9

Rotational Motion

In 1967, a group of astrophysicists from the University of Cambridge in England was looking for quasars using an enormous radio telescope. Jocelyn Bell, a physics graduate student, noticed a series of regular radio pulses in the midst of a lot of receiver noise. It looked like somebody was sending a radio message, turning the signal on and off every 1.33 seconds. This is an incredibly small time for astronomical objects. At first, the astrophysicists believed that they had found signals from extraterrestrial life. The group had, in fact, discovered a new class of astronomical objects, called pulsars, which emit radio signals every second or so. The study of rotational motion explains how pulsars can emit signals so rapidly.

- **How can a star rotate 1000 times faster than a merry-go-round?**
- **Why is it more difficult to balance on a stopped bike than on a moving bike?**
- **How is the Moon slowing Earth's rate of rotation?**

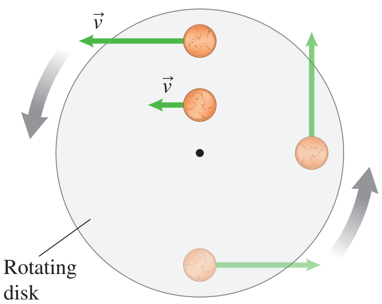
IN THE LAST CHAPTER, we only analyzed rigid bodies that remained at rest. In many cases, however, objects do not remain at rest when torques are exerted—they rotate. Think about a car tire that rotates around the axle as the car moves. In this chapter, we will learn how to describe, explain, and predict such motions.

BE SURE YOU KNOW HOW TO:

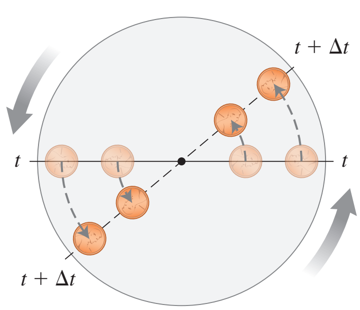
- Draw a force diagram for a system (Section 3.1).
- Determine the torque produced by a force (Section 8.2).
- Apply conditions of static equilibrium for a rigid body (Section 8.3).

FIGURE 9.1 Top views comparing the velocities of coins traveling on a rotating disk.

(a) The direction of the velocity $\vec{v}$ for each coin changes continually.



(b) Coins at the edge travel farther during Δt than those near the center. The speed v will be greater for coins near the edge than for coins near the center.



(c) All coins turn through the same angle in Δt, regardless of their position on the disk.

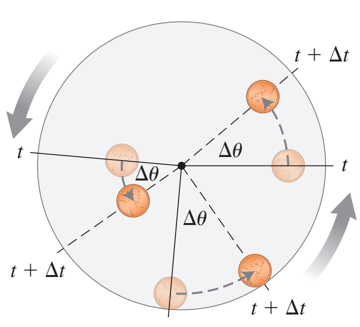
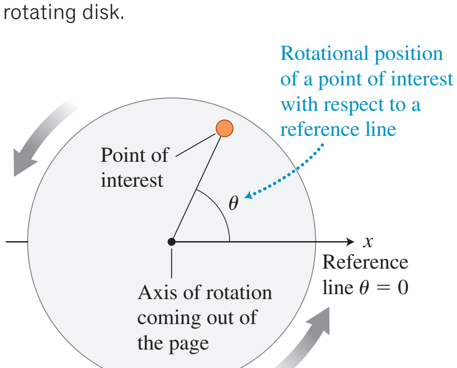


FIGURE 9.2 The rotational position of a point on a rotating disk.



9.1 Rotational kinematics

In order to understand the motion of rotating rigid bodies, we will follow the same strategy that we used for linear motion. We start by investigating how to describe rotational motion and to explain how forces and torques cause objects to rotate in the way they do. This will help us understand why a bicycle is so stable when moving, how the gravitational pull of the Moon slows the rotation of Earth, and many other interesting phenomena.

One common example of a rotating rigid body is a spinning disk, such as a turning DVD. Suppose a horizontal disk is rotating on a lab bench in front of you and you are looking down on it. You wish to describe the counterclockwise motion of the disk quantitatively. This is trickier than it might seem at first. When we investigated the motion of point-like objects, we did not have to specify which part of the object we were describing, since the object was located at a single point. With a rigid body, there are infinitely many points to choose from. For example, imagine that you place small coins at different locations on the disk, as shown in **Figure 9.1a**. As the disk turns, you observe that the direction of the velocity of each coin changes continually (see the coins on the outer edge of the disk in Figure 9.1a). In addition, a coin that sits closer to the edge moves faster and covers a longer distance during a particular time interval than a coin closer to the center (Figure 9.1b). This means that different parts of the disk move not only in different directions, but also at different speeds relative to you.

On the other hand, there are similarities between the motions of different points on a rotating rigid body. In Figure 9.1c, we see that during a particular time interval, all coins at the different points on the rotating disk turn through the same angle. Perhaps we should describe the rotational position of a rigid body using an angle.

Rotational (angular) position θ

Consider again a disk that rotates on a lab bench about a fixed point. The axis of rotation passes through the center of and is perpendicular to the disk (**Figure 9.2**). A fixed line *perpendicular* to the axis of rotation (like the positive x-axis in Figure 9.2) is used as a reference line. We can draw another line on the disk from the axis of rotation to a point of interest, for example, to a coin sitting on the rotating disk. The angle θ in the counterclockwise direction between the reference line and the line to the point of interest is the **rotational position** (or **angular position**) of the point of interest. The observer is stationary beside the lab bench and looking down on the disk.

> **Rotational position θ** The rotational position θ of a point on a rotating object (sometimes called the angular position) is defined as an angle in the counterclockwise direction between a reference line (usually the positive x-axis) and a line drawn from the axis of rotation to that point. The units of rotational position can be either degrees or radians.

Units of rotational position

The degree (°) is the most familiar unit of rotational position. There are 360° in a circle. If a point on the turning object is at the top of the circle, its position is 90° from a horizontal, positive x-axis. When at the bottom of the circle, its position is 270° or, equivalently, −90°.

The unit for rotational position that is most useful in physics is the **radian**. It is defined in terms of the two lengths shown in **Figure 9.3**. The **arc length** s is the path length in the counterclockwise (CCW) direction along the circumference of the circle from the positive x-axis to the position of a point on the circumference of the rotating object. The other length is the radius r of the circle. The angle θ in units of radians (rad) is the ratio of s and r:

$$\theta \text{ (in radians)} = \frac{s}{r} \tag{9.1}$$

Note that the radian unit has no dimensions; it is the ratio of two lengths. We can multiply by the radian unit or remove the radian unit from an equation with no consequence. If we put the unit **rad** in the equation, it is usually because it is a reminder that we are using radians for angles.

TIP From Eq. (9.1) we see that the arc length for a 1-rad angle equals the radius of the circle. For example, the 1-rad angle shown in **Figure 9.4** is the ratio of the 2-cm arc length and the 2-cm radius and is simply 1. If you use a calculator to work with radians, make sure it is in the radian mode.

One complete rotation around a circle corresponds to a change in arc length of $2\pi r$ (the circumference of the circle) and a change in rotational position of

$$\theta \text{ (one complete rotation)} = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$

Thus, there are 2π radians in one circle. We can now relate the two rotational position units:

$$360° = 2\pi \text{ rad}$$

We can use this equation to convert between degrees and radians.

We can use Eq. (9.1) to find the arc length s if the radius and rotational position θ are known:

$$s = r\theta \text{ (for } \theta \text{ in radians only)}$$

For example, if a car travels 2.0 rad around a highway curve of radius 100 m, the car travels a path length along the arc equal to

$$s = r\theta = (100 \text{ m})(2.0 \text{ rad}) = 200 \text{ m}$$

We dropped the radian unit in the answer because angles measured in radians are dimensionless.

FIGURE 9.3 The rotational position θ in radians is the ratio of the arc length s and the radius r.

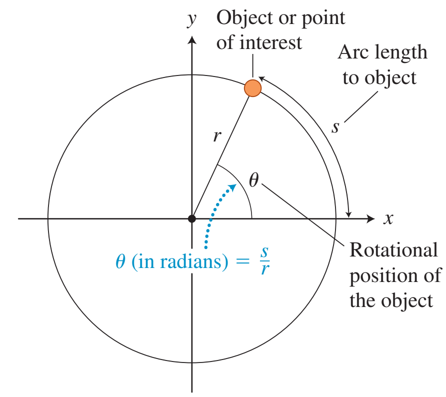
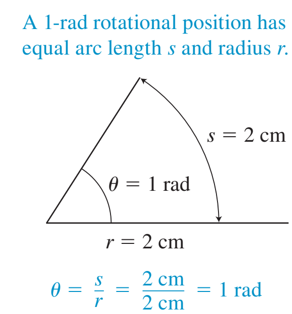


FIGURE 9.4 The arc length for a 1-rad angle equals the radius of the circle.



TIP You cannot calculate arc length using $s = r\theta$ when θ is measured in degrees. You must first convert θ to radians.

Rotational (angular) velocity ω

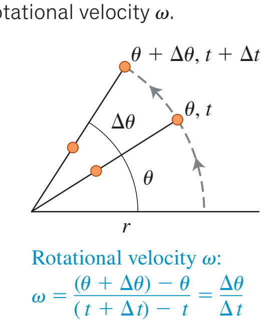
When we were investigating the motion of a point-like object along a single axis, we defined the translational velocity of that object as the rate of change of its linear position. Thus, it seems natural to define the **rotational (angular) velocity** ω of a rigid body as the rate of change of each point's rotational position. Because all points on the rigid body rotate through the same angle in the same period of time (see Figure 9.1c), each point on the rigid body has the same rotational velocity. This means we can just refer to the rotational velocity of the rigid body itself, rather than to any specific point within it.

Rotational velocity ω The average rotational velocity (sometimes called angular velocity) of a turning rigid body is the ratio of its change in rotational position $\Delta\theta$ and the time interval Δt needed for that change (see **Figure 9.5**):

$$\omega = \frac{\Delta\theta}{\Delta t} \tag{9.2}$$

The sign of ω (omega) is positive for counterclockwise turning and negative for clockwise turning, as seen looking along the axis of rotation. *Rotational (angular) speed* is the magnitude of the rotational velocity. The most common units for rotational velocity and speed are radians per second (rad/s) and revolutions per minute (rpm).

FIGURE 9.5 Each point on a rigid body has the same rotational velocity ω.



TIP Rotational velocity is the same for all points of a rotating rigid body. It is independent of the radius—the distance of a chosen point on the rigid body from the axis of rotation.

To distinguish the rotational velocity from the familiar velocity that characterizes the linear motion of an object, the latter is called linear velocity. When an object rotates, each point of the object has *linear* velocity. If you examine Figure 9.1a you see that the linear velocity vectors are tangent to the circle. Thus the linear velocity of a point on a rotating object is sometimes called **tangential velocity**.

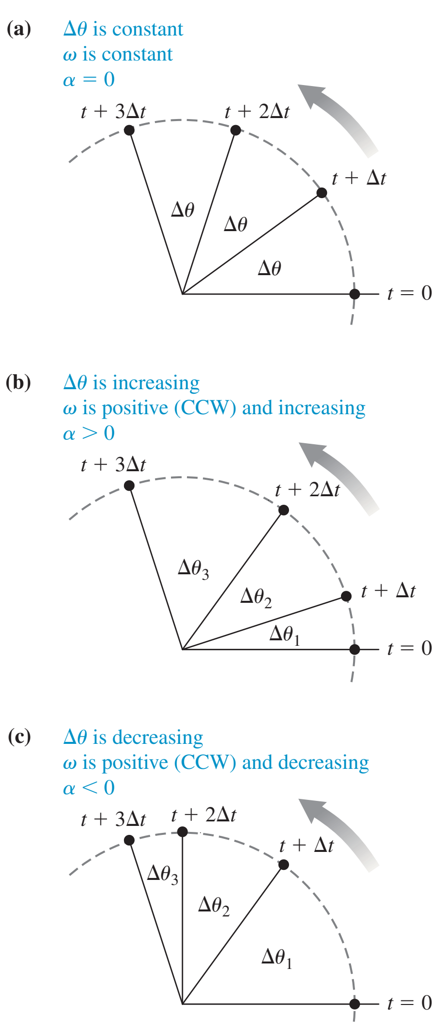
The **revolution** is a familiar unit from everyday life. One revolution (rev) corresponds to a complete rotation about a circle and equals 360°. The revolution is not a unit of rotational position. It is a unit of *change* in rotational position $\Delta\theta$. Revolutions are usually used to indicate change in rotational position per unit time. For example, a motor that makes 120 complete turns in 1 min is said to have a rotational speed of 120 revolutions per minute (120 rpm). Automobile engines rotate at about 2400 rpm.

TIP The definition of average rotational velocity or rotational speed becomes the instantaneous values of these quantities if you consider a small time interval in Eq. (9.2) and the corresponding small change in the rotational position. See our earlier discussion of instantaneous velocity in Section 2.7.

Rotational (angular) acceleration α

When we investigated the linear motion of a point-like object along a single axis, we developed the physical quantity acceleration to describe the object's change in velocity. This was translational acceleration, as it described the changing velocity of the object while moving from one position to another. We could apply the same translational acceleration idea to the center of mass of a rigid body that is moving as a whole from one position to another. But usually we are interested in the rate of change of the rigid body's rotational velocity, that is, its **rotational acceleration**. In other words, when the rotation rate of a rigid body increases or decreases, it has a nonzero rotational acceleration.

FIGURE 9.6 Three rotational motion diagrams and the corresponding signs of the rotational accelerations.



Rotational acceleration α The average rotational acceleration α (alpha) of a rotating rigid body (sometimes called angular acceleration) is its change in rotational velocity $\Delta\omega$ during a time interval Δt divided by that time interval:

$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{9.3}$$

The unit of rotational acceleration is $(\mathrm{rad/s})/\mathrm{s} = \mathrm{rad/s^2}$.

Figure 9.6 shows motion diagrams for three different types of rotational motion. Let's consider these rotational motion diagrams and try to develop a rule for how the sign of the rotational acceleration relates to the rotational velocity for a CCW-turning disk. Note that when the disk's rotational velocity is constant (the lengths of arcs between the dots are the same), its rotational acceleration is zero (Figure 9.6a). When its CCW rotational velocity (positive) is increasing (note that in Figure 9.6b the arcs between the dots increase in length), its rotational acceleration has the same sign (positive) as the rotational velocity. If the disk's CCW rotational velocity (positive) is decreasing (note that in Figure 9.6c the arcs between the dots are shrinking), its rotational acceleration has the opposite sign (negative). Similarly, when a disk is rotating CW (negative rotational velocity) faster and faster, its rotational acceleration has the same sign (negative). If the disk's clockwise rotational velocity (negative) is decreasing, its rotational acceleration has the opposite sign (positive).

