and translate The situation is sketched here. When the diver is moving at terminal speed, the forces that the air exerts on the diver and that Earth exerts on the diver balance—the net force is zero. We choose the diver as the system of interest with vertical y-axis pointing upward.



Simplify and diagram A force diagram for the diver is shown to the right. Assume that the buoyant force that the air exerts on the diver is negligible in comparison to the other forces exerted on her and that the drag force involves turbulent airflow past the diver.



Represent mathematically Use the force diagram to help apply Newton's second law for the diver:

$$\begin{aligned} ma_y &= \Sigma F_y \\ \Rightarrow 0 &= F_{\text{D Air on D}} + (-F_{\text{E on D}}) \\ &= \tfrac{1}{2} C_{\text{D}} \rho A v_{\text{terminal}}^2 - mg \end{aligned}$$

Solve and evaluate Solving for the diver's terminal speed gives

$$v_{\text{terminal}} = \sqrt{\frac{2mg}{C_D \rho A}}$$

All of the quantities in the above expression are known except the cross-sectional area of the diver along her line of motion. If we assume that she is 1.5 m tall and 0.3 m wide, her cross-sectional area is about 0.5 m^2. We find that her terminal speed is

$$v_{\text{terminal}} = \sqrt{\frac{2(60\text{ kg})(9.8\text{ N/kg})}{(0.6)(1.3\text{ kg/m}^3)(0.5\text{ m}^2)}} = 55\text{ m/s}$$

The unit is correct. The magnitude seems reasonable—about 120 mi/h.

Note that we assumed a turbulent drag force (Eq. (14.11). Was this assumption appropriate? To assess this, we will estimate the maximum speed of the diver for the drag force to be laminar. According to Eq. (14.10),

$$R_e = \frac{vl\rho}{\eta} \quad \Rightarrow \quad v = \frac{R_e \eta}{l\rho}$$

Estimating the characteristic dimension for the person to be about 1 m (somewhere between her 1.5-m height and 0.3-m width) and the critical value of the Reynolds number to be 1 for a laminar drag force, we obtain the following estimate for the greatest speed of an object for which the drag force exerted by the fluid is laminar:

$$v = \frac{R_e \eta}{l\rho} \approx \frac{1 \times (1.9 \times 10^{-5}\text{ N}\cdot\text{s/m}^2)}{(1\text{ m}) \times (1.3\text{ kg/m}^3)} \approx 1.5 \times 10^{-5}\text{ m/s}$$

This is a tiny speed—much too slow for a skydiver. Therefore, we could safely use the equation for turbulent drag force to estimate her speed.

Try it yourself Suppose the diver pulled her legs to her chest so she was more in the shape of a ball. How qualitatively would that affect her terminal speed? Explain.

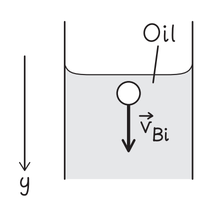
Answer Her cross-sectional area would be smaller, and according to the above equation, her terminal speed would be greater.

Below is an example of a problem whose solution depends on our assumptions. Being able to consider the effects of assumptions is a very useful skill.

EXAMPLE 14.7 Multiple possibility problem

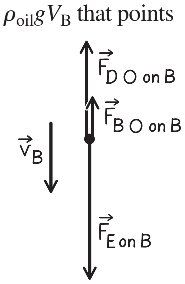
A small metal ball is launched vertically downward, with initial velocity $\vec{v}_{Bi}$, from just below the surface of oil that fills a very deep container. Draw a qualitative velocity-versus-time and an acceleration-versus-time graph for the motion of the ball.

Sketch and translate We draw a sketch of the situation; let's choose the y-axis to point down. In order to draw velocity- and acceleration-versus-time graphs, we need to know the sum of the forces exerted on the ball.



Simplify and diagram The term "simplify" relates to the assumptions that we need to make to solve the problem. Given that one of the forces exerted on the ball is variable (the drag force), the assumptions we make about the initial velocity will determine the outcome. Therefore, in this case it is more useful to first identify the forces exerted on the ball and then consider different assumptions.

First, we need to identify the forces that are exerted on the ball and draw a force diagram (at right). Earth exerts a force $F_{E\text{ on }B} = m_B g$ that points down. The oil exerts a buoyant force $F_{B\text{ O on }B} = \rho_{\text{oil}} g V_B$ that points up. Since the ball is made of metal we know that $F_{E\text{ on }B} > F_{B\text{ O on }B}$. The oil also exerts a drag force $F_{D\text{ O on }B}$ that is proportional to v_B and points opposite to $\vec{v}_B$ (up in our case). Given that we are interested in qualitative graphs, it is enough to remember that the drag force exerted by the oil on the ball is proportional to the speed of the ball with respect to the oil. This is true for a laminar as well as for a turbulent drag force.

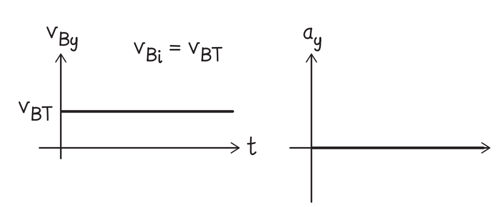


Since $a_y = \dfrac{\Sigma F_{O\text{ on }B\,y}}{m_B}$, and $F_{E\text{ on }B\,y}$ and $F_{B\text{ O on }B\,y}$ are constant, the sign and magnitude of $\vec{a}_y$ will depend on the magnitude of $F_{D\text{ O on }B\,y}$ or on the speed of the ball. Let us consider the following assumptions.

1. The ball is initially moving such that the magnitude of the drag force is exactly equal to the difference between the gravitational force and the buoyant force:

$$F_{D\text{ O on }B\,y} = F_{E\text{ on }B\,y} - F_{B\text{ O on }B\,y} \Rightarrow \Sigma F_{O\text{ on }B\,y} = 0;\ a_y = 0$$

In this special case, the ball will continue to move with the same speed with which it was launched. This is the terminal speed, which we will denote v_{BT}. The graphs for this motion are shown below.