What could we conclude about the signs of Earth's rotational velocity and acceleration if we were looking down on Earth from above the North Pole? The rotational velocity would have a positive sign (turning counterclockwise), and because the rotational velocity is constant, the rotational acceleration would be zero. (We will learn later that Earth's rotational velocity is not exactly constant.)

Relating translational and rotational quantities

Are there mathematical connections between physical quantities describing the rotational motion of a rigid body and the translational motion of different points on the body? Recall that the rotational position θ of a point on a turning object depends on the radial distance r of that point from the axis of rotation and the path length s measured along the arc connecting that point to the reference axis (see Figure 9.3):

$$s = r\theta \tag{9.1}$$

If the angle changes by $\Delta\theta$, the distance of the point of the object along the arc changes by Δs, so that

$$\Delta s = r\,\Delta\theta$$

Do similar relations exist for other quantities? Suppose, for example, that a point on the object changes rotational position by $\Delta\theta$ in a time interval Δt. Its rotational velocity is $\omega = \Delta\theta/\Delta t$. The change in arc length is Δs along its circular path, and its tangential speed (the speed of the object tangent to the circle, sometimes called linear speed) is $v_{\mathrm{t}} = \Delta s/\Delta t$. Substituting for Δs, we get

$$v_{\mathrm{t}} = \frac{\Delta s}{\Delta t} = \frac{r\,\Delta\theta}{\Delta t} = r\left(\frac{\Delta\theta}{\Delta t}\right) = r\omega \tag{9.4}$$

Notice that while the rotational speed of all points of the same rigid body is the same, the tangential (linear) speed of different points increases as their distance from the axis of rotation increases. A similar relationship can be derived that relates that point's acceleration a_{t} tangent to the circle and its rotational acceleration α:

$$a_{\mathrm{t}} = \frac{\Delta v_{\mathrm{t}}}{\Delta t} = \frac{r\,\Delta\omega}{\Delta t} = r\left(\frac{\Delta\omega}{\Delta t}\right) = r\alpha \tag{9.5}$$

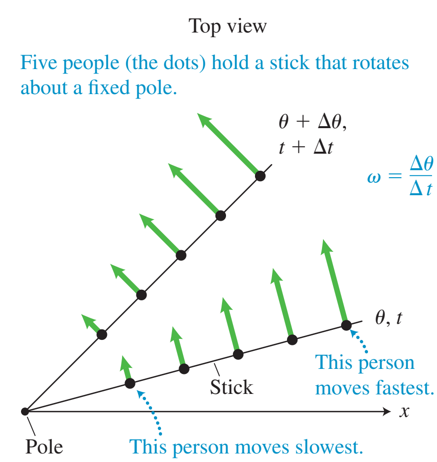
The signs of the rotational position and velocity are positive for counterclockwise turning, and the signs of the translational position and velocity are also positive for counterclockwise motion.

To visualize this relationship, imagine five people (the point objects in **Figure 9.7**) holding on to a long stick that can rotate horizontally about a vertical pole to which it is attached on one end. These people hold on to the stick as it completes a full circle. The person closest to the pole moves the slowest, the next person moves a little faster, and the one at the free end has to almost run to keep the stick in his hands. At a particular time, all of them have the same rotational position θ and the same rotational velocity ω. However, the linear distances and speeds are larger for the people farther from the axis of rotation (larger values of r).

Black holes are an extreme case of rotational motion. Black holes form when some stars at the end of their lives collapse, forming small, very dense objects. If the star was spinning when it was young, it continues to spin when it becomes a black hole, only much faster. The matter near the outer edge of the black hole is usually extremely hot gas that orbits the black hole with a tangential speed near the speed of light. The figure below is an artist's rendition of what such a black hole might look like, if we could see it.

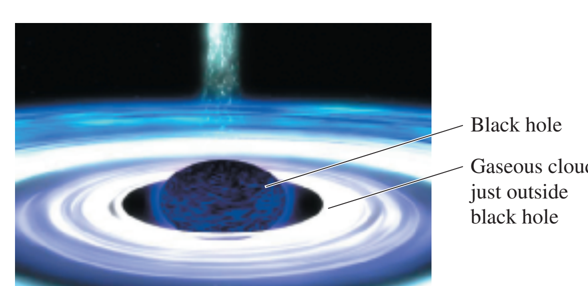
TIP You get the familiar translational quantities for motion along the circular path by multiplying the corresponding angular rotational quantities by the radius r of the circle.

FIGURE 9.7 A top-view diagram of five people (represented by dots) holding on to a stick that rotates about a fixed pole.



QUANTITATIVE EXERCISE 9.1 Orbiting a black hole

Black hole GRS 1915+105 in the constellation Aquila (the Eagle) is about 35,000 light-years from Earth. It was formed when the core of a star with about 14 solar masses (mass of 14 times the mass of our Sun) collapsed. The boundary of the black hole, called the event horizon, is a sphere with radius about 25 km. Surrounding the black hole is a stable gaseous cloud with an innermost 30-km-radius stable circular orbit. This cloud moves in a circle about the black hole about 970 times per second. Determine the tangential speed of matter in this innermost stable orbit.



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Represent mathematically We model the cloud as a rotating disk with radius $r = 30{,}000$ m; to find the rotational speed we convert the rotations in revolutions per second into radians per second: $\omega = (970\ \text{rev/s})(2\pi\ \text{rad/rev}) = 6100\ \text{rad/s}$. We need to find the tangential speed v_t of the particles of gas in orbit around the black hole, which is related to the radius r of the circular orbit and the rotational speed ω of the matter:

$$v_t = r\omega$$

Solve and evaluate The speed of the matter in the innermost stable orbit is

$$v_t = r\omega = (30{,}000\ \text{m})(6100\ \text{rad/s}) = 1.8 \times 10^8\ \text{m/s}$$

This is slightly more than half the speed of light! Actually, the physics we have developed is only moderately applicable in this environment of extreme gravitational forces and high speeds. Our answer is about 20% high compared to a more sophisticated analysis done using Einstein's theory of general relativity. Nevertheless, using ideas from uniform circular motion, we estimate that the radial acceleration of matter moving in that circular orbit is about $v^2/r \approx 10^{11}g$. It's not a good place to visit.

Try it yourself You ride a carnival merry-go-round and are 4.0 m from its center. A motion detector held by your friend next to the merry-go-round indicates that you are traveling at a tangential speed of 5.0 m/s. What are the rotational speed and time interval needed to complete one revolution on the merry-go-round?

Answer $\omega = 1.3\ \text{rad/s}$; $T = 5.0\ \text{s}$.

Rotational motion at constant acceleration

Earlier, we developed equations that related the physical quantities t, x, v, and a, which we used to describe the translational motion of a point-like object along a single axis with constant acceleration. Similar equations relate the rotational kinematics quantities t, θ, ω, and α, assuming the rotational acceleration is constant. We're not going to develop them based on observations in the way we did for the equations of translational motion, since the process will be nearly the same. Instead, we will rely on the connections we have seen between the translational and rotational quantities. The analogous rotational motion equations are provided in **Table 9.1** along with the corresponding translational motion equations. Because the quantities that describe motion depend on the choice of the reference frame, always note the location of the observer in a particular situation.

TABLE 9.1 Equations of kinematics for translational motion with constant acceleration and the analogous equations for rotational motion with constant rotational acceleration

Translational motion	Rotational motion	
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$	(9.3)
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(9.6)
$2a_x(x - x_0) = v_x^2 - v_{0x}^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$	(9.7)

For rotational motion, θ_0 is an object's rotational position at time $t_0 = 0$; ω_0 is the object's rotational velocity at time $t_0 = 0$; θ and ω are the rotational position and rotational velocity at some later time t; and α is the object's constant rotational acceleration during the time interval from time zero to time t.

- The sign of the rotational position is positive for counterclockwise θ and negative for clockwise θ from the reference axis.
- The sign of the rotational velocity ω depends on whether the object is rotating counterclockwise (+) or clockwise (−).
- The sign of the rotational acceleration α depends on how the rotational velocity is changing; α has the same sign as ω if the magnitude of ω is increasing and the opposite sign of ω if ω's magnitude is decreasing.

REVIEW QUESTION 9.1 Visualize an ice skater rotating faster and faster in a clockwise direction. What are the signs of rotational velocity and rotational acceleration? As the skater starts slowing down, what are the signs of rotational velocity and acceleration?

9.2 Physical quantities affecting rotational acceleration

What causes a rigid body to have a particular rotational acceleration? When we investigated translational motion we learned that the acceleration of a point-like object was determined by its interactions with other objects, that is, forces that objects in the environment exerted on it. Perhaps there is an analogous way to think about what causes rotational acceleration. In the last chapter, we learned that the net torque produced by forces exerted on a system had to equal zero for the object to remain in static equilibrium, to not start rotating. What happens when the net torque isn't zero? Let's investigate this. In Observational Experiment **Table 9.2**, we perform experiments with a metal arm that can rotate freely around a vertical axis. We attach to the arm two fans that we can turn on and off and move along the arm (**Figure 9.8**).

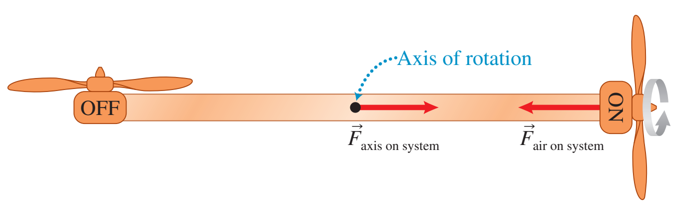
FIGURE 9.8 The equipment for the experiments in Table 9.2.

OBSERVATIONAL EXPERIMENT TABLE 9.2 Turning effects of forces exerted on a rotating arm

VIDEO OET 9.2

Observational experiment / Analysis

Experiment 1. Two fans are fixed on the arm. One fan is switched on, and it pushes air along the arm. The arm does not rotate.
(Note that all figures show the top view of the experimental setup.)

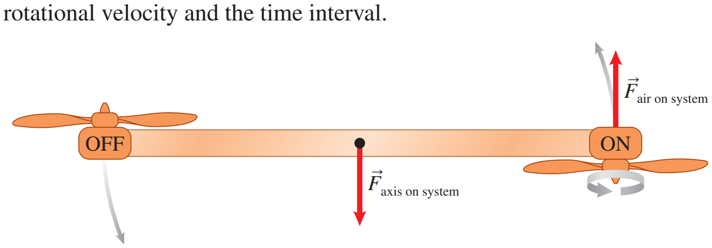


Analysis: The arm with the fans is the system. The horizontal forces are shown in the figure at left.

Because the arm does not accelerate translationally, $\vec{a}_1 = 0$. This is only possible when $\Sigma\vec{F} = 0$. The force exerted by the air on the system and the force exerted by the axis on the system therefore add to zero.

The arm does not rotate ($\omega_1 = 0$) or accelerate rotationally ($\alpha_1 = 0$). This is only possible when $\Sigma\tau = 0$; in this case, the torques produced by both forces are equal to zero.

Experiment 2. The turned-on fan rotates so that it pushes air perpendicular to the arm. When the fan is on, the arm rotates faster and faster. We determine its rotational acceleration by measuring the change in rotational velocity and the time interval.

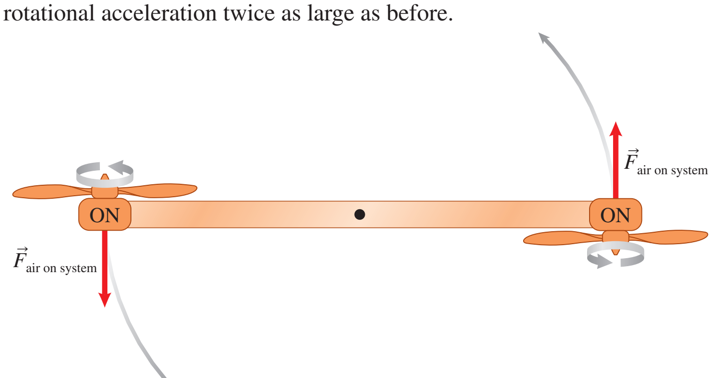


Analysis: The air exerts a force on the fan and the arm and causes a positive torque, $\tau_2 > 0$.

The arm has increasing positive rotational velocity and a positive rotational acceleration, $\alpha_2 > 0$.

The arm does not accelerate translationally, $\vec{a}_2 = 0$; this is only possible when the sum of the forces exerted on it is zero, $\Sigma\vec{F} = 0$.

Experiment 3. Both fans are turned on so that air pushes on the fans and the arm in opposite directions. The arm rotates faster and faster, with the rotational acceleration twice as large as before.



Analysis: The air pushing on the fans and the arm creates two torques of the same magnitude that are both positive:

$$\Sigma\vec{F} = 0$$
$$\vec{a}_3 = 0$$
$$\tau_2 + \tau_3 = 2\tau_2$$
$$\alpha_3 = 2\alpha_2$$

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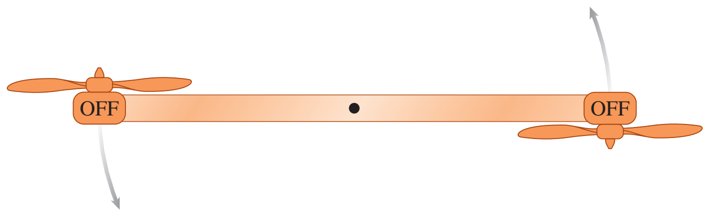
Observational experiment	Analysis
Experiment 4. While the arm is rotating, you simultaneously turn off the fans. The arm continues to rotate at a constant rotational velocity. 	$\Sigma\vec{F} = 0$ $\vec{a}_4 = 0$ $\Sigma\tau = 0$ $\alpha_4 = 0$ $\omega \neq 0$

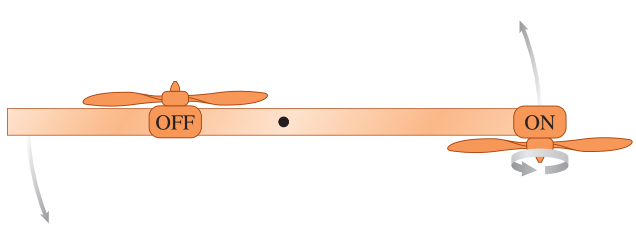
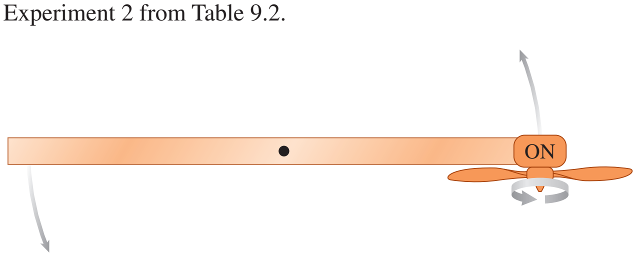
Patterns

- An external force that produces a zero torque on the arm does not change the arm's rotational velocity. If the arm is at rest, it remains at rest.
- When there are no external forces exerting torques on a rotating arm, its rotational velocity remains constant.
- External forces that produce a nonzero net torque on the arm cause rotational acceleration. Doubling the net torque doubles the rotational acceleration of the arm.

Using these patterns, we can hypothesize that the rotational acceleration of an object with fixed axis of rotation depends on the external net torque exerted on the object. But is it just the torque that determines the acceleration? In translational motion, the other quantity affecting translational acceleration was the mass of the object. What is the analogous quantity in rotational motion? Could it be the mass, too? Let's test the hypothesis that the rotational acceleration depends on the sum of the external torques and the mass of the object in Testing Experiment **Table 9.3**.

TESTING EXPERIMENT TABLE 9.3 Testing the hypothesis that mass affects rotational acceleration

VIDEO TET 9.3

Testing experiment	Prediction	Outcome
Experiment 1. Repeat Experiment 2 from Table 9.2, but this time move the turned-off fan closer to the axis of rotation. 	If the rotational acceleration depends on the external torques and the mass of the system and we have the same fans as in Experiment 2 from Table 9.2, then changing the location of the turned-off fan should not change the rotational acceleration of the system.	The rotational acceleration of the arm is greater than in Experiment 2 from Table 9.2.
Experiment 2. Remove the turned-off fan from the arm and repeat Experiment 2 from Table 9.2. 	Because the mass of the system decreases, the rotational acceleration should increase.	The rotational acceleration of the arm is greater than in Experiment 2 from Table 9.2 and greater than in Experiment 1 above.

Conclusion

Although we found that the mass of the system affects its rotational acceleration, we rejected the hypothesis that the rotational acceleration depends *only* on the net exerted torque and the mass of the object. We found that the distribution of mass with respect to the axis of rotation is important, too.

We found that the distribution of mass with respect to the axis of rotation affects an object's rotational acceleration. A simple experiment that you can perform in your kitchen helps you "feel" the effect of mass distribution with respect to the axis of rotation. Lay a broom on a hard, smooth floor. First, try to increase the broom's rotational speed about a vertical axis by spinning it with one hand holding the broomstick near the end opposite the broom head. Then do it again while holding the broom in the middle, nearer the broom head. It is much easier to increase the rotational speed while holding it in the middle. You are turning the same mass, but the location of that mass with respect to the axis of rotation makes a difference—just as we found in Table 9.3.

We can now summarize two patterns that we have discovered so far:

Changes in rotational velocity Rotational acceleration depends on net torque. The greater the net torque, the greater the rotational acceleration. Rotational acceleration also depends on the total mass of the object and the mass distribution with respect to the axis of rotation.

Let us focus on the *distribution* of mass of the rotating object. The closer the mass of the object to the axis of rotation, the easier it is to change its rotational motion. We call the physical quantity characterizing the location of the mass relative to the axis of rotation the **rotational inertia** (also known as the **moment of inertia**) of the object. Rotational inertia I depends on both the total mass of the object and the distribution of that mass about its axis of rotation. For objects of the same mass, the more mass that is located near the axis of rotation, the smaller the object's rotational inertia will be. Likewise, the more mass that is located farther away from the axis of rotation, the greater the object's rotational inertia will be. For objects of different mass but the same mass distribution, the more massive object has more rotational inertia. The higher the rotational inertia of an object, the harder it is to change its rotational motion. In summary, this quantity is the rotational equivalent of mass.

We have now found two factors that affect the rotational acceleration of an object:

- The rotational inertia of the object
- The net torque produced by forces exerted on the object

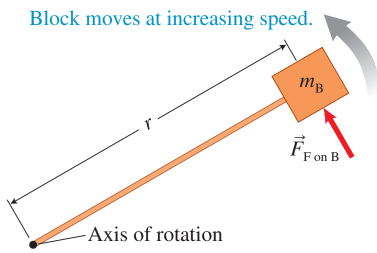
Notice how this is similar to what we learned when studying translational motion. A nonzero net force (sum of the forces) needs to be exerted on an object to cause its velocity to change. The greater the net force, the greater the translational acceleration of the object. The mass of the object affects its acceleration, too—the greater the mass, the smaller the acceleration for the same net force.

Remember that in all of the experiments we have performed so far in this chapter, rotational motion was all that was possible. Each rigid body was rotating about a fixed axis through its center of mass. If the rigid body were not held fixed, then a change in both translational and rotational motion could occur. The translational acceleration of the center of mass of such an object is determined by Newton's second law $\vec{a} = \Sigma\vec{F}/m$. The rotational acceleration around its center of mass will be determined by the ideas we will investigate over the next several sections.

REVIEW QUESTION 9.2 A solid wooden ball and a smaller solid metal ball have equal mass (the metal ball is smaller because it is much denser than wood). Both can rotate on an axis going through their centers. You exert a force on each that produces the same torque about the axis of rotation. Which sphere's rotational motion will change the least? Explain.

9.3 Newton's second law for rotational motion

FIGURE 9.9 A top view of an experiment to relate torque and rotational acceleration.



To construct a quantitative relation between rotational acceleration, net torque, and rotational inertia we start with a simple example of rotational motion: a small block attached to a light stick that can move on a smooth surface in a circular path (**Figure 9.9**). The axis of rotation passes through a pin at the other end of the stick. After we analyze this case, we will generalize the result to the rotation of an extended rigid body. You push the block with your finger, exerting a small force $\vec{F}_{\text{F on B}}$ on the block tangent to the circular path. This push causes a torque, which in turn causes the block and stick's rotational velocity about the pin to increase.

The torque produced by the force $\vec{F}_{\text{F on B}}$ is

$$\tau = rF_{\text{F on B}} \sin\theta = rF_{\text{F on B}} \sin 90^\circ = rF_{\text{F on B}}$$

Since the block is small, we can reasonably model it as a point-like object. This allows us to apply Newton's second law. Since the mass of the block is much larger than the mass of the stick, we assume that the stick has no mass. The finger exerts a force of constant magnitude pushing lightly in a direction tangent to the block's circular path. Thus, the tangential component of Newton's second law for the block is

$$a_t = \frac{F_{\text{F on B}}}{m_B}$$

There is a mathematical way to get the torque produced by the pushing force. Rearrange the above equation to get

$$m_B a_t = F_{\text{F on B}}$$

Then multiply both sides of the equation by r, the radius of the circular path:

$$m_B\, r\, a_t = rF_{\text{F on B}}$$

Recall from Eq. (9.5) that $a_t = r\alpha$. Thus,

$$m_B\, r\,(r\alpha) = rF_{\text{F on B}}$$

The right side of this equation equals the torque τ caused by $\vec{F}_{\text{F on B}}$:

$$(m_B r^2)\,\alpha = \tau$$

$$\Rightarrow \alpha = \frac{\tau}{m_B r^2}$$

Examine the above equation and compare it to Newton's second law for the same object—the block not only acquires translational acceleration a_t due to the force exerted on it by the finger, but also acquires rotational acceleration around the axis caused by the torque produced by that same force. This rotational acceleration is directly proportional to the torque produced by the force and inversely proportional to the mass of the block times the square of the distance between the block and the axis of rotation. The latter makes sense—we found experimentally that the farther the mass of the object is from the axis of rotation, the harder it is to change its rotational velocity. Thus the denominator in the equation above is an excellent candidate for the rotational inertia of the block about the pin (the axis of rotation in this situation).

In the above thought experiment, there was just a single force exerted on the object producing a torque about the axis of rotation. More generally, there could be several forces producing torques. It's reasonable that we should add the torques produced by all forces exerted on the object to determine its rotational acceleration:

$$\alpha = \frac{1}{m_B r^2}\Sigma\tau = \frac{1}{m_B r^2}(\tau_1 + \tau_2 + \ldots) \tag{9.8}$$

where $\tau_1, \tau_2, \ldots$ are the torques produced by forces $\vec{F}_{1\text{ on O}}, \vec{F}_{2\text{ on O}}, \ldots$ exerted on the object.

Analogy between translational motion and rotational motion

Notice how similar Eq. (9.8)

$$\alpha = \frac{1}{m_B r^2}\Sigma\tau$$

is to Newton's second law for translational motion

$$\vec{a} = \frac{1}{m}\Sigma\vec{F}$$

When forces are exerted on a point-like object, we can describe its motion using two acceleration-type quantities—translational and rotational acceleration when the axis around which the object rotates is located outside the object. The translational acceleration is determined by Newton's second law, and the rotational acceleration is determined by Eq. (9.8), called **Newton's second law for rotational motion**. There is a strong analogy between each of the three quantities in the two equations (see **Table 9.4**).

For translational motion, mass is a measure of an object's resistance to the changes in its translational motion. For the rotational motion of a point-like object, the object's mass times the square of its distance r from the axis of rotation (mr^2) is a measure of the object's resistance to the changes in its rotational motion. In summary, the quantity mr^2 is the rotational inertia I of a point-like object of mass m around the axis that is the distance r from the location of the object.

For translational motion, the net force $\Sigma\vec{F}$ exerted on an object of interest by other objects causes that object's velocity to change—it has a translational acceleration $(\vec{a} = \Delta\vec{v}/\Delta t)$. For rotational motion, the net torque $\Sigma\tau$ produced by forces exerted on the object causes its rotational velocity to change—it has a rotational acceleration $(\alpha = \Delta\omega/\Delta t)$.

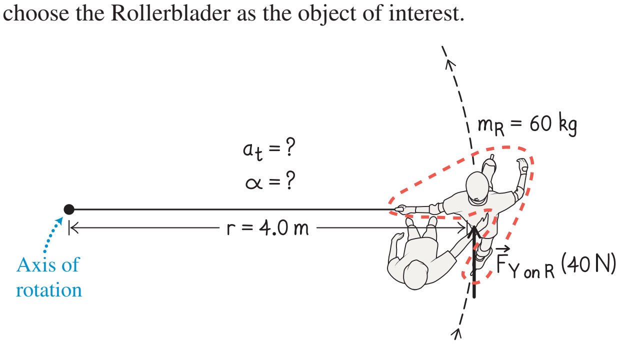
TABLE 9.4 Analogy between translational and rotational quantities in Newton's second law

	Translational motion	Rotational motion
Inertia of a point-like object	m	mr^2
Cause of acceleration	$\Sigma\vec{F}$	$\Sigma\tau$
Acceleration	$\vec{a}$	α

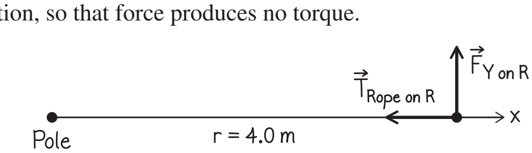
EXAMPLE 9.2 Pushing a Rollerblader

A 60-kg Rollerblader holds a 4.0-m-long rope that is loosely tied around a metal pole. You push the Rollerblader, exerting a 40-N force on her, which causes her to move increasingly fast in a counterclockwise circle around the pole. The surface she skates on is smooth, and the wheels of her Rollerblades are well oiled. Determine the tangential and rotational acceleration of the Rollerblader.

Sketch and translate We sketch the situation as shown below. We choose the Rollerblader as the object of interest.



Simplify and diagram Since the size of the Rollerblader is small compared to the length of the rope, we can model her as a point-like object. The following figure shows a force diagram for the Rollerblader (viewed from above). Her tangential acceleration has no vertical component along a vertical axis (which would extend up and out of the page in the figure). The upward normal force $\vec{N}_{F\ on\ R}$ that the floor exerts on her balances the downward gravitational force $\vec{F}_{E\ on\ R}$ exerted by Earth on her (these forces are not shown in the figure). The tension force exerted by the rope on the Rollerblader $\vec{T}_{Rope\ on\ R}$ points directly toward the axis of rotation, so that force produces no torque.



Represent mathematically From the force diagram we conclude that your push $\vec{F}_{Y\ on\ R}$ on the Rollerblader is the only force that produces a nonzero torque $\tau = rF_{Y\ on\ R}\sin 90° = rF_{Y\ on\ R}$, where r is the radius of the Rollerblader's circular path.

Use Newton's second law in the tangential direction to determine the Rollerblader's tangential acceleration:

$$a_t = \frac{1}{m_R}\Sigma F_t = \frac{F_{Y\ on\ R}}{m_R}$$

Use Newton's second law for rotational motion [Eq. (9.8)] to determine the Rollerblader's rotational acceleration:

$$\alpha = \frac{1}{m_R r^2}\Sigma\tau = \frac{1}{m_R r^2}\tau_{Y\ on\ R} = \frac{1}{m_R r^2}(rF_{Y\ on\ R}) = \frac{F_{Y\ on\ R}}{m_R r}$$

Solve and evaluate For the tangential acceleration:

$$a_t = \frac{F_{Y\ on\ R}}{m_R} = \frac{40\ \text{N}}{60\ \text{kg}} = 0.67\ \text{m/s}^2$$

(CONTINUED)

For the rotational acceleration:

$$\alpha = \frac{F_{\text{Y on R}}}{m_{\text{R}} r} = \frac{40\ \text{N}}{(60\ \text{kg})(4.0\ \text{m})} = 0.17\ \text{rad/s}^2$$

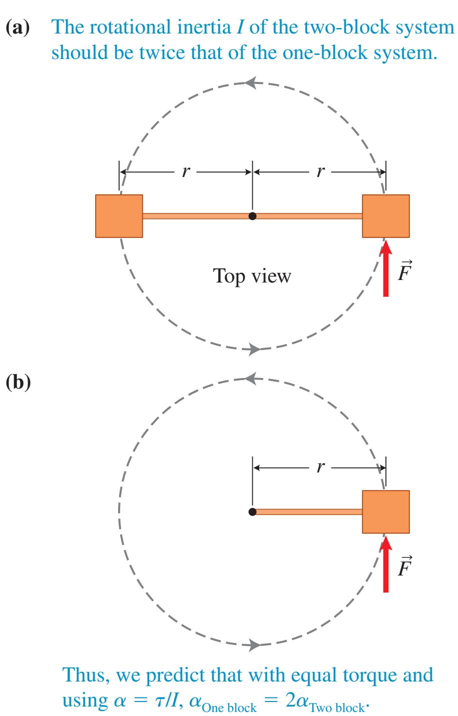
Let's check the units; note that $\frac{\text{N}}{\text{kg m}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{kg m}} = \frac{1}{\text{s}^2}$. Remember that the radian is not an actual unit. It is dimensionless. It is just a reminder that this is the angle unit appropriate for these calculations. So the units are correct, and the magnitudes for both results are reasonable. The Rollerblader would have a rotational velocity of 0.17 rad/s after 1 s, 0.34 rad/s after 2 s, and so forth—the rotational velocity increases 0.17 rad/s each second.