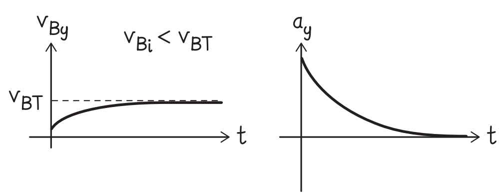
2. The ball is initially moving such that the magnitude of the drag force is *smaller* than the difference between the gravitational force and the buoyant force:

$$F_{D\text{ O on }B\,y} < F_{E\text{ on }B\,y} - F_{B\text{ O on }B\,y} \Rightarrow \Sigma F_{O\text{ on }B\,y} > 0;\ a_y > 0$$

Since $F_{D\text{ O on }B}$ is proportional to v_B, this case will occur when $v_{Bi} < v_{BT}$. In this case, the ball will initially accelerate downward,

(CONTINUED)

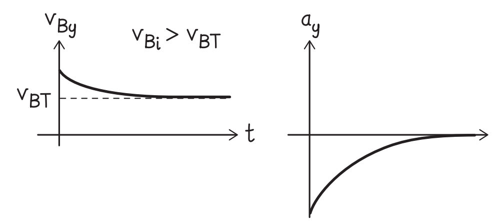
but as the speed of the ball increases, $F_{\text{D O on B}}$ will increase until the sum of the forces is zero. As explained in case 1, at this point the ball reaches terminal velocity and continues moving with constant speed (zero acceleration). The graphs for this motion are shown below.



3. The ball is initially moving such that the magnitude of the drag force is *larger* than the difference between the gravitational force and the buoyant force:

$$F_{\text{D O on B}\,y} > F_{\text{E on B}\,y} - F_{\text{B O on B}\,y} \Rightarrow \Sigma F_{\text{O on B}\,y} < 0\,;\, a_y < 0$$

Since $F_{\text{D O on B}}$ is proportional to v_{B}, this case will occur when $v_{\text{Bi}} > v_{\text{BT}}$. In this case, the acceleration of the ball will initially point up (the ball will slow down). As the speed of the ball decreases, $F_{\text{D O on B}}$ will decrease until the sum of the forces is zero. Therefore, the ball will slow down until it reaches the terminal velocity and then continues moving with constant speed (zero acceleration). The graphs for this motion are shown below.



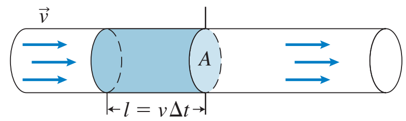
Try it yourself Describe what will happen if we shoot the ball upward from the bottom of the oil-filled container. How does the scenario depend on the initial speed of the ball?

Answer No matter what the initial speed of the ball, the ball will move upward and slow down because all the forces exerted on it point in the direction opposite to its motion. Then it will start moving downward in the same way as described in case 2 in this example.

REVIEW QUESTION 14.7 When a skydiver falls at constant terminal speed, shouldn't the magnitude of the resistive drag force that the air exerts on the skydiver be a little less than the magnitude of the downward gravitational force that Earth exerts on the diver? If they are equal, shouldn't the diver stop falling? Explain.

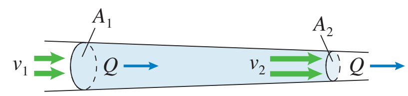
Summary

Flow rate The flow rate Q of a fluid is the volume V of fluid that passes a cross section in a tube divided by the time interval Δt needed for that volume to pass. The flow rate also equals the product of the average speed v of the fluid and the cross-sectional area A of the vessel. (Section 14.2)



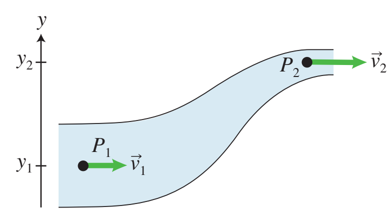
$$Q = vA \quad \text{Eq. (14.2)}$$

Continuity equation If fluid does not accumulate, the flow rate into a region (position 1) must equal the flow rate out of the region (position 2). At position 1 the fluid has speed v_1 and the tube has cross-sectional area A_1. At position 2 the fluid has speed v_2 and the tube has cross-sectional area A_2. (Section 14.2)



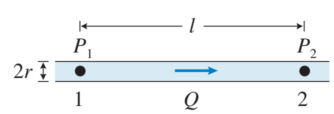
$$Q = v_1A_1 = v_2A_2 \quad \text{Eq. (14.3)}$$

Bernoulli's equation For a fluid flowing without resistive forces or turbulence, the sum of the kinetic energy density $(1/2)\rho v^2$, the gravitational potential energy density ρgy, and pressure P of the fluid is a constant. (Section 14.4)



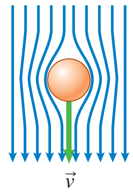
$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \text{Eq. (14.6)}$$

Poiseuille's law For viscous fluid flow, the pressure drop $(P_1 - P_2)$ across a fluid of viscosity η flowing in a tube depends on the length l of the tube, its radius r, and the fluid flow rate Q. (Section 14.6)



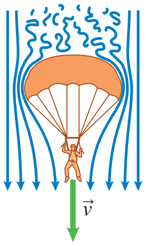
$$P_1 - P_2 = \left(\frac{8}{\pi}\right)\frac{\eta l}{r^4}Q \quad \text{Eq. (14.7)}$$

Laminar drag force When a spherical object (like a marble sinking in oil) moves slowly through a fluid, the fluid exerts a resistive drag force on the object that is proportional to the object's speed v. **Stokes's law** describes the force. (Section 14.7)



$$F_D = 6\pi\eta r v \quad \text{Eq. (14.9)}$$

Turbulent drag force For an object moving at faster speed through a fluid (like a parachutist with an open parachute), turbulence occurs and the resistive drag force is proportional to the square of the speed. (Section 14.7)



$$F_D = \tfrac{1}{2}C_D\rho A v^2 \quad \text{Eq. (14.11)}$$

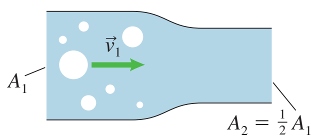
Questions

Multiple Choice Questions

1. A roof is blown off a house during a tornado. Why does this happen?
   (a) The air pressure in the house is lower than that outside.
   (b) The air pressure in the house is higher than that outside.
   (c) The wind is so strong that it blows the roof off.
2. A river flows downstream and widens, and the flow speed slows. As a result, the pressure of the water against a dock downstream compared to upstream will be
   (a) higher. (b) lower. (c) the same.
3. Why does the closed top of a convertible bulge when the car is riding along a highway?
   (a) The volume of air inside the car increases.
   (b) The air pressure is greater outside the car than inside.
   (c) The air pressure inside the car is greater than the pressure outside.
   (d) The air blows into the front part of the roof, lifting the back part.