Try it yourself Suppose you exerted the same force on your friend, but the friend is holding an 8.0-m-long rope instead of a 4.0-m-long rope. How will this affect the rotational acceleration?

Answer The tangential acceleration will not change, but the rotational acceleration will be half as large. The torque will be doubled because the distance from the axis of rotation to the point where the force is applied will be doubled. But the rotational inertia (mr^2) in the denominator will be quadrupled because of the r^2. The combination will be a reduction of the rotational acceleration by half—0.08 rad/s^2.

Newton's second law for rotational motion applied to rigid bodies

FIGURE 9.10 Comparing the effect of rotational inertia on rotational acceleration.



We know that the mass of an object composed of many small objects with masses m_1, m_2, m_3, etc. is the sum of the masses of its parts: $m = m_1 + m_2 + m_3 + \ldots.$ Mass is a scalar quantity and therefore is always positive. The rotational inertia of a point-like object with respect to some axis of rotation is a scalar quantity. Thus, it is reasonable to think that the same rule applies to rigid bodies: the rotational inertia of a rigid body about some axis of rotation is the sum of the rotational inertias of the individual point-like objects that make up the rigid body.

To test this idea, we can use it to calculate the rotational inertia of a lightweight stick with a block attached to each end, as shown in **Figure 9.10a**. The axis of rotation is at the middle of the stick. If our reasoning is correct, the rotational inertia of this two-block rigid body should be twice the rotational inertia of a single block at the end of a stick that is half the length (Figure 9.10b): $I_{\text{Two block}} = 2I_{\text{One block}}$.

If this is correct, and we exert the same torque $(\tau = rF \sin 90^\circ = rF)$ on the two-block–stick system, the rotational acceleration of this system should be half the rotational acceleration of the one-block–stick system:

$$\alpha_{\text{Two block}} = \frac{\tau}{I_{\text{Two block}}} = \frac{\tau}{2I_{\text{One block}}} = \frac{1}{2}\left(\frac{\tau}{I_{\text{One block}}}\right) = \frac{1}{2}\alpha_{\text{One block}}$$

We check this prediction by performing a testing experiment. We exert the same force on the two-block system and the one-block system, thus producing the same torque, and measure the angular acceleration of each system. The outcome matches the above prediction. The rotational inertia of the two-block system is twice that of the one-block system.

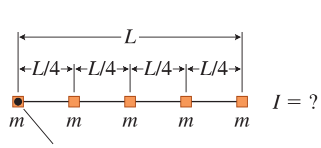
It appears that the rotational inertia of a rigid body that consists of several point-like parts located at different distances from the axis of rotation is the sum of the mr^2 terms for each part:

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots$$

Let's apply this idea.

QUANTITATIVE EXERCISE 9.3 Rotational inertia of a rigid body

Use what you learned about rotational inertia to write an expression for the rotational inertia of the rigid body shown at right. Each of the five blocks has mass m. They are connected with lightweight sticks of equal length $L/4$.



Represent mathematically Each block of mass m contributes differently to the rotational inertia of the system. The farther the block from the axis of rotation, the greater its contribution to the rotational inertia of the system of blocks. We can add the rotational inertia of each block of mass m about the axis of rotation:

$$I = m(0)^2 + m(L/4)^2 + m(2L/4)^2 + m(3L/4)^2 + m(4L/4)^2$$

Solve and evaluate When added together, the rotational inertia of the five-block system is $I = 1.88mL^2$. Each block (except the one located at the axis of rotation) contributes to the system's rotational inertia. However, blocks farther from the axis of rotation contribute much more than those near the axis. In fact, the block at the right side of the rod contributes more to the rod's rotational inertia (mL^2) than the other four blocks combined ($0.88mL^2$).

Try it yourself Calculate the rotational inertia of the same system altered so that the axis of rotation passes perpendicular through the central block.

Answer $0.625mL^2$.

Calculating rotational inertia

We can calculate the rotational inertia of a rigid body about a specific axis of rotation in about the same way we determined the rotational inertia of the five-block system in Quantitative Exercise 9.3, by adding the rotational inertias of each part of the entire system. However, for most rigid bodies, the parts are not separate objects but are instead parts in a continuous distribution of mass—like in a door or baseball bat. In such a case, we break the continuous distribution of mass into very small pieces and add the rotational inertias for all of the pieces. Consider the person's leg shown in **Figure 9.11**, which we model as a rigid body if none of the joints bend. For example, mass element 7 contributes an amount $m_7r_7^2$ to the rotational inertia of the leg. The rotational inertia of the whole leg is then

$$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_7r_7^2 + \ldots + m_{18}r_{18}^2 \quad (9.9)$$

All r's in the above are from the same axis of rotation. The rotational inertia would be different if we chose a different axis of rotation. There are other ways to do the summation process in Eq. (9.9); often it is done using integral calculus, and sometimes I is determined experimentally.

Table 9.5 gives the rotational inertias of some common *uniform* objects (a uniform object is made of the same material throughout) for specific axes of rotation. Notice the coefficients in front of the mR^2 and mL^2 expressions for the objects of different shapes. The value of the coefficient is determined by the mass distribution inside the object and the location of the axis of rotation.

FIGURE 9.11 Add the mr^2 of all the small parts to find the rotational inertia I of the leg.

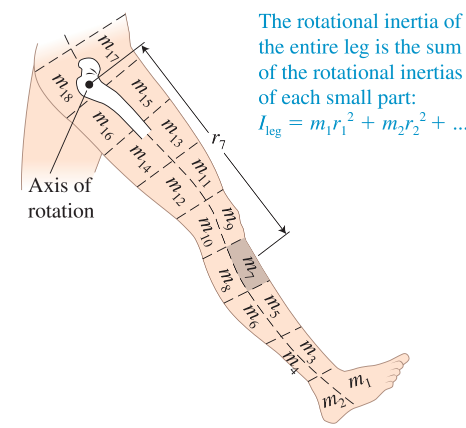
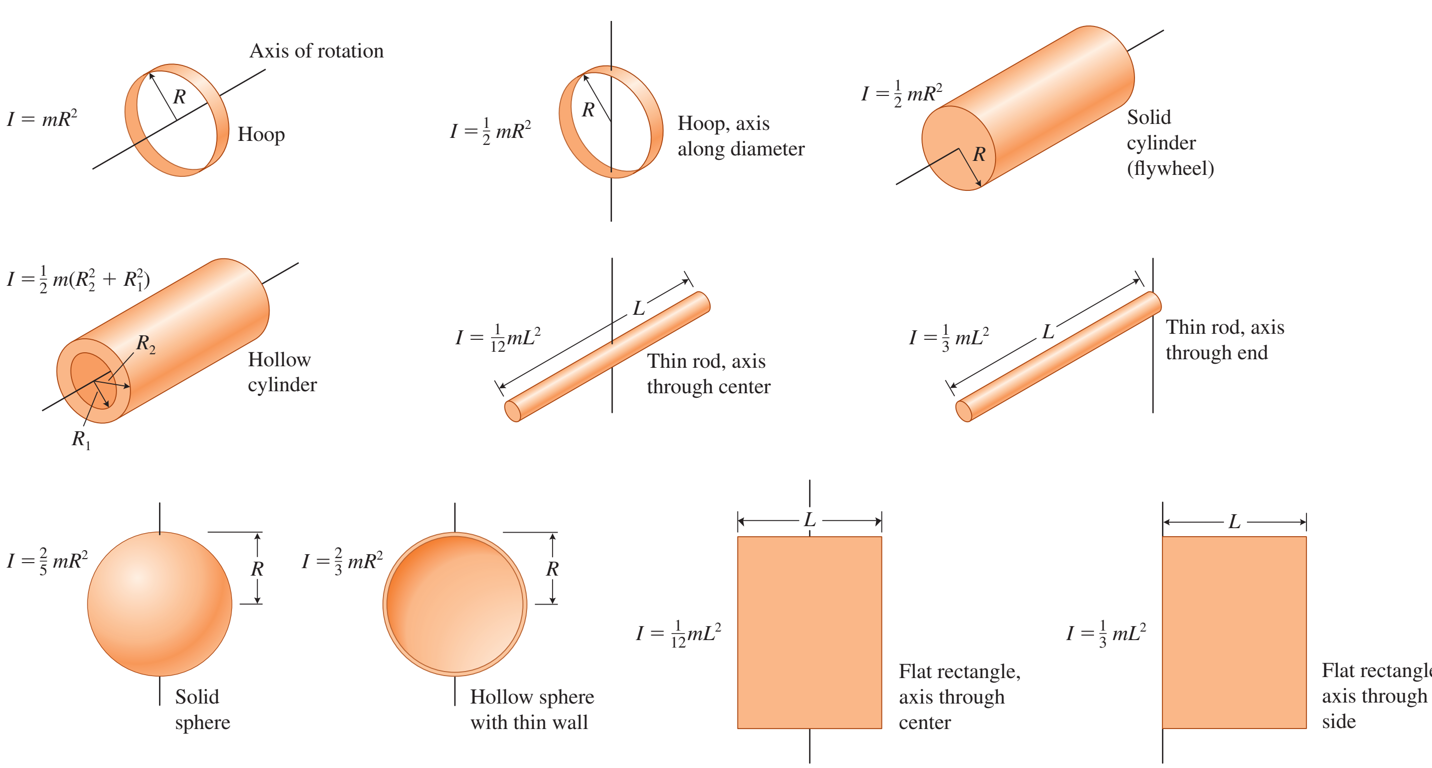


TABLE 9.5 Expressions for the rotational inertia of standard shape objects



We can now rewrite the rotational form of Newton's second law in terms of the rotational inertia of the rigid body.

Rotational form of Newton's second law One or more objects exert forces on a rigid body with rotational inertia I that can rotate about some axis. The sum of the torques $\Sigma\tau$ due to these forces about that axis causes the object to have a rotational acceleration α:

$$\alpha = \frac{1}{I}\Sigma\tau \qquad (9.10)$$

TIP By writing Newton's second law in the form

$$\vec{a}_S = \frac{1}{m_S}\Sigma\vec{F}_{\text{on S}} = \frac{\vec{F}_{O_1\text{ on S}} + \vec{F}_{O_2\text{ on S}} + \ldots + \vec{F}_{O_n\text{ on S}}}{m_S}$$

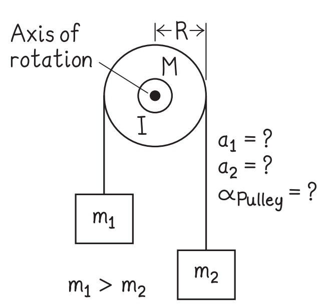
we see the cause-effect relationship between the net force $\Sigma\vec{F}_{\text{on S}}$ exerted on the system and the system's resulting translational acceleration $\vec{a}_S$. The same idea is seen in Eq. (9.10), only applied to the rotational acceleration:

$$\alpha_S = \frac{1}{I_S}\Sigma\tau = \frac{\tau_1 + \tau_2 + \ldots + \tau_n}{I_S}$$

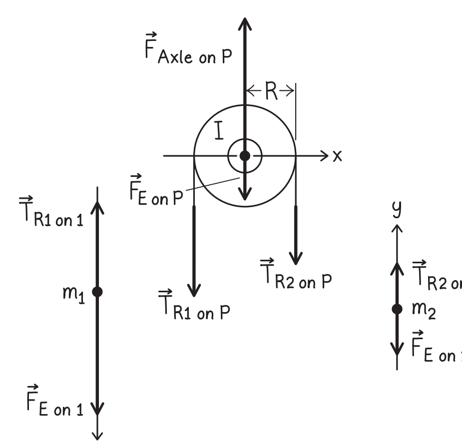
EXAMPLE 9.4 Atwood machine

In an Atwood machine, a block of mass m_1 and a less massive block of mass m_2 are connected by a string that passes over a pulley of mass M and radius R. What are the translational accelerations a_1 and a_2 of the two blocks and the rotational acceleration α of the pulley?

Sketch and translate A sketch of the situation is shown here. You might recall that we analyzed a similar situation previously (in Section 4.4). However, at that time we assumed that the pulley had negligible (zero) mass. We had no choice but to make that assumption because we had not yet developed the physics for rotating rigid bodies. Now that we have, we can analyze the situation in different ways depending on the choice of system. We analyze the situation using three separate systems: block 1, block 2, and the pulley, and then combine the analyses to answer the questions.



Simplify and diagram We model the blocks as point-like objects and the pulley as a rigid body. Force diagrams for all three objects are shown at top right. A string wrapped around the rim of a pulley (or any disk/cylinder) pulls purely tangentially, so the torque it produces is simply the product of the magnitude of the force and the radius of the pulley. Previously, we assumed that the force exerted by the string pulling down on each side of the massless and frictionless pulley was the same—the pulley just changed the direction of the string but not the tension it exerted on the blocks below. Now, with a pulley with nonzero mass, the forces that the string exerts on two sides are different. If they were not, the pulley would not have a rotational acceleration. We assume that the string does not stretch (the translational acceleration of the blocks will have the same magnitude $a_1 = a_2 = a$) and that the string does not slip on the pulley (a point on the edge of the pulley has the same translational acceleration as the blocks). We also assume that the pulley's axle is oiled enough that the frictional torque can be ignored.



The translational acceleration of the hanging objects is due to the difference between the gravitational force that Earth exerts on them and the tension force that the string exerts on them. The rotational acceleration of the pulley is due to a nonzero net torque produced by the two tension forces exerted on the pulley.

We consider the pulley to be similar to a solid cylinder. Then according to Table 9.5, its rotational inertia around the axis that passes through its center is $I = \frac{1}{2}MR^2$, where R is the radius of the pulley and M is its mass.

Represent mathematically The force diagrams help us apply Newton's second law in component form for the two blocks and the rotational form for the pulley. The coordinate systems used in each case are shown. We choose the coordinate systems so the translational accelerations of both blocks are positive.

Block on the left: $m_1 a = +m_1 g + (-T_{\text{R1 on 1}})$

Block on the right: $m_2 a = -m_2 g + T_{\text{R2 on 2}}$

The pulley: $T_{\text{R1 on P}}R + (-T_{\text{R2 on P}}R) = I\alpha = \left(\dfrac{MR^2}{2}\right)\left(\dfrac{a}{R}\right)$

$$\Rightarrow (T_{\text{R1 on 1}})R - (T_{\text{R2 on 2}})R = \frac{MRa}{2}$$

$$\Rightarrow T_{\text{R1 on 1}} - T_{\text{R2 on 2}} = \frac{Ma}{2}$$

We now have three equations with three unknowns—the two tension forces exerted by the rope on the pulley and the magnitude of the acceleration a of the blocks. We can write expressions for $T_{\text{R1 on 1}}$ and $T_{\text{R2 on 2}}$ using the first two equations. We then have

$$T_{\text{R1 on 1}} = m_1 g - m_1 a$$

$$T_{\text{R2 on 2}} = m_2 g + m_2 a$$

After substituting these expressions for the rope forces into the pulley equation, we get

$$(m_1 g - m_1 a) - (m_2 g + m_2 a) = Ma/2$$

Solve and evaluate This equation can be rearranged to get an expression for the translational acceleration of the blocks:

$$a = \frac{m_1 - m_2}{\frac{1}{2}M + m_1 + m_2}g$$

Notice that if we neglect the mass of the pulley, the acceleration becomes

$$a = \frac{m_1 - m_2}{m_1 + m_2}g$$

a larger acceleration than with the pulley (and consistent with the result we got in Chapter 4). Thus, the massive pulley decreases the acceleration of the blocks, which makes sense. If the pulley mass M is much heavier than the masses of the hanging objects m_1 and m_2, the acceleration becomes very small—it is almost like the blocks are hanging from a fixed massive object and not moving at all. We can find the rotational acceleration of the pulley by dividing the translational acceleration by the radius of the pulley because the string does not slip on the pulley.

$$\alpha = \frac{a}{R} = \left(\frac{m_1 - m_2}{\frac{1}{2}M + m_1 + m_2}\right)\frac{g}{R}$$

Try it yourself Determine the translational acceleration of the blocks and the rotational acceleration of the pulley for the following given information: $m_1 = 1.2$ kg, $m_2 = 0.8$ kg, $M = 1.0$ kg, and $R = 0.20$ m.

Answer $a = 1.6\ \text{m/s}^2$ and $\alpha = 7.8\ \text{rad/s}^2$.

Notice the calculations of the translational acceleration of the blocks in Example 9.4. The acceleration is equal to the sum of the forces divided by an effective total mass of the system.

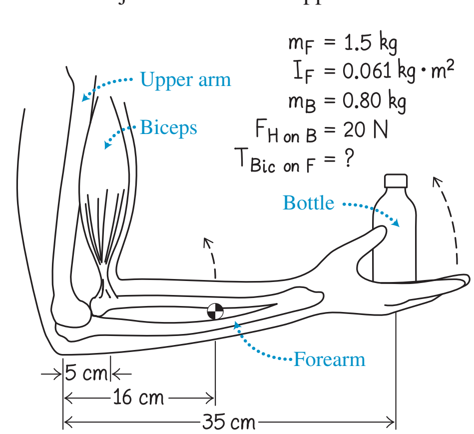
$$a = \frac{m_1 - m_2}{\frac{1}{2}M + m_1 + m_2}g$$

Here you see that the pulley only contributes half of its mass due to its mass distribution. We can say that the pulley's "effective mass" is $M/2$ because of how its mass is distributed.

EXAMPLE 9.5 Throwing a bottle

A woman tosses a 0.80-kg soft drink bottle vertically upward to a friend on a balcony above. At the beginning of the toss, her forearm rotates upward from the horizontal so that her hand exerts a 20-N upward force on the bottle. Determine the force that her biceps exerts on her forearm during this initial instant of the throw. The mass of her forearm is 1.5 kg, and its rotational inertia about the elbow joint is 0.061 kg • m^2. The attachment point of the biceps muscle is 5.0 cm from the elbow joint, the hand is 35 cm away from the elbow, and the center of mass of the forearm/hand is 16 cm from the elbow.

Sketch and translate A sketch of the situation is shown at right. There is no information given about the kinematics of the process (for example, no way to directly determine the rotational acceleration of her arm). How can we use the rotational form of Newton's second law to determine the unknown force that the biceps muscle exerts on the woman's forearm during the throw? We do know the force her hand exerts on the bottle and the bottle's mass. So, in the first part of the problem, we can first use the translational form of Newton's second law with the bottle as the system to find the bottle's vertical acceleration at the beginning of the throw. We can then use this acceleration to find the angular acceleration of the arm and then finally use the rotational form of Newton's second law to find the force that the biceps muscle exerts on her arm during the throw. For this second part of the problem, we choose the lower arm and hand as the system of interest. The axis of rotation is at the elbow joint between the upper arm and the forearm.



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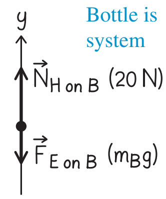
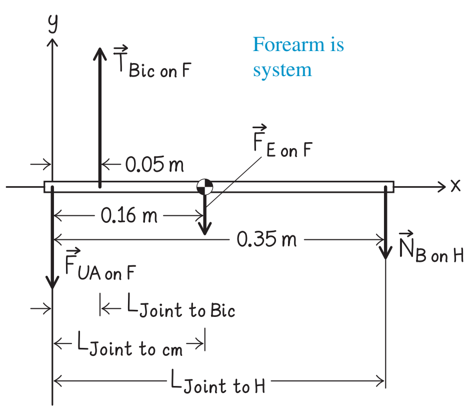
Simplify and diagram The figure at right is a force diagram for the bottle as a system. Earth exerts a downward 7.8-N gravitational force $\vec{F}_{\text{E on B}}$ on the bottle and the woman's hand exerts an upward 20-N normal force $\vec{N}_{\text{H on B}}$ on the bottle. Since these forces do not cancel, the bottle has an initial upward acceleration. Next, consider the forearm and hand as the system. Assume that the forearm and hand form a rigid body. The bottle exerts a downward 20-N force on her hand $\vec{N}_{\text{B on H}}$. Earth exerts a downward gravitational force $\vec{F}_{\text{E on F}}$ on the forearm at its center of mass. Her biceps muscle exerts an upward tension force $\vec{T}_{\text{Bic on F}}$. The upper arm presses down on the forearm at the joint, exerting a force $\vec{F}_{\text{UA on F}}$. If the upper arm did not push down, the forearm at the joint would fly upward when the biceps muscle pulled up on it.





Represent mathematically We first analyze the bottle's motion to determine its translational acceleration; then determine the rotational acceleration of the forearm and hand system; and finally apply the rotational form of Newton's second law to find the force that the biceps needs to exert on the system to cause this rotational acceleration. Consider the initial instant of the bottle's upward trip. The y-component form of Newton's second law applied to the bottle can be used to determine the vertical acceleration $a_{\text{B}y}$ for the bottle:

$$a_{\text{B}y} = \frac{N_{\text{H on B }y} + F_{\text{E on B }y}}{m_{\text{B}}} = \frac{N_{\text{H on B}} + (-m_{\text{B}}g)}{m_{\text{B}}}$$

The rotational acceleration of the forearm/hand system at that instant is related to the vertical acceleration of the bottle:

$$\alpha_{\text{F}} = \frac{a_{\text{B}y}}{r}$$

where r is the distance from the axis of rotation to the hand. The magnitude of the force that the biceps muscle exerts on the forearm ($T_{\text{Bic on F}}$) can be determined using the rotational form of Newton's second law applied to the forearm/hand. Notice that here the system consists of different parts joined together.

$$T_{\text{Bic on F}}L_{\text{Joint to Bic}} + (-F_{\text{E on F}}L_{\text{Joint to cm}}) + (-N_{\text{B on H}}L_{\text{Joint to H}}) = I_{\text{F}}\alpha_{\text{F}}$$

Solve and evaluate We now use the known values of the quantities to solve the problem:

$$a_{\text{B}y} = \frac{N_{\text{H on B}} - m_{\text{B}}g}{m_{\text{B}}} = \frac{20\text{ N} - (0.80\text{ kg})(9.8\text{ N/kg})}{(0.80\text{ kg})} = 15.2\text{ m/s}^2$$

$$\alpha_{\text{F}} = \frac{a_{\text{B}y}}{r} = \frac{(15.2\text{ m/s}^2)}{(0.35\text{ m})} = +43.4\text{ rad/s}^2$$

$$T_{\text{Bic on F}}(0.05\text{ m}) - [(1.5\text{ kg})(9.8\text{ N/kg})](0.16\text{ m}) - (20\text{ N})(0.35\text{ m}) = (0.061\text{ kg}\cdot\text{m}^2)(43.4\text{ rad/s}^2)$$

Solving the above equation, we find that $T_{\text{Bic on F}} = 240\text{ N} = 54\text{ lb}$, a reasonable magnitude for this force.

Try it yourself Determine the force that the woman's biceps exerts on her forearm during the initial instant of a vertical toss of a 100-g rubber ball if she is exerting a 10-N force on the ball.

Answer 430 N.

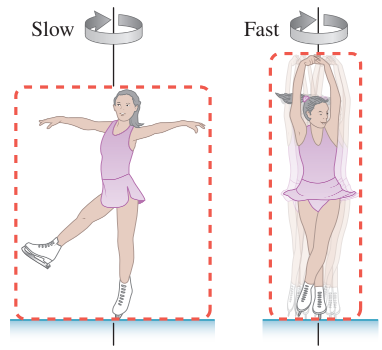
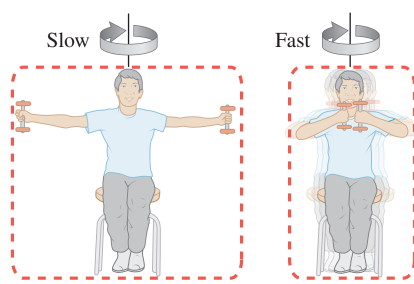
REVIEW QUESTION 9.3 How is Newton's second law for rotational motion similar to Newton's second law for translational motion? How is it different?

9.4 Rotational momentum

Earlier in this textbook (Chapters 6 and 7), we constructed powerful principles for momentum and energy that allowed us to analyze complex processes that involved translational motion. Is it possible to find analogous principles for the rotational (angular) momentum and rotational energy of extended bodies? Consider the experiments in Observational Experiment **Table 9.6**.

OBSERVATIONAL EXPERIMENT TABLE 9.6 Observations concerning rotational motion

VIDEO OET 9.6

Observational experiment	Analysis
Experiment 1. A figure skater initially spins slowly with a leg and two arms extended. Then she pulls her leg and arms close to her body, and her spinning rate increases dramatically. 	*Initial situation:* Large rotational inertia I and small rotational speed ω. *Final situation:* Smaller rotational inertia I and larger rotational speed ω.
Experiment 2. A man sitting on a chair that can spin with little friction initially holds dumbbells far from his body and spins slowly. When he pulls the dumbbells close to his body, the spinning rate increases dramatically. 	*Initial situation:* Large rotational inertia I and small rotational speed ω. *Final situation:* Smaller rotational inertia I and larger rotational speed ω.

Pattern

- There are no external forces exerted on either person—no torques.
- As the mass distribution of the system moves closer to the axis of rotation, the system's rotational inertia I decreases and the system's rotational speed ω increases (even though the net torque on the system is zero).

For each experiment in Table 9.6, the rotational inertia I of the spinning person decreased (the mass moved closer to the axis of rotation). Simultaneously, the rotational speed ω of the person increased. When I increases and ω decreases (or vice versa), $I\omega$ remains constant. We tentatively propose that when the rotational inertia I of an extended body in an isolated system decreases, its rotational speed ω increases, and vice versa.

Below we will continue to explore this idea quantitatively and then test it experimentally.

Rotational momentum is constant for an isolated system

Note that I is the rotational analog of the mass m of a point-like object and ω is the rotational analog of the translational velocity $\vec{v}$. The linear momentum of an object is the product of its mass m and its velocity $\vec{v}$. Let's propose that a turning object's **rotational momentum** L (analogous to linear momentum $\vec{p} = m\vec{v}$) is defined as

$$L = I\omega \tag{9.11}$$

In the chapter on linear momentum (Chapter 6) we derived a relationship [Eq. (6.4)] between the net force exerted on an object and the change in its linear momentum:

$$\Sigma\vec{F}(t_f - t_i) = \vec{p}_f - \vec{p}_i \tag{6.4}$$

where $\vec{p} = m\vec{v}$. Torque τ is analogous to force $\vec{F}$. Thus, using the analogy between rotational and translational motion, we write

$$\Sigma\tau(t_f - t_i) = L_f - L_i$$

where $L = I\omega$. If a system with one rotating body is isolated, then the external torque exerted on the object is zero. In such a case, the rotational momentum of the object does not change $(0 = L_f - L_i)$, and the object's rotational momentum is constant $(L_f = L_i)$, or

$$I_i\omega_i = I_f\omega_f$$

Note that this is consistent with our tentative qualitative rule. If the final value of one quantity (I or ω) increases for an isolated system, then the other quantity must decrease.

The similarity between the reasoning concerning rotational momentum of an isolated system and the reasoning we used to study linear momentum leads us to believe that we can use the bar chart representation to analyze rotational situations. We can also hypothesize that the change of rotational momentum of a system that is not isolated should be equal to the rotational equivalent of impulse. When a sum of forces $\Sigma\vec{F}$ is exerted on an system over a time interval Δt, the momentum of the system changes by the amount of $\Sigma\vec{F} \cdot \Delta t$. We can use the analogy between translational and rotational motion to say that **when a sum of torques $\Sigma\tau$ is exerted on a system over a time interval Δt, the rotational momentum of the system changes by the rotational impulse $\Sigma\tau\,\Delta t$.**

Rotational momentum and rotational impulse We now have a quantitative relation between rotational momentum $L = I\omega$ and rotational impulse.

$$L_i + \Sigma\tau\Delta t = L_f \tag{9.12}$$

The initial rotational momentum of a turning object plus the product of the net external torque exerted on the object and the time interval during which it is exerted equals the final rotational momentum of the object.

If the net torque that external objects exert on the turning object is zero, or if the torques add to zero, then the rotational momentum L of the turning object remains constant:

$$L_f = L_i \quad \text{or} \quad I_f\omega_f = I_i\omega_i \tag{9.13}$$

TIP Rotational momentum is sometimes called angular momentum.

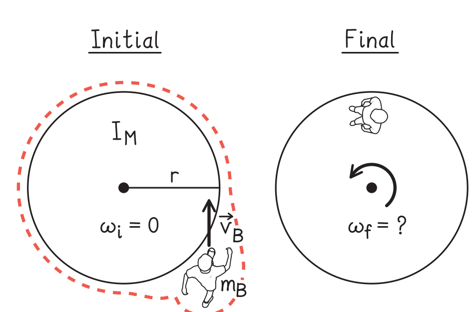
To explain most of the applications of torque and rotational momentum in this book, we account for their directions using positive or negative signs:

- A torque is positive if it tends to rotate the object counterclockwise and negative if it tends to rotate the object clockwise about the axis of rotation.
- A body rotating counterclockwise has positive rotational momentum and one rotating clockwise has negative rotational momentum.