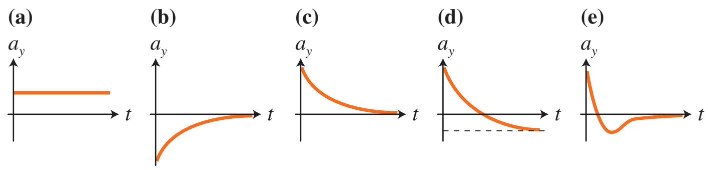
4. How does Bernoulli's principle help explain air going up the chimney of a house?
   (a) Air blowing across the top of the chimney reduces the pressure above the chimney.
   (b) The air above the chimney attracts the ashes.
   (c) The hot ashes seek the cooler outside air.
   (d) The gravitational potential energy is lower above the chimney.
5. As a river approaches a dam, the width of the river increases and the speed of the flowing water decreases. What can explain this effect?
   (a) Bernoulli's equation
   (b) The continuity equation
   (c) Poiseuille's law
6. What is an incompressible fluid?
   (a) A law of physics
   (b) A physical quantity
   (c) A model of an object
7. What is viscous flow?
   (a) A physical phenomenon
   (b) A law of physics
   (c) A physical quantity
8. The heart does about 1 J of work pumping blood during one heartbeat. What is the immediate first and main type of energy that increases due to the heart's work?
   (a) Kinetic energy
   (b) Thermal energy
   (c) Elastic potential energy
9. Several air bubbles are present in water flowing through a pipe of variable cross-sectional area (**Figure Q14.9**). What happens to the volume of an air bubble when it arrives at the narrow part of the pipe, where the cross-sectional area is half that of the wider part? Assume the temperature of the gas in the bubble stays constant.

FIGURE Q14.9



   (a) The volume of the bubble remains unchanged because water is incompressible.
   (b) The volume of the bubble remains unchanged because the temperature of the gas in the bubble is constant.
   (c) The volume of the bubble decreases because the pressure in water increases.
   (d) The volume of the bubble increases because the pressure in water decreases.
10. A small metal ball is released from just below the surface of oil that fills a very deep container. The y-axis points down. Which of the acceleration-versus-time graphs in **Figure Q14.10** represents the motion of the ball after it is released?

FIGURE Q14.10



11. A small metal ball is launched downward from just below the surface of oil that fills a very deep container. The initial speed of the ball is larger than the terminal speed of the ball in the oil. The y-axis points down. Which of the graphs in Figure Q14.10 represents the motion of the ball after it is launched?

Conceptual Questions

12. You have two identical large jugs with small holes on the side near the bottom. One jug is filled with water and the other with liquid mercury. The liquid in each jug, sitting on a table, squirts out the side hole into a container on the floor. Which container, the one catching the water or the one catching the mercury, must be closer to the table in order to catch the fluid? Or should they be placed at the same distance? Which jug will empty first, or do they empty at the same time? Explain. Indicate any assumptions that you made.
13. Why does much of the pressure drop in the circulatory system occur across the arterioles (small vessels carrying blood to the capillaries) and capillaries as opposed to across the much larger diameter arteries?
14. If you partly close the end of a hose with your thumb, the water squirts out farther. Give at least one explanation for why this phenomenon occurs.
15. Compare and contrast work-energy bar charts, which you learned about in Chapter 7, with Bernoulli bar charts.
16. Consider Bernoulli's equation, Poiseuille's law, and Stokes's law. Which of these are applicable to viscous fluids? Explain.
17. You need a liquid that will exhibit turbulent flow in a tube even at lower speeds. Which properties of liquids will you evaluate when choosing a liquid? Explain.

Problems

Below, BIO indicates a problem with a biological or medical focus. Problems labeled EST ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems. Unless stated otherwise, assume in these problems that atmospheric pressure is 1.01×10^5 N/m^2 and that the densities of water and air are 1000 kg/m^3 and 1.3 kg/m^3, respectively.

14.1 and 14.2 Fluids moving across surfaces—qualitative analysis and Flow rate and fluid speed

1. **Watering plants** You water flowers outside your house. (a) Determine the flow rate of water moving at an average speed of 32 cm/s through a garden hose of radius 1.2 cm. (b) Determine the speed of the water in a second hose of radius 1.0 cm that is connected to the first hose.
2. **Irrigation canal** You live near an irrigation canal that is filled to the top with water. (a) It has a rectangular cross section of 5.0-m width and 1.2-m depth. If water flows at a speed of 0.80 m/s, what is its flow rate? (b) If the width of the stream is reduced to 3.0 m and the depth to 1.0 m as the water passes a flow-control gate, what is the speed of the water past the gate?
3. **Fire hose** During a fire, a firefighter holds a hose through which 0.070 m^3 of water flows each second. The water leaves the nozzle at an average speed of 25 m/s. What information about the hose can you determine using these data?
4. The main waterline for a neighborhood delivers water at a maximum flow rate of 0.010 m^3/s. If the speed of this water is 0.30 m/s, what is the pipe's radius?
5. * BIO **Blood flow in capillaries** The average flow rate of blood in the aorta is 80 cm^3/s. Beyond the aorta, this blood eventually travels through about 6×10^9 capillaries, each of radius 8.0×10^{-4} cm. What is the average speed of the blood in the capillaries?
6. * **Irrigating a field** It takes a farmer 2.0 h to irrigate a field using a 4.0-cm-diameter pipe that comes from an irrigation canal. How long would the job take if he used a 6.0-cm pipe? What assumption did you make? If this assumption is not correct, how will your answer change?

14.4 Bernoulli's equation

7. Represent the process sketched in **Figure P14.7** using a qualitative Bernoulli bar chart and an equation (include only terms that are not zero).

FIGURE P14.7

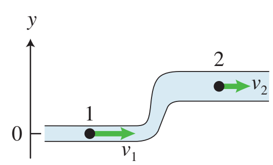
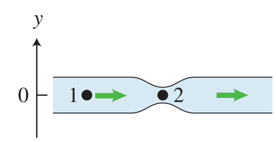


FIGURE P14.8



8. * Represent the process sketched in **Figure P14.8** using a qualitative Bernoulli bar chart and an equation (include only terms that are not zero).
9. **Fluid flow problem** Write a symbolic equation (include only terms that are not zero) and draw a sketch of a situation that could be represented by the qualitative Bernoulli bar chart shown in **Figure P14.9** (there are many possibilities).

FIGURE P14.9

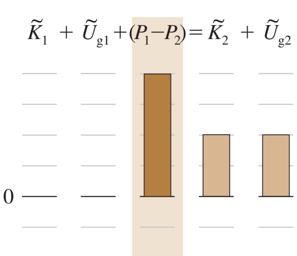
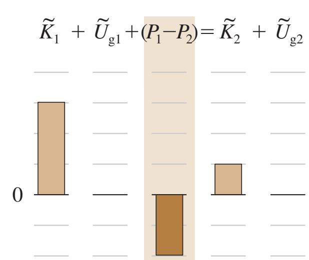


FIGURE P14.10



10. Repeat Problem 14.9 using the bar chart in **Figure P14.10**.
11. * Repeat Problem 14.9 using the bar chart in **Figure P14.11**.

FIGURE P14.11

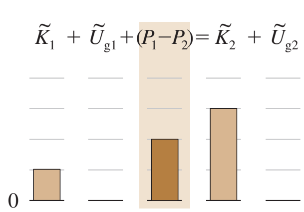
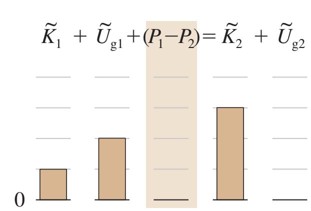


FIGURE P14.12



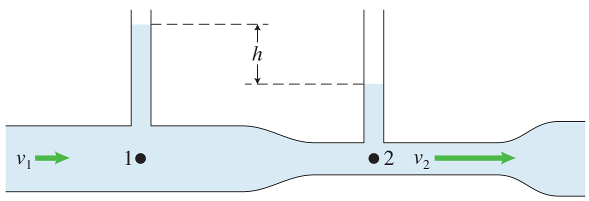
12. Repeat Problem 14.9 using the bar chart in **Figure P14.12**.
13. An application of Bernoulli's equation is shown below. Construct a qualitative Bernoulli bar chart that is consistent with the equation and draw a sketch of a situation that could be represented by the equation (there are many possibilities).
$\rho g y_2 = 0.5\rho v_1^2$
14. Repeat Problem 14.13 using the equation $0.5\rho v_1^2 + (P_1 - P_2) = 0.5\rho v_2^2$ and $P_1 < P_2$.
15. * Repeat Problem 14.13 using the equation below.
$0.5\rho v_1^2 + (P_1 - P_2) =$
$0.5\rho v_2^2 + \rho g y_2$ and $P_1 < P_2$.
16. * **Wine flow from barrel** While visiting a winery, you observe wine shooting out of a hole in the bottom of a barrel. The top of the barrel is open. The hole is 0.80 m below the top surface of the wine. Represent this process in multiple ways (a sketch, a bar chart, and an equation) and apply Bernoulli's equation to a point at the top surface of the wine and another point at the hole in the barrel.
17. **Water flow in city water system** Water is pumped at high speed from a reservoir into a large-diameter pipe. This pipe connects to a smaller diameter pipe. There is no change in elevation. Represent the water flow from the large pipe to the smaller pipe in multiple ways—a sketch, a bar chart, and an equation.

14.5 Skills for analyzing processes using Bernoulli's equation

18. * The pressure of water flowing through a 0.060-m-radius pipe at a speed of 1.8 m/s is $2.2 \times 10^5\ \mathrm{N/m^2}$. What is (a) the flow rate of the water and (b) the pressure in the water after it goes up a 5.0-m-high hill and flows in a 0.050-m-radius pipe?
19. * **Siphoning water** You want to siphon rainwater and melted snow from the cover of an above-ground swimming pool. The cover is 1.4 m above the ground. You have a plastic hose of 1.0-cm radius with one end in the water on the pool cover and the other end on the ground. (a) At what speed does water exit the hose? (b) If you want to empty the pool cover in half the time, what new hose radius should you use? (c) How much faster does the water flow through this wider pipe?
20. * **Cleaning skylights** You are going to wash the skylights in your kitchen. The skylights are 8.0 m above the ground. You connect two garden hoses together—a 0.80-cm-radius hose to a 1.0-cm-radius hose. The smaller hose is held on the roof of the house and the wider hose is attached to the faucet on the ground. The pressure at the opening of the smaller hose is 1 atm, and you want the water to have the speed of 6.0 m/s. What should be the pressure at ground level in the large hose? What should be the speed?
21. * **BIO** **Blood flow in artery** Blood flows at an average speed of 0.40 m/s in a horizontal artery of radius 1.0 cm. The average pressure is $1.4 \times 10^4\ \mathrm{N/m^2}$ above atmospheric pressure (the gauge pressure). (a) What is the average speed of the blood past a constriction where the radius of the opening is 0.30 cm? (b) What is the gauge pressure of the blood as it moves past the constriction? What assumption did you make to answer these questions?
22. * **Straw aspirator** A straw extends out of a glass of water by a height h. How fast must air blow across the top of the straw to draw water to the top of the straw?
23. * **Gate for irrigation system** You observe water at rest behind an irrigation dam. The water is 1.2 m above the bottom of a gate that, when lifted, allows water to flow under the gate. Determine the height h from the bottom of the dam that the gate should be lifted to allow a water flow rate of $1.0 \times 10^{-2}\ \mathrm{m^3/s}$. The gate is 0.50 m wide.
24. * **BIO** **Flutter in blood vessel** A person has a 5200-$\mathrm{N/m^2}$ gauge pressure of blood flowing at 0.50 m/s inside a 1.0-cm-radius main artery. The gauge pressure outside the artery is 3200 $\mathrm{N/m^2}$. When using his stethoscope, a physician hears a fluttering sound farther along the artery. The sound is a sign that the artery is vibrating open and closed, which indicates that there must be a constriction in the artery that has reduced its radius and subsequently reduced the internal blood pressure to less than the external 3200-$\mathrm{N/m^2}$ pressure. What is the maximum artery radius at this constriction?
25. * **BIO** **Effect of smoking on arteriole radius** The average radius of a smoker's arterioles, the small vessels carrying blood to the capillaries, is 5% smaller than those of a nonsmoker. (a) Determine the percent change in flow rate if the pressure across the arterioles remains constant. (b) Determine the percent change in pressure if the flow rate remains constant.
26. * **Roof of house in wind** The mass of the roof of a house is 2.1×10^4 kg and the area of the roof is 160 $\mathrm{m^2}$. At what speed must air move across the roof of the house so that the roof is lifted off the walls? Indicate any assumptions you made.
27. * You have a U-shaped tube open at both ends. You pour water into the tube so that it is partially filled. You have a fan that blows air at a speed of 10 m/s. (a) How can you use the fan to make water rise on one side of the tube? Explain your strategy in detail. (b) To what maximum height can you get the water to rise? Note: You cannot touch the water yourself.