EXAMPLE 9.6 Jumping on a merry-go-round

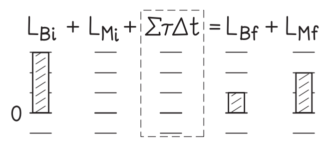
A boy of mass m_B running at speed v_B steps tangentially onto the stationary circular platform of a merry-go-round that can rotate on a frictionless bearing about its central shaft. The radius of the platform is r and the rotational inertia of the merry-go-round is I_M. After stepping onto the platform, the boy stops moving with respect to the merry-go-round. Derive an expression for the rotational velocity of the merry-go-round ω_f after the boy steps on it.

Sketch and translate A sketch of the situation is shown at right. We choose the boy and the merry-go-round as the system and place the axis of rotation along the shaft of the merry-go-round. The initial state is just before the running boy steps onto the platform, and the final state is when the boy and the platform are rotating together.



Simplify and diagram We model the boy as a point-like object. The process in the problem is similar to an inelastic collision of two objects.

The vertical forces exerted on the system by Earth and the ground cannot exert torques on the system because they are parallel to the axis of rotation. At the moment when the boy steps onto the platform, the ground exerts a horizontal static friction force on the axis (preventing the merry-go-round from moving translationally), but because this force is exerted on the axis of the merry-go-round, it exerts no torque on the system. Therefore, the sum of all torques exerted on the system is zero, and the rotational momentum of the boy and the merry-go-round should remain constant. We can represent the process with a rotational momentum bar chart as shown in the figure below. The external impulse is zero because the torque is zero; thus the initial momentum of the boy is equal to the final momentum of the boy and merry-go-round together. Because the boy does not move with respect to the merry-go-round after that, and there are no external torques, the rotational momentum of the system and therefore the speed of the system are constant. Here we have a process in which the linear momentum of the system is not constant but the rotational momentum is.



Represent mathematically We use the bar chart to write the mathematical representation of the process: the initial and final rotational momenta of the system should be equal ($L_i = L_f$). Thus,

$$L_{\text{Bi}} + L_{\text{Mi}} = L_{\text{B and M f}}$$

From the bar chart: $L_{\text{Mi}} = 0$. The initial rotational momentum of the boy with respect to the axis of rotation (which is outside his body), just before he jumps onto the merry-go-round, can be expressed as

$$L_{\text{Bi}} = I_{\text{B}}\omega_{\text{B}} = m_{\text{B}}r^2 \cdot \frac{v_{\text{B}}}{r} = m_{\text{B}}v_{\text{B}}r$$

In the final state, the boy and the merry-go-round rotate as one rigid body. Therefore, the final rotational momentum of the system can be expressed as $L_{\text{B and M f}} = (m_{\text{B}}r^2 + I_{\text{M}})\omega_{\text{f}}$.

Solve and evaluate Combining the equations above, we get

$$m_{\text{B}}v_{\text{B}}r = (m_{\text{B}}r^2 + I_{\text{M}})\omega_{\text{f}}$$

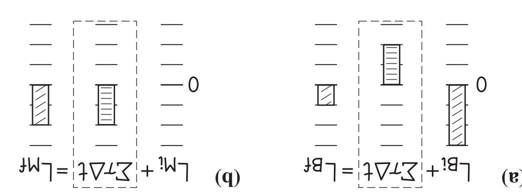
and finally

$$\omega_{\text{f}} = \frac{m_{\text{B}}v_{\text{B}}r}{m_{\text{B}}r^2 + I_{\text{M}}}$$

To evaluate the expression we need to examine it for extreme cases. The expression predicts that the faster the boy runs, the larger the final rotational velocity of the merry-go-round—this makes sense. The larger the rotational inertia of the merry-go-round, the smaller the final rotational velocity—this prediction makes sense, too. Checking the units for ω_{f} in our expression, we get $\frac{\text{kg} \times \text{m/s} \times \text{m}}{\text{kg} \times \text{m}^2} = \frac{1}{\text{s}}$. The units for rotational velocity are rad/s, but as we noted earlier, radians have no dimension and so our unit analysis confirms the expression for ω_{f}.

Try it yourself In the example, we chose the boy and the merry-go-round as the system. How will the rotational momentum bar charts change if we choose (a) the boy as the system or (b) the merry-go-round as the system? Draw these two bar charts.

Answer



Rotational momentum of a shrinking object

At the beginning of the chapter, we described the discovery of pulsars, astronomical objects that rotate very quickly and emit repetitive radio signals with a very small time interval between them. The signals from the first discovered pulsar had a period of approximately 1.33 s. Astronomers could not explain at first how pulsars could rotate so rapidly. Most stars, including our Sun, rotate very much the way Earth does, usually taking several days to complete one rotation (about a month for our Sun).

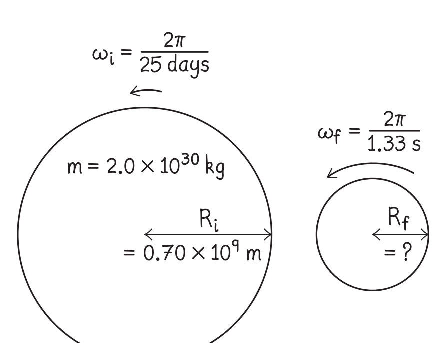
However, as a star's core collapses and its mass moves closer to the axis of rotation, its rotational velocity increases because its rotational momentum is constant (assuming the star does not interact with any other objects). How much does a star need to shrink so that its period of rotation becomes seconds instead of days?

TIP Remember that the notation for period is T. Do not confuse this with T also used for the tension force.

EXAMPLE 9.7 A pulsar

Imagine that our Sun ran out of nuclear fuel and collapsed. What would its radius have to be in order for its period of rotation to be the same as the first discovered pulsar described above? The Sun's current period of rotation is 25 days.

Sketch and translate First, sketch the process. The Sun is the system. We can convert the present period of rotation of the Sun into seconds ($T_{\text{i}} = 25$ days $= 2.16 \times 10^6$ s). Its mass is $m = 2.0 \times 10^{30}$ kg and its radius is $R_{\text{i}} = 0.70 \times 10^9$ m. After the Sun collapses, its period of rotation will be $T_{\text{f}} = 1.33$ s. What will be its radius?



(CONTINUED)

Simplify and diagram Assume that the Sun is a sphere with its mass distributed uniformly. Assume also that it does not lose any mass as it collapses.

Represent mathematically Now, apply the principle of rotational momentum conservation (Eq. 9.13) to the Sun's collapse:

$$I_i\omega_i = I_f\omega_f$$

From Table 9.5 we find that the rotational inertia of a sphere rotating around an axis passing through its center is

$$I = \tfrac{2}{5}mR^2$$

The rotational velocity of an object is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

where T is the period for one rotation. Combining the above three equations, we get

$$\left(\tfrac{2}{5}mR_i^2\right)\frac{2\pi}{T_i} = \left(\tfrac{2}{5}mR_f^2\right)\frac{2\pi}{T_f}$$

Dividing by the $2/5$, 2π, and m on each side of the equation, we get

$$\frac{R_i^2}{T_i} = \frac{R_f^2}{T_f}$$

Solve and evaluate Multiply both sides of the above by T_f and take the square root:

$$\begin{aligned} R_f &= \sqrt{\frac{R_i^2 T_f}{T_i}} \\ &= \sqrt{\frac{(0.70 \times 10^9\ \text{m})^2(1.33\ \text{s})}{2.16 \times 10^6\ \text{s}}} \\ &= 5.5 \times 10^5\ \text{m} = 550\ \text{km} \end{aligned}$$

Although this is much smaller than the radius of Earth, models of stellar evolution actually do predict that the Sun's core will eventually shrink to this size and possibly smaller.

Try it yourself When massive stars explode, the collapse can shrink their radii to about 10 km. What would be the period of rotation of such a star if it originally had a mass twice the mass of the Sun, a radius that was 1.3 times the Sun's radius, and the same initial period of rotation as the Sun (25 days)?

Answer 2.6×10^{-4} s.

Vector nature of torque, rotational velocity, and rotational momentum

So far, we have treated torque, rotational velocity, and rotational momentum as scalar quantities, but at the same time we assigned signs to them reflecting CW or CCW direction of rotation. Thus the value of these quantities depends on the direction of rotation. In fact, all of those quantities are vector quantities, but for most applications in this chapter we do not need to treat them as such. However, there are a few applications for which the vector nature of torque, rotational velocity, and rotational momentum is important. The vector direction of both rotational velocity and rotational momentum can be determined using **a right-hand rule**.

Right-hand rule for determining the direction of torque, rotational velocity, and rotational momentum Curl the four fingers of your right hand in the direction of rotation of the turning object. Your thumb, held perpendicular to the fingers, then points in the direction of both the object's rotational velocity and rotational momentum (**Figure 9.12**). To determine the **vector direction** of the torque that a force produces on an object (as opposed to the clockwise/counterclockwise way of describing it) about an axis of rotation, first imagine that the object is at rest and that the torque you are interested in is the only torque exerted on the object. Next, curl the fingers of your right hand in the direction that the torque would make the object rotate. Your thumb, held perpendicular to the fingers, shows the direction of this torque.

Bicycling We can use the vector nature of torque and rotational momentum to understand why a bicycle is much more stable when moving fast—especially if the bicycle has massive tires. Consider an axis of rotation parallel to the ground that passes through the two contact points of the tires with the ground. This axis of rotation is below the center of mass of the stationary bike—an unstable equilibrium (**Figure 9.13**). When the bicycle is moving quickly, the rotating tires (and therefore the bicycle + rider system) have considerable rotational momentum, which will change only when an unbalanced torque

FIGURE 9.12 Using the right-hand rule to determine the direction of an object's rotational momentum $\vec{L}$.

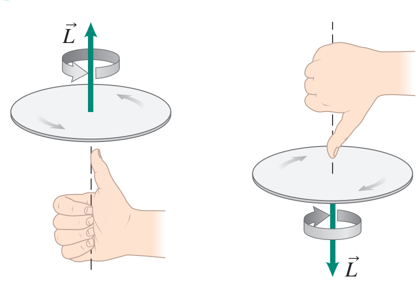
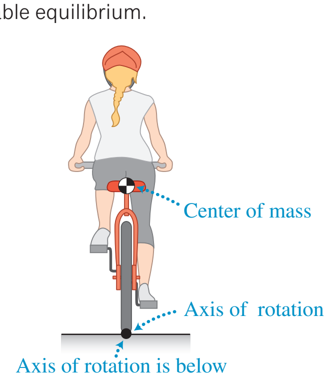


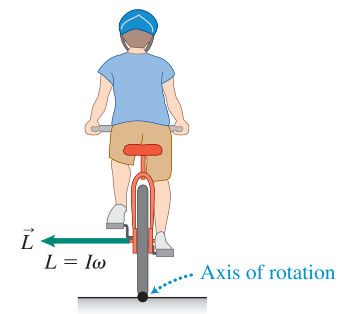
FIGURE 9.13 A bicycle that is not moving is in unstable equilibrium.



is exerted on the system. When a bicycle is moving on a smooth road, the rotational velocity and the rotational momentum vectors are perpendicular to the plane of rotation of the bike tires (**Figure 9.14**), and they are large due to the rapid rotation of the tires.

When the bike + rider system is balanced, the gravitational force exerted by Earth on the system produces no torque since that force points directly at the axis of rotation. If the rider's balance shifts a bit, or the wind blows, or the road is uneven, the system will start tilting. As a result, the gravitational force exerted on the system will produce a torque. However, since the rotational momentum of the system is large, this torque does not change its direction by much right away, but it takes only several tenths of a second for the torque to change the rotational momentum significantly. This is enough time for an experienced rider to make corrections to rebalance the system. The faster the person is riding the bike, the greater the rotational momentum of the system and the more easily the rider can keep the system balanced.

FIGURE 9.14 Rotating bicycle tires have rotational momentum that stabilizes the bicycle.



Gyroscopes Guidance systems for spaceships rely on the constancy of rotational momentum in isolated systems to help them maintain their chosen course. Once the ship is pointed in the desired direction, one or more heavy gyroscopes start rotating. The gyroscope is a wheel whose axis of rotation keeps the ship oriented in the chosen direction. The gyroscope is similar to the rotating bicycle tires that help keep a rider upright without tipping or changing direction. Gyroscopes are also used in cameras to prevent them from vibrating or moving while the camera lens is open.

REVIEW QUESTION 9.4 After a playground merry-go-round is set in motion, its rotational speed decreases noticeably if another person jumps on it. However, if a person riding the merry-go-round steps off, the rotational speed seems not to change at all. Explain.

9.5 Rotational kinetic energy

We are familiar with the kinetic energy $(1/2)mv^2$ of a single particle moving along a straight line or in a circle. It would be useful to calculate the kinetic energy of a rotating body—like Earth. Doing so would allow us to use the work-energy approach to solving problems involving rotation. Let's start by deriving an expression for the **rotational kinetic energy** of a single particle of mass m moving in a circle of radius r at speed v. According to the kinematics in Section 9.1, its linear speed v and rotational speed ω are related:

$$v = r\omega$$

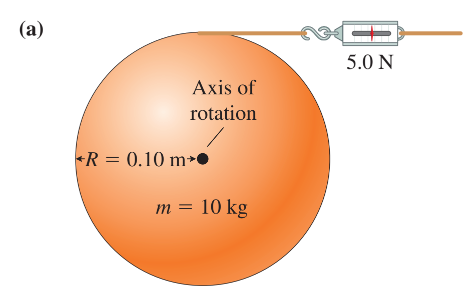
Thus, the kinetic energy of this particle moving in a circle can be written as

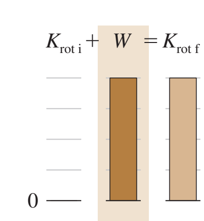
$$K_{\text{rot}} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(r\omega)^2 = \tfrac{1}{2}(mr^2)\omega^2 = \tfrac{1}{2}I\omega^2$$

where $I = mr^2$ is the rotational inertia of a particle moving a distance r from the center of its circular path. The expression for the translational kinetic energy $(1/2)mv^2$ of a particle is similar to the rotational version, which involves the product of a mass-like term I and the square of a speed-like term ω. Can we use the expression $\frac{1}{2}I\omega^2$ for the rotational kinetic energy of a rotating rigid body?