28. * Engineers use a venturi meter to measure the speed of a fluid traveling through a pipe (see **Figure P14.28**). Positions 1 and 2 are in pipes with surface areas A_1 and A_2, with A_1 greater than A_2, and are at the same vertical height. How can you determine the relative speeds at positions 1 and 2 and the pressure difference between positions 1 and 2?

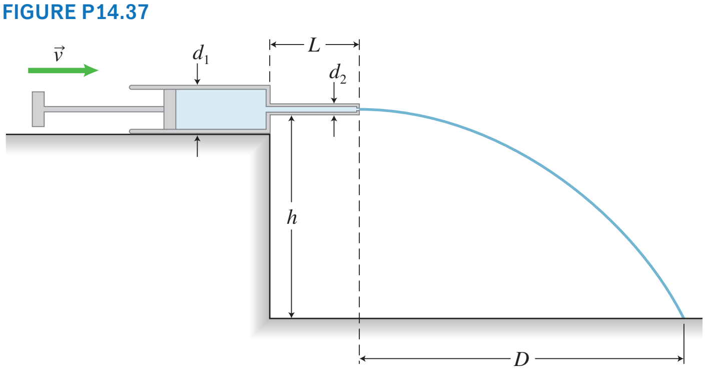
FIGURE P14.28



14.6 Viscous fluid flow

29. * A 5.0-cm-radius horizontal water pipe is 500 m long. Water at 20 °C flows at a rate of $1.0 \times 10^{-2}\ m^3/s$. (a) Determine the pressure drop due to viscous friction from the beginning to the end of the pipe. (b) What radius pipe must you use if you want to keep the pressure difference constant and double the flow rate?
30. **Fire hose** A volunteer firefighter uses a 5.0-cm-diameter fire hose that is 60 m long. The water moves through the hose at 12 m/s. The temperature outside is 20 °C. What is the pressure drop due to viscous friction across the hose?
31. **Another fire hose** The pump for a fire hose can develop a maximum pressure of $6.0 \times 10^5\ N/m^2$. A horizontal hose that is 50 m long is to carry water of viscosity $1.0 \times 10^{-3}\ N \cdot s/m^2$ at a flow rate of $1.0\ m^3/s$. What is the minimum radius for the hose?
32. * **Solar collector water system** Water flows in a solar collector through a copper tube of radius R and length l. The average temperature of the water is T °C and the flow rate is $Q\ cm^3/s$. Explain how you would determine the viscous pressure drop along the tube, assuming the water does not change elevation.
33. * BIO **Blood flow through capillaries** Your heart pumps blood at a flow rate of about $80\ cm^3/s$. The blood flows through approximately 9×10^9 capillaries, each of radius 4×10^{-4} cm and 0.1 cm long. Determine the viscous friction pressure drop across a capillary, assuming a blood viscosity of $4 \times 10^{-3}\ N \cdot s/m^2$.
34. * Determine the ratio of the flow rate through capillary tubes A and B (that is, Q_A/Q_B). The length of A is twice that of B, and the radius of A is one-half that of B. The pressure across both tubes is the same.
35. * A piston pushes 20 °C water through a horizontal tube of 0.20-cm radius and 3.0-m length. One end of the tube is open and at atmospheric pressure. (a) Determine the force needed to push the piston so that the flow rate is $100\ cm^3/s$. (b) Repeat the problem using SAE 10 oil instead of water.
36. * How can you use the venturi meter system (see Problem 14.28) to determine whether viscous fluid needs an additional pressure difference to flow at the same speed as a nonviscous fluid?
37. * A syringe is filled with water and fixed at the edge of a table at height $h = 1.0$ m above the floor (**Figure P14.37**). The diameter of the piston is $d_1 = 20.0$ mm, the needle has length $L = 50.00$ mm, and the inner diameter of the needle is $d_2 = 1.0$ mm. You press on the piston so that it moves at constant speed of 10.00 mm/s. Determine (a) the distance D at which the water jet hits the floor and (b) the pressure difference between the ends of the needle. Assume the viscosity of water is $1.0 \times 10^{-3}\ N \cdot s/m^2$.

FIGURE P14.37



14.7 Drag force

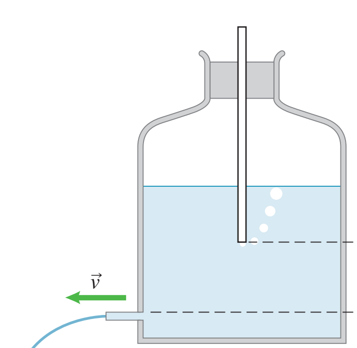
38. **Car drag** A 2300-kg car has a drag coefficient of 0.60 and an effective frontal area of $2.8\ m^2$. Determine the air drag force on the car when traveling at (a) 24 m/s (55 mi/h) and (b) 31 m/s (70 mi/h).
39. * EST **Air drag when biking** Estimate the drag force opposing your motion when you ride a bicycle at 8 m/s.
40. BIO **Drag on red blood cell** Determine the drag force on an object the size of a red blood cell with a radius of 1.0×10^{-5} m that is moving through 20 °C water at speed 1.0×10^{-5} m/s. (Assume laminar flow.)
41. * BIO EST **Protein terminal speed** A protein of radius 3.0×10^{-9} m falls through a tube of water with viscosity $\eta = 1.0 \times 10^{-3}\ N \cdot s/m^2$. Earth exerts a constant downward 3.0×10^{-22}-N force on the protein. (a) Use Stokes's law and the information provided to estimate the terminal speed of the protein. Assume no buoyant force is exerted on the protein. (b) How many hours would be required for the protein to fall 0.10 m?
42. * EST Earth exerts a constant downward force of 7.5×10^{-13} N on a clay particle falling in water. The particle settles 0.10 m in 820 min. Estimate the radius of a clay particle. Assume no buoyant force is exerted on the clay particle. The viscosity of water is $1.0 \times 10^{-3}\ N \cdot s/m^2$.
43. * A sphere falls through a fluid. Earth exerts a constant downward 0.50-N force on the sphere. The fluid exerts an opposing drag force on the fluid given by $F_D = 2v$, where F_D is in newtons if v is in meters per second. Determine the terminal speed of the sphere.
44. * **Terminal speed of balloon** A balloon of mass m drifts down through the air. The air exerts a resistive drag force on the balloon described by the equation $F_D = 0.03v^2$ where F_D is in newtons if v is in meters per second. What is the terminal speed of the balloon?
45. You observe four different liquids (listed with their viscosities and densities in the table below) as they flow with the same speed through identical tubes. You gradually increase the speed of the liquids until their flows become turbulent. Rank the liquids in the order in which you see their flow become turbulent.

Liquid	Viscosity $(N \cdot s/m^2)$	Density (kg/m^3)
Blood	4×10^{-3}	1.05×10^3
Water	1×10^{-3}	1.00×10^3
Ethanol	1×10^{-3}	0.79×10^3
Acetone	3×10^{-4}	0.79×10^3

General Problems

46. ** Describe how you will design an experimental procedure that will help you decide whether the drag force exerted on a coffee filter falling at terminal velocity depends on the filter's speed v or on its speed squared v^2.
47. * If the speed of air that is blowing across the upper end of a vertical tube is great enough, a thin plate that is placed at the lower end of the tube will remain touching the end of the tube, without falling down. Derive an expression for the minimum speed of air that needs to blow across a tube of diameter d to keep the plate of mass m from falling down.
48. **Mariotte's bottle** **Figure P14.48** shows a device called Mariotte's bottle that can deliver a constant flow rate. It consists of a tube that releases air bubbles into a sealed bottle or tank at a height b above the point where the liquid exits. (a) Under what conditions does Mariotte's bottle deliver a constant flow rate and thus a constant speed of liquid? Explain. (b) How does this constant speed of liquid leaving the bottle depend on b, assuming that there is no friction at the opening where liquid leaves the bottle? (Hint: Note that the pressure at the point where the bubbles are entering the bottle is equal to atmospheric pressure.)

FIGURE P14.48



49. ** **BIO** **Pressure needed for intravenous needle** A glucose solution of viscosity $2.2 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$ and density $1030\ \mathrm{kg/m^3}$ flows from an elevated open bag into a vein. The needle into the vein has a radius of 0.20 mm and is 3.0 cm long. All other tubes leading to the needle have much larger radii, and viscous forces in them can be ignored. The pressure in the vein is $1000\ \mathrm{N/m^2}$ above atmospheric pressure. (a) Determine the pressure relative to atmospheric pressure needed at the entrance of the needle to maintain a flow rate of $0.10\ \mathrm{cm^3/s}$. (b) To what elevation should the bag containing the glucose be raised to maintain this pressure at the needle?

50. ** **Viscous friction with Bernoulli** We can include the effect of viscous friction in Bernoulli's equation by adding a term for the thermal energy generated by the viscous retarding force exerted on the fluid. Show that the term to be added to Eq. (14.5) for flow in a vessel of uniform cross-sectional area A is

$$\frac{\Delta U_{\text{Th}}}{V} = \frac{4\pi\eta l v}{A}$$

where v is the average speed of the fluid of viscosity η along the center of a pipe whose length is l.

51. ** (a) Show that the work W done per unit time Δt by viscous friction in a fluid with a flow rate Q across which there is a pressure drop ΔP is

$$\frac{W}{\Delta t} = \Delta P Q = Q^2 R = \frac{\Delta P^2}{R}$$

where $R = 8\eta l/\pi r^4$ is called the *flow resistance* of the fluid moving through a vessel of radius r. (b) By what percentage must the work per unit time increase if the radius of a vessel decreases by 10% and all other quantities including the flow rate remain constant (the pressure does not remain constant)?

52. ** **BIO** **EST** **Thermal energy in body due to viscous friction** Estimate the thermal energy generated per second in a normal body due to the viscous friction force in blood as it moves through the circulatory system.

53. ** **BIO** **Essential hypertension** Suppose your uncle has hypertension that causes the radii of his 40,000,000 arterioles to decrease by 20%. Each arteriole initially was 0.010 mm in radius and 1.0 cm long. By what factor does the resistance $R = 8\eta l/\pi r^4$ to blood flow through an arteriole change because of these decreased radii? The pressure drop across all of the arterioles is about 60 mm Hg. If the flow rate remains the same, what now is the pressure drop change across the arteriole part of the circulatory system?