To test this idea, consider a solid sphere of known radius R and mass m that can rotate freely on an axis. We wrap a string around the sphere and pull the string with a force probe exerting a constant force of a known magnitude so that the sphere starting at rest completes 5.0 revolutions (**Figure 9.15a**). After we stop pulling, we measure the rotational speed ω of the sphere. But before measuring it, we predict its value using this expression for rotational kinetic energy. If we choose the sphere as the system, the string is the only external object that exerts a force that causes a nonzero torque on the sphere. This string force does work, which changes the sphere's kinetic energy from zero to some new value (Figure 9.15b). Thus, the initial rotational kinetic energy of

FIGURE 9.15 A string pulls a solid sphere (top view).





the sphere (zero) plus the work done by the string on the sphere during these five turns equals the final rotational kinetic energy of the sphere:

$$K_{\text{rot i}} + W = K_{\text{rot f}}$$

The string pulls parallel to the displacement of the edge of the sphere during the entire time. We can use the expression for the rotational kinetic energy under test to predict the magnitude of the final rotational speed:

$$0 + F_{\text{String on Sphere}}(5 \cdot 2\pi R) \cos 0 = \tfrac{1}{2} I\omega^2$$

where $5 \cdot 2\pi R$ is the distance the string is pulled—five circumferences of the sphere. The above leads to a prediction of the final rotational speed:

$$\omega_{\text{f}} = \sqrt{\frac{F_{\text{String on Sphere}} \cdot 20\pi R}{I}}$$

From Table 9.5 we know that the rotational inertia of a solid sphere of radius R and mass m is $I = (2/5)mR^2$ (the axis passes through the center of the sphere); in our sphere, $m = 10$ kg and $R = 0.10$ m. Thus, the rotational inertia is

$$I = (2/5)mR^2 = (2/5)(10\text{ kg})(0.10\text{ m})^2 = 0.040\text{ kg} \cdot \text{m}^2$$

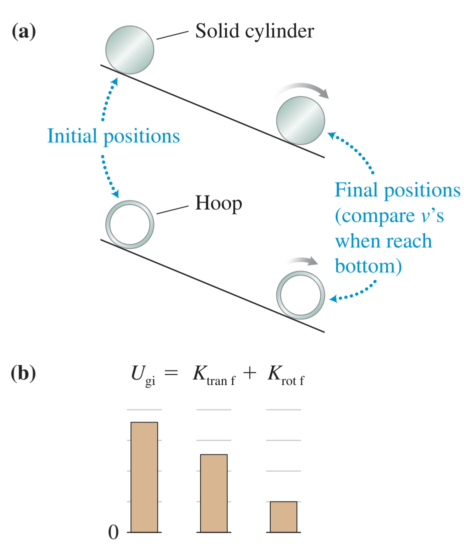
We pull the string so that it exerts a 5.0-N force on the edge of the sphere. Thus, the sphere's final speed should be

$$\omega_{\text{f}} = \sqrt{\frac{(5.0\text{ N})(20\,\pi)(0.10\text{ m})}{(0.040\text{ kg} \cdot \text{m}^2)}} = \left(28\,\frac{\text{rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 4.5\,\frac{\text{rev}}{\text{s}}$$

When we measure the final angular velocity, it is about 4.5 rev/s.

We can apply our new understanding of rotational kinetic energy to predict the outcome of the experiment shown in **Figure 9.16a**. A solid cylinder and a hoop of the same radius and mass start rolling at the top of an inclined plane. Which object reaches the bottom of the plane first? What is the ratio of their speeds at the bottom?

FIGURE 9.16 A race between a hoop and a cylinder of the same mass.



Both Earth-object systems start with the same gravitational potential energy. As they roll, both acquire translational and rotational kinetic energies. We can represent the process in a work-energy bar chart (Figure 9.16b). The bar chart helps us construct a mathematical description:

$$mgy_{\text{i}} = \tfrac{1}{2}mv_{\text{f}}^2 + \tfrac{1}{2}I\omega_{\text{f}}^2$$

Since the objects are rolling without skidding, the rotational speed ω and translational speed v are related as $\omega = v/r$. We substitute this into the above energy equation and then rearrange:

$$mgy_{\text{i}} = \tfrac{1}{2}\left(m + \frac{I}{r^2}\right)v_{\text{f}}^2$$

$$\Rightarrow v_{\text{f}} = \sqrt{\frac{2mgy_{\text{i}}}{\left(m + \dfrac{I}{r^2}\right)}}$$

This expression for v_{f} suggests that at the bottom of the inclined plane, the object with greater rotational inertia I will have the smaller translational speed v_{f}. Using the information in Table 9.5, we have $I_{\text{cylinder}} = \frac{1}{2}mr^2$ and $I_{\text{hoop}} = mr^2$. Thus $I_{\text{hoop}} > I_{\text{cylinder}}$, giving the conclusion that the cylinder should reach the bottom first.

We can now find the ratio of their final speeds. We insert the expressions for the rotational inertia of the solid cylinder and of the hoop to find their final speeds. For the cylinder, $v_{\text{f}} = \sqrt{\frac{4}{3}gy_{\text{i}}}$, and for the hoop, $v_{\text{f}} = \sqrt{gy_{\text{i}}}$. The ratio of their final speeds is $\sqrt{\frac{4}{3}}$, with the cylinder moving faster. When we perform the experiment (see the video), we see that the cylinder reaches the bottom of the plane first.

VIDEO 9.1

This is the second testing experiment involving rotational kinetic energy in which the outcome matched the prediction. Given this support for our prediction and the lack of counterevidence, we will use this mathematical expression for a rigid body's rotational kinetic energy.

Rotational kinetic energy The rotational kinetic energy of an object with rotational inertia I turning with rotational speed ω is

$$K_{\text{rot}} = \tfrac{1}{2} I\omega^2 \tag{9.14}$$

TIP When you encounter a new physical quantity, always check whether its units make sense. In this particular case, the units for I are $\text{kg} \cdot \text{m}^2$ and the units for ω^2 are $1/\text{s}^2$. Thus, the unit for kinetic energy is $\text{kg} \cdot \text{m}^2/\text{s}^2 = (\text{kg} \cdot \text{m}/\text{s}^2)\,\text{m} = \text{N} \cdot \text{m} = \text{J}$, the correct unit for energy.

Flywheels for storing and providing energy

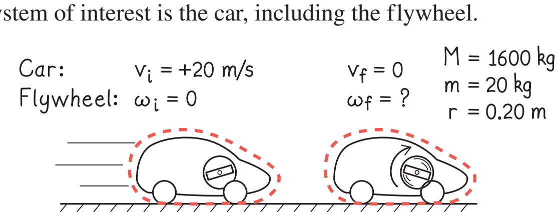
You stop your car at a stoplight. Before stopping, the car had considerable kinetic energy; after stopping, the kinetic energy is zero. It has been converted to internal energy due to friction in the brake pads. Unfortunately, this thermal energy cannot easily be converted back into a form that is useful. Is there a way to convert that translational kinetic energy into some other form of energy that would help the car regain translational kinetic energy when the light turns green?

Efforts are under way to use the rotational kinetic energy of flywheels (rotating disks) for this purpose. Instead of rubbing a brake pad against the wheel and slowing it down, the braking system would, through a system of gears or through an electric generator, convert the car's translational kinetic energy into the rotational kinetic energy of a flywheel. As the car's translational speed decreases, the flywheel's rotational speed increases. This rotational kinetic energy could then be used to help the car start moving, rather than relying entirely on the chemical potential energy of gasoline.

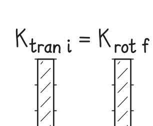
EXAMPLE 9.8 Flywheel rotational speed

A 1600-kg car traveling at 20 m/s approaches a stop sign. If it could transfer all of its translational kinetic energy to a 0.20-m-radius, 20-kg flywheel while stopping, what rotational speed would the flywheel acquire?

Sketch and translate A sketch of the situation is shown below. The system of interest is the car, including the flywheel.



Simplify and diagram The process is represented at right with a bar chart. The initial energy of the system is the car's translational kinetic energy; the final energy is the flywheel's rotational kinetic energy. Braking converts car's initial kinetic energy into the flywheel's rotational energy, which is saved for future use. Assume that the flywheel is a solid disk with rotational inertia of $(1/2)mr^2$ (see Table 9.5).



Represent mathematically Use the bar chart to help construct an energy conservation equation:

$$K_{\text{tran i}} = K_{\text{rot f}}$$

$$\tfrac{1}{2} M v_{\text{i}}^2 = \tfrac{1}{2} I \omega_{\text{f}}^2$$

Multiplying both sides of the equation by 2 and dividing by I, we get

$$\omega_{\text{f}}^2 = \frac{M v_{\text{i}}^2}{I}$$

The rotational inertia of the disk (a solid cylinder) is $I_{\text{cylinder}} = (1/2)mr^2$. Thus,

$$\omega_{\text{f}}^2 = \frac{M v_{\text{i}}^2}{I} = \frac{M v_{\text{i}}^2}{\frac{1}{2} m r^2} = \frac{2 M v_{\text{i}}^2}{m r^2}$$

Solve and evaluate To find the rotational speed, take the square root of both sides of the above equation:

$$\omega_{\text{f}} = \frac{v_{\text{i}}}{r}\sqrt{\frac{2M}{m}} = \frac{20\ \text{m/s}}{0.20\ \text{m}}\sqrt{\frac{2(1600\ \text{kg})}{(20\ \text{kg})}} = 1300\ \text{rad/s}$$

$$= 200\ \text{rev/s} = 12{,}000\ \text{rpm}$$

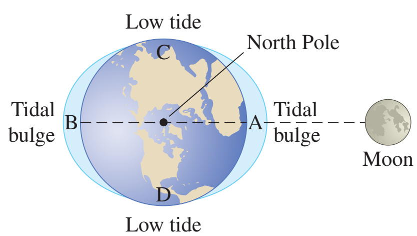
Try it yourself Could you store more energy in a rotating hoop or in a rotating solid cylinder, assuming they have the same mass, radius, and rotational speed?

Answer The hoop has a greater rotational inertia and therefore would have a greater rotational kinetic energy at the same rotational speed.

REVIEW QUESTION 9.5 Will a can of watery chicken noodle soup roll slower or faster down an inclined plane than an equal-mass can of thick English clam chowder?

9.6 Tides and Earth's day

FIGURE 9.17 The ocean bulges on both sides of Earth along a line toward the Moon.



The level of the ocean rises and falls by an average of 1 m twice each day, a phenomenon known as the tides. Many scientists, including Galileo, tried to explain this phenomenon and suspected that the Moon was a part of the answer. Isaac Newton was the first to explain how the motion of the Moon actually creates tides. He noted that at any moment, different parts of Earth's surface are at different distances from the Moon and that the distance from a given location on Earth to the Moon varied as Earth rotated. As illustrated in **Figure 9.17**, point A is closer to the Moon than the center of Earth or point B are, and therefore the gravitational force exerted by the Moon on point A is greater than the gravitational force exerted on point B. Due to the difference in forces, Earth elongates along the line connecting its center to the Moon's. This makes water rise to a high tide at point A and surprisingly also at B. The water "sags" a little at points C and D, forming low tides at those locations. When the Sun is aligned with the Moon and Earth, the bulging is especially pronounced.

As the solid Earth rotates beneath the tidal bulges, it attempts to drag the bulges with it. A large amount of friction is produced, which converts the rotational kinetic energy of Earth into internal energy. The time interval needed for Earth to complete one turn on its axis increases by 0.0016 s every 100 years. In other words, the Earth day is slowly getting longer. In a very long time, Earth will stop turning relative to the Moon and an unmoving tidal bulge will face toward and away from the Moon. This "tidal locking" has already occurred on the Moon (although it is solid, the same principle applies), which is why on Earth we only see one side of the Moon. It rotates around its axis with the same period as it moves around Earth.

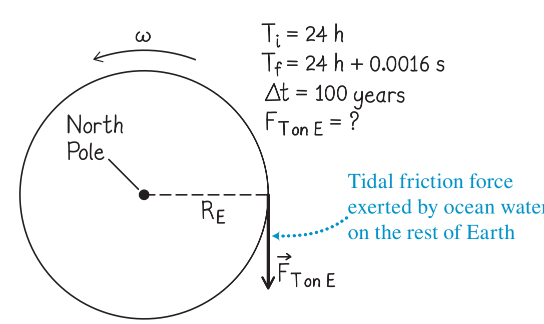
Let's use our new understanding of rotational dynamics to estimate the friction force the tides exert on Earth, causing Earth's rotation rate to decrease.

EXAMPLE 9.9 Tides slow Earth's rotation

Estimate the effective tidal friction force exerted by ocean water on Earth that causes a 0.0016-s increase in Earth's rotation time every 100 years.

Sketch and translate The situation is already sketched in Figure 9.17. The solid Earth is the system; the water covering most of its surface is considered an external object for this estimate. Earth rotates counterclockwise, taking 24 hours for one revolution (one period), as seen looking down on the North Pole. Remember that when an object rotates counterclockwise, its angular velocity is considered to be positive. Since Earth's rotation is gradually slowing, the tidal friction force is producing a negative torque, opposite the positive sign of the rotational velocity. We need to find the magnitude of the force that would increase the time of one revolution by 0.0016 s in 100 years.

Simplify and diagram Assume that the solid Earth is a sphere covered uniformly with water on its surface. Assume also that the frictional force exerted by the water on Earth is constant in magnitude and exerted at the equator. The force exerted by the tidal bulge on Earth produces a torque that opposes Earth's rotation.



Represent mathematically To estimate the friction force exerted by the oceans on Earth, we need to determine the torque produced by that force. We can use the rotational form of Newton's second law if we can determine the rotational inertia of Earth and its rotational acceleration. The rotational acceleration of Earth is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\left(+\dfrac{2\pi}{T_f}\right) - \left(+\dfrac{2\pi}{T_i}\right)}{\Delta t}$$

$$= \frac{2\pi}{\Delta t}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)$$

where $T_i = 24\text{ h}\left(\dfrac{3600\text{ s}}{1\text{ h}}\right) = 86{,}400\text{ s}$, $T_f = T_i + \Delta T = 86{,}400\text{ s} + 0.0016\text{ s}$, and $\Delta t = 100\text{ years}\left(\dfrac{365\text{ days}}{1\text{ year}}\right)\left(\dfrac{86{,}400\text{ s}}{1\text{ day}}\right) = 3.15 \times 10^9\text{s}$.