54. * **Parachutist** A parachutist weighing 80 kg, including the parachute, falls with the parachute open at a constant 8.5-m/s speed toward Earth. The drag coefficient $C_D = 0.50$. What is the area of the parachute?

55. A 0.20-m-radius balloon falls at terminal speed 40 m/s. If the drag coefficient is 0.50, what is the mass of the balloon?

56. ** **Terminal speed of skier** A skier going down a slope of angle θ below the horizontal is opposed by a turbulent drag force that the air exerts on the skier and by a kinetic friction force that the snow exerts on the skier. Show that the terminal speed is

$$v_T = \left[\frac{2mg(\sin\theta - \mu\cos\theta)}{C_D \rho A}\right]^{1/2}$$

where μ is the coefficient of kinetic friction between the skis and the snow, ρ is the density of air, A is the skier's frontal area, and C_D is the drag coefficient.

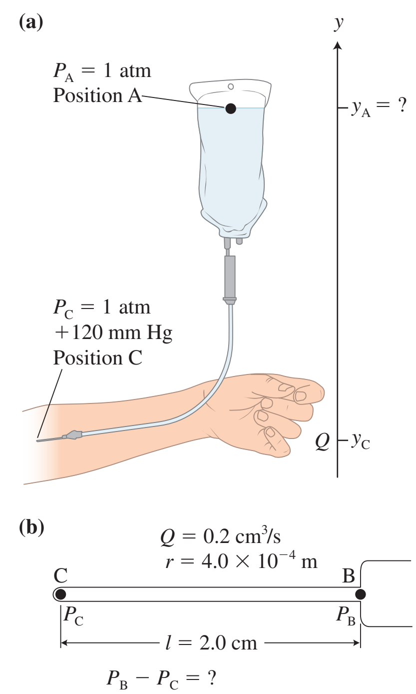
57. ** A grain of sand of radius 0.15 mm and density $2300\ \mathrm{kg/m^3}$ is placed in a 20 °C lake. Determine the terminal speed of the sand as it sinks into the lake. Do not forget to include the buoyant force that the water exerts on the grain.

58. ** **EST** **Comet crash** On June 30, 1908, a monstrous comet fragment of mass greater than 10^9 kg is thought to have devastated a 2000-km^2 area of remote Siberia (this impact was called the Tunguska event). Estimate the terminal speed of such a comet in air of density $0.70\ \mathrm{kg/m^3}$. State all of your assumptions.

Reading Passage Problems

BIO **Intravenous (IV) feeding** A patient in the hospital needs fluid from a glucose nutrient bag. The glucose solution travels from the bag down a tube and then through a needle inserted into a vein in the patient's arm (Figure 14.12a). Your study of fluid dynamics makes you think that the bag seems a little low above the arm and the narrow needle seems long. You wonder if the glucose is actually making it into the patient's arm. What height should the bag (open at the top) be above the arm so that the glucose solution (density $1000\ \mathrm{kg/m^3}$ and viscosity $1.0 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$) drains from the open bag down the 0.6-m-long, 2.0×10^{-3}-m radius tube and then through the 0.020-m-long, 4.0×10^{-4}-m radius needle and into the vein? The gauge pressure in the vein in the arm is $+930\ \mathrm{N/m^2}$ (or 7 mm Hg). The nurse says the flow rate should be $0.20 \times 10^{-6}\ \mathrm{m^3/s}$ ($0.2\ \mathrm{cm^3/s}$).

FIGURE 14.12 (a) A glucose solution flowing from an open container into a vein. (b) The analysis of the needle in this system.



59. Which answer below is closest to the speed with which the glucose should flow out of the end of the needle at position C in Figure 14.12b?
(a) 0.0004 m/s (b) 0.004 m/s (c) 0.04 m/s
(d) 0.4 m/s (e) 4 m/s

60. Which answer below is closest to the speed with which the glucose should flow through the end of the tube just to the right of position B in Figure 14.12b?
(a) 0.0002 m/s (b) 0.002 m/s (c) 0.02 m/s
(d) 0.2 m/s (e) 2 m/s

61. Assume that there is no resistive friction pressure drop across the needle (as could be determined using Poiseuille's law). Use the Bernoulli equation and the results from Problems 14.59 and 14.60 to determine which answer below is closest to the change in pressure between positions B and C ($P_B - P_C$) in Figure 14.12b.
(a) $8\ \mathrm{N/m^2}$ (b) $80\ \mathrm{N/m^2}$ (c) $800\ \mathrm{N/m^2}$
(d) $8000\ \mathrm{N/m^2}$ (e) $80{,}000\ \mathrm{N/m^2}$

62. Now, in addition to the Bernoulli pressure change from position B to position C calculated in Problem 14.61, there may be a Poiseuille resistive friction pressure change across the needle from position B to position C. Which answer below is closest to that pressure change?
(a) $0.4\ \mathrm{N/m^2}$ (b) $4\ \mathrm{N/m^2}$ (c) $40\ \mathrm{N/m^2}$
(d) $400\ \mathrm{N/m^2}$ (e) $4000\ \mathrm{N/m^2}$

63. The blood pressure in the vein at position C in Figure 14.12b at the exit of the needle into the blood is $930\ \mathrm{N/m^2}$. Use this value and the results of Problems 14.61 and 14.62 to determine which answer below is closest to the gauge pressure at position B in the tube carrying the glucose to the needle.
(a) $1010\ \mathrm{N/m^2}$ (b) $1410\ \mathrm{N/m^2}$ (c) $1980\ \mathrm{N/m^2}$
(d) $2800\ \mathrm{N/m^2}$ (e) $4620\ \mathrm{N/m^2}$

64. Suppose that there is no Poiseuille resistive friction pressure decrease from the top of the glucose solution in the open bag (position A in Figure 14.12a) through the tube and down to position C near the entrance to the needle. Which answer below is closest to the minimum height of the top of the bag in order for the glucose to flow down from the tube and through the needle into the blood? Remember that the pressure at position A is atmospheric pressure, which is zero gauge pressure.
(a) 0.04 m (b) 0.08 m (c) 0.14 m
(d) 0.27 m (e) 0.60 m

65. Suppose there is a Poiseuille resistive friction pressure decrease from the top of the glucose solution (position A in Figure 14.12a) through the tube and down to position C near the entrance to the needle. How will this affect the placement of the bag relative to the arm?
(a) The bag will need to be higher.
(b) The bag can remain the same height above the arm.
(c) The bag can be placed lower relative to the arm.
(d) Too little information is provided to answer the question.

BIO The human circulatory system In the human circulatory system, depicted in **Figure 14.13**, the heart's left ventricle pumps about 80 cm^3 of blood into the aorta every second. The blood then moves into a larger and larger number of smaller radius vessels (aorta, arteries, arterioles, and capillaries). After the capillaries, which deliver nutrients to the body cells and absorb waste products, the vessels begin to combine into a smaller number of larger radius vessels (venules, small veins, large veins, and finally the vena cava). The vena cava returns blood to the heart (see **Table 14.5**).

FIGURE 14.13 A schematic representation of the circulatory system including the pressure variation across different types of vessels.

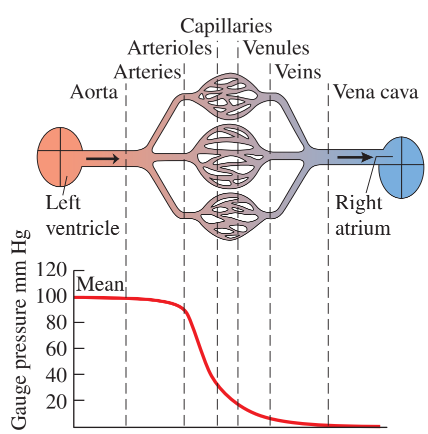


TABLE 14.5 The different types of vessels in the circulatory system

Vessel type	Number of vessels	Approximate radius (mm)
Aorta	1	5
Large arteries	40	2
Smaller arteries	2400	0.4
Arterioles	40,000,000	0.01
Capillaries	1,200,000,000	0.004
Venules	80,000,000	0.02
Small veins	2400	1
Large veins	40	3
Vena cava	1	6

The flow rate Q of blood through the arteries equals the flow rate through the arterioles, which equals the flow rate through the capillaries, and so forth. The average blood gauge pressure in the aorta is about 100 mm Hg. The pressure drops as blood passes through the different groups of vessels and is approximately 0 mm Hg when it returns to the heart at the vena cava.

A working definition of the resistance R to flow by a group of vessels is the ratio of the gauge pressure drop ΔP across those vessels divided by the flow rate Q through the vessels:

$$R = \frac{\Delta P}{Q} \tag{14.12}$$

The gauge pressure drop across the whole system is (100 mm Hg – 0), and the total resistance is

$$R_{\text{total}} = \frac{\Delta P_{\text{total}}}{Q} = \frac{100 \text{ mm Hg}}{80 \text{ cm}^3/\text{s}} = 1.25 \frac{\text{mm Hg}}{\text{cm}^3/\text{s}}$$

The gauge pressure drop across the whole system is the sum of the drops across each type of vessel:

$$\Delta P_{\text{total}} = \Delta P_{\text{aorta}} + \Delta P_{\text{arteries}} + \Delta P_{\text{arterioles}} + \Delta P_{\text{capillaries}} + \cdots + \Delta P_{\text{vena cava}}$$

Now rearrange and insert Eq. (14.12) into the above for the pressure drop across each part:

$$QR_{\text{total}} = QR_{\text{aorta}} + QR_{\text{arteries}} + QR_{\text{arterioles}} + QR_{\text{capillaries}} + \cdots + QR_{\text{vena cava}}$$

Canceling the common flow rate through each group of vessels, we have an expression for the total resistance of the circulatory system:

$$R_{\text{total}} = R_{\text{aorta}} + R_{\text{arteries}} + R_{\text{arterioles}} + R_{\text{capillaries}} + \cdots + R_{\text{vena cava}}$$

The measured gauge pressure drop across the arterioles is about 50 mm Hg, and the arteriole resistance is

$$R_{\text{arterioles}} = \frac{\Delta P_{\text{arterioles}}}{Q} = \frac{50 \text{ mm Hg}}{80 \text{ cm}^3/\text{s}} = 0.62 \frac{\text{mm Hg}}{\text{cm}^3/\text{s}}$$

or about 50% of the total resistance. The next most resistive group of vessels is the capillaries, at about 25% of the total resistance. These percentages vary significantly from person to person. A person with essential hypertension has arterioles and capillaries that are reduced in radius. The resistance to blood flow increases dramatically (the resistance has a $1/r^4$ dependence). The blood pressure has to be greater (for example, double the normal value) in order to produce reasonable flow to the body cells. Even with the increased pressure, the flow rate may still be lower than normal.

66. The capillaries typically produce about 25% of the resistance to blood flow. Which pressure drop below is closest to the pressure drop across the group of capillaries?
(a) 5 mm Hg (b) 15 mm Hg (c) 25 mm Hg
(d) 35 mm Hg (e) 45 mm Hg

67. We found that the arteriole resistance to fluid flow was about $0.62 \text{ mm Hg}/(\text{cm}^3/\text{s})$. By what factor would you expect the resistance of all the arterioles to change if the radius of each arteriole decreased to 80% of the original value?
(a) 1.3 (b) 1.6 (c) 2.4
(d) 0.4 (e) 0.6

68. Why is the resistance to fluid flow through unobstructed arteries relatively small compared to resistance to fluid flow through the arterioles and capillaries?
(a) The arteries are nearer the heart.
(b) There is a relatively small number of arteries.
(c) The artery radii are relatively large.
(d) b and c
(e) a, b, and c

69. The huge number of capillaries and venules is needed to
(a) provide nutrients (such as O_2) and remove waste products from all of the body cells.
(b) distribute water uniformly throughout the body.
(c) reduce the resistance of the circulatory system.
(d) b and c
(e) a, b, and c

70. Which number below best represents the ratio of the resistance of a single capillary to the resistance of a single arteriole, assuming they are equally long?
(a) 40 (b) 6 (c) 2.5
(d) 0.4 (e) 0.026