Because T_f and T_i are so close, your calculator will likely evaluate the rotational acceleration of Earth to be zero. To deal with this, we put the equation into another form.

$$\alpha = \frac{2\pi}{\Delta t}\left(\frac{1}{T_f} - \frac{1}{T_i}\right) = \frac{2\pi}{\Delta t}\left(\frac{T_i - T_f}{T_i T_f}\right)$$

$$= \frac{2\pi}{\Delta t}\left(\frac{-\Delta T}{T_i(T_i + \Delta T)}\right) = \frac{2\pi}{\Delta t}\left(\frac{-\Delta T}{T_i^2 + T_i\,\Delta T}\right)$$

Now look at the two terms in the denominator, T_i^2 and $T_i\,\Delta T$. Because ΔT is so small, the second term is much less than the first term. This means the second term can be dropped without affecting the result in any significant way. Thus,

$$\alpha = -\frac{2\pi\,\Delta T}{\Delta t T_i^2}$$

We can use the rotational form of Newton's second law to get an alternative expression for Earth's rotational acceleration:

$$\alpha = \frac{1}{I}\Sigma\tau = \frac{1}{\frac{2}{5}mR_E^2}\left(F_{\text{T on E}}R_E \sin 90°\right)$$

The R_E in the numerator is the distance from the axis of rotation to the surface where the friction force is exerted, the radius of Earth. The R_E in the denominator is also the radius of Earth.

$$\alpha = \frac{1}{\frac{2}{5}mR_E^2}\left(F_{\text{T on E}}R_E \sin 90°\right) = \frac{5F_{\text{T on E}}}{2mR_E}$$

Setting the two expressions for the magnitude of the rotational acceleration equal to each other, we get

$$\frac{2\pi\,\Delta T}{\Delta t T_i^2} = \frac{5F_{\text{T on E}}}{2mR_E}$$

Solve and evaluate Solve the previous equation for the force of the tides on Earth:

$$F_{\text{T on E}} = \frac{4\pi m R_E\,\Delta T}{5\,\Delta t T_i^2}$$

$$= \frac{4\pi(5.97 \times 10^{24}\text{ kg})(6.38 \times 10^6\text{ m})(0.0016\text{ s})}{5(3.15 \times 10^9\text{s})(86{,}400\text{ s})^2}$$

$$= 6.5 \times 10^9\text{ N}$$

The magnitude of this friction force seems big, and it is. But when exerted on an object of such large mass $(5.97 \times 10^{24}\text{ kg})$, the effect is extremely tiny. By comparison, the gravitational force that the Sun exerts on Earth is 3.5×10^{22} N.

Try it yourself Estimate the time interval in years that it will take for Earth's rotation to change from 24 hours to 27 days.

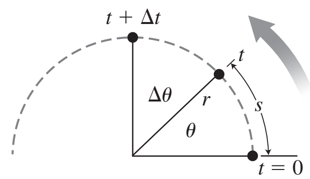
Answer

Using Eq. (9.3), we find $t = (\omega - \omega_0)/\alpha \approx -\omega_0/\alpha = (-7 \times 10^{-5}\text{ rad/s})/(-4 \times 10^{-22}\text{ rad/s}^2) = 2 \times 10^{17}$ s, almost 10 billion years.

REVIEW QUESTION 9.6 How can you explain the increasing length of a day on Earth?

Summary

Rotational kinematics The rotational motion of a rigid body can be described using quantities similar to those for translational motion—rotational position θ, rotational velocity ω, and rotational acceleration α. (Section 9.1)



- Rotational position (in radians)

$$\theta = s/r \qquad \text{Eq. (9.1)}$$

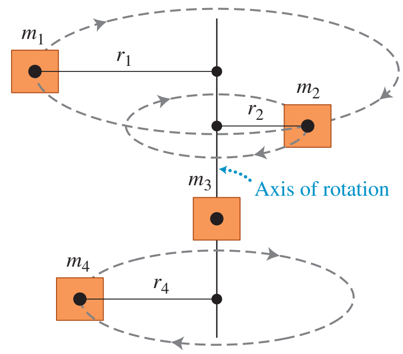
- Rotational velocity (in rad/s)

$$\omega = \Delta\theta/\Delta t \qquad \text{Eq. (9.2)}$$

- Rotational acceleration (in rad/s^2)

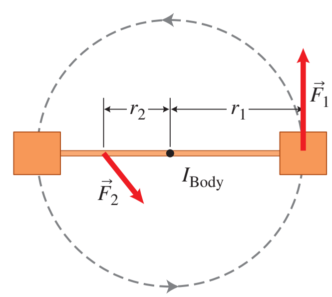
$$\alpha = \Delta\omega/\Delta t \qquad \text{Eq. (9.3)}$$

Rotational inertia I is the physical quantity equal to the sum of the mr^2 terms for each part of an object and depends on the distribution of mass relative to an axis of rotation. (Sections 9.2 and 9.3)



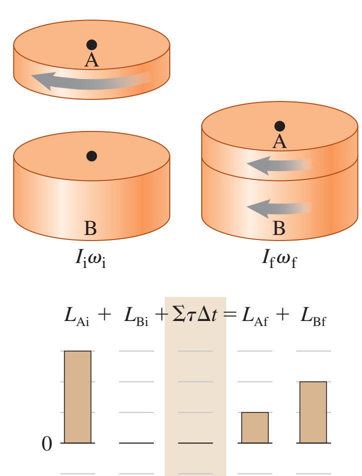
$$I = \Sigma mr^2 \qquad \text{Eq. (9.9)}$$

Rotational dynamics A rigid body's rotational acceleration equals the net torque produced by forces exerted on the body divided by its rotational inertia. (Section 9.3)



$$\alpha = \frac{1}{I}\Sigma\tau \qquad \text{Eq. (9.10)}$$

Rotational momentum L is the product of the rotational inertia I of an object and its rotational velocity ω, positive for counterclockwise rotation and negative for clockwise rotation. For an isolated system (zero net torque exerted on it), the rotational momentum of the system is constant. (Section 9.4)



$$L = I\omega \qquad \text{Eq. (9.11)}$$

Translational motion equivalent:

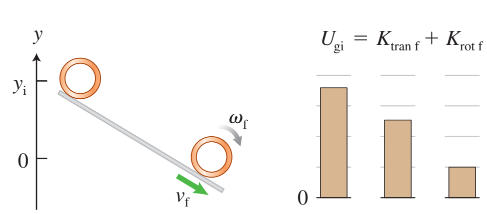
$$\vec{p} = m\vec{v} \qquad \text{Eq. (6.1)}$$

For isolated system,

$$I_i\omega_i = I_f\omega_f \qquad \text{Eq. (9.13)}$$

For nonisolated systems, rotational impulse $\Sigma\tau\,\Delta t$ is nonzero.

Rotational kinetic energy K_{rot} of a rigid body is energy due to the rotation of the object about a particular axis. This is another form of kinetic energy that is included in the work-energy principle. (Section 9.5)



$$K_{rot} = \tfrac{1}{2}I\omega^2 \qquad \text{Eq. (9.14)}$$

Translational motion equivalent:

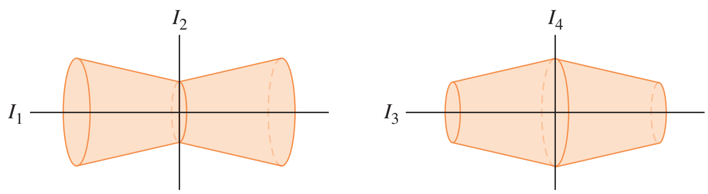
$$K_{tran} = \tfrac{1}{2}mv^2 \qquad \text{Eq. (7.5)}$$

Questions

Multiple Choice Questions

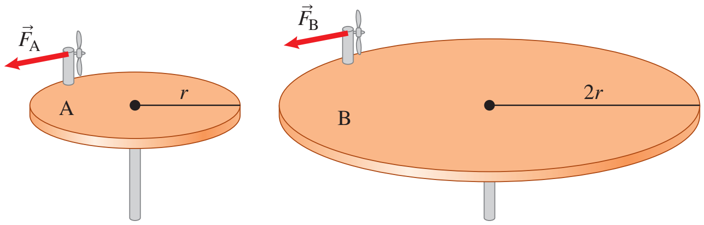
1. Is it easier to open a door that is made of a solid piece of wood or a door of the same mass made of light fiber with a steel frame?
   (a) Wooden door
   (b) Fiber door with a steel frame
   (c) The same difficulty
   (d) Not enough information to answer
2. You push a child on a swing. Why doesn't the child continue in a vertical loop over the top of the swing?
   (a) The torque of the force that Earth exerts on the child pulls him back.
   (b) The swing does not have enough kinetic energy when at the bottom.
   (c) The swing does not have enough rotational momentum.
   (d) All of the above are correct.
3. In terms of the torque needed to rotate your leg as you run, would it be better to have a long calf and short thigh, or vice versa?
   (a) Long calf and short thigh
   (b) Short calf and long thigh
   (c) Does not matter
4. Suppose that two bicycles have equal overall mass, but one has thin lightweight tires while the other has heavier tires made of the same material. Why is the bicycle with thin tires easier to accelerate?
   (a) Thin tires have less area of contact with the road.
   (b) With thin tires, less mass is distributed at the rims.
   (c) With thin tires, you don't have to raise the large mass of the tire at the bottom to the top.
5. When riding a 10-speed bicycle up a hill, a cyclist shifts the chain to a larger-diameter gear attached to the back wheel. What must be true when the cyclist shifts to a larger gear?
   (a) The torque exerted by the chain on the gear is larger.
   (b) The force exerted by the chain on the gear is larger.
   (c) The cyclist pedals more frequently to travel the same distance.
   (d) Both a and c are correct.
6. The objects in **Figure Q9.6** are made of two identical paper cups glued together. Rank the rotational inertias I_1, I_2, I_3, and I_4 about the indicated axes using the signs $=$, $>$, and $<$. Explain.

FIGURE Q9.6



7. Select all the pairs below in which the two physical quantities have the same units.
   (a) Rotational velocity and translational velocity
   (b) Rotational kinetic energy and translational kinetic energy
   (c) Linear momentum and rotational momentum
   (d) Work and torque
   (e) Power and energy
   (f) Impulse (that changes linear momentum) and rotational impulse
8. If you turn on a coffee grinding machine sitting on a smooth tabletop, what do you expect it to do?
   (a) Start rotating in the same direction as the blades rotate
   (b) Start rotating in the direction opposite the blade rotation
   (c) Grind the coffee without any rotation of the machine
9. A bowling ball is rolling without skidding down an incline. While accelerating down the incline, the ratio of translational kinetic energy to rotational kinetic energy of the ball
   (a) remains constant and equal to about 0.4.
   (b) remains constant and equal to about 2.5.
   (c) remains constant; the value depends on the radius of the ball.
   (d) decreases.
   (e) increases.
10. The Mississippi River carries sediment from higher latitudes toward the equator. How does this affect the length of the day?
   (a) Increases the day
   (b) Decreases the day
   (c) Does not affect the day
   (d) There is no relation between the mass distribution and the length of the day.
11. Two disks are cut from the same uniform board. The radius of disk B is twice the radius of the disk A. The disks can rotate around axes with negligible friction. Two very light battery-powered fans are attached to the disks, as shown in **Figure Q9.11**. When switched on, the fans exert equal forces on the disks. Which of the following correctly compares and explains the rotational accelerations of the disks after the fans are switched on?
   (a) $\alpha_A = 4\alpha_B$ because $F_B = F_A$ and $m_B = 4m_A$.
   (b) $\alpha_A = \alpha_B$ because $\tau_B = 2\tau_A$ and $I_B = 2I_A$.
   (c) $\alpha_A = 2\alpha_B$ because $\tau_B = 2\tau_A$ and $I_B = 4I_A$.
   (d) $\alpha_A = 4\alpha_B$ because $\tau_B = 2\tau_A$ and $I_B = 8I_A$.
   (e) $\alpha_A = 8\alpha_B$ because $\tau_B = 2\tau_A$ and $I_B = 16I_A$.

FIGURE Q9.11



Conceptual Questions

12. Explain your choices for Questions 1–11 (your instructor will choose which ones).
13. A spinning raw egg, if stopped momentarily and then released by the fingers, will resume spinning. Explain. Will this happen with a hard-boiled egg? Explain.
14. Compare the magnitude of Earth's rotational momentum about its axis to that of the Moon about Earth. The tides exert a torque on Earth and the Moon so that eventually they will rotate with the same period. The object with the greater rotational momentum will experience the smaller percent change in the period of rotation. Will Earth's solar day increase more than the Moon's period of rotation decreases? Explain.
15. You lay a pencil on a smooth desk (ignore sliding friction). You push the pencil, exerting a constant force first directly at its center of mass and then close to the tip of the pencil. In both cases, the force is exerted perpendicular to the body of the pencil. If the forces that you exert on the pencil are exactly the same in magnitude and direction, in which case is the translational acceleration of the pencil greater in magnitude?
16. If you watch the dive of an Olympic diver, you note that she continues to rotate after leaving the board. However, her center of mass follows a parabolic curve. Explain why.
17. Explain why you do not tip over when riding a bicycle but do tip when stationary at a stoplight.
18. Sometimes a door is not attached properly and it will open by itself or close by itself. But it will never do both. Why?
19. Why do tightrope walkers carry long, heavy bars?

Problems

Below, BIO indicates a problem with a biological or medical focus. Problems labeled EST ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

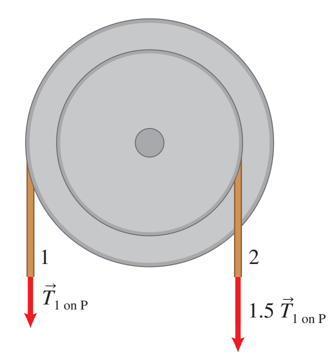
9.1 Rotational kinematics

1. The sweeping second hand on your wall clock is 20 cm long. What is (a) the rotational speed of the second hand, (b) the translational speed of the tip of the second hand, and (c) the rotational acceleration of the second hand? Assume the second hand moves smoothly.
2. You find an old record player in your attic. The turntable has two readings: 33 rpm and 45 rpm. What do they mean? Express these quantities in different units.
3. * Consider again the turntable described in the last problem. Determine the magnitudes of the rotational acceleration in each of the following situations. Indicate the assumptions you made for each case. (a) When on and rotating at 33 rpm, it is turned off and slows and stops in 60 s. (b) When off and you push the play button, the turntable attains a speed of 33 rpm in 15 s. (c) You switch the turntable from 33 rpm to 45 rpm, and it takes about 2.0 s for the speed to change. (d) In the situation in part (c), what is the magnitude of the average tangential acceleration of a point on the turntable that is 15 cm from the axis of rotation?
4. You step on the gas pedal in your car, and the car engine's rotational speed changes from 1200 rpm to 3000 rpm in 3.0 s. What is the engine's average rotational acceleration?
5. You pull your car into your driveway and stop. The drive shaft of your car engine, initially rotating at 2400 rpm, slows with a constant rotational acceleration of magnitude 30 rad/s^2. How long does it take for the drive shaft to stop turning?
6. An old wheat-grinding wheel in a museum actually works. The sign on the wall says that the wheel has a rotational acceleration of 190 rad/s^2 as its spinning rotational speed increases from zero to 1800 rpm. How long does it take the wheel to attain this rotational speed?
7. **Centrifuge** A centrifuge at the same museum is used to separate seeds of different sizes. The average rotational acceleration of the centrifuge according to a sign is 30 rad/s^2. If starting at rest, what is the rotational velocity of the centrifuge after 10 s?
8. * **Potter's wheel** A fly sits on a potter's wheel 0.30 m from its axle. The wheel's rotational speed decreases from 4.0 rad/s to 2.0 rad/s in 5.0 s. Determine (a) the wheel's average rotational acceleration, (b) the angle through which the fly turns during the 5.0 s, and (c) the distance traveled by the fly during that time interval.
9. * During your tennis serve, your racket and arm move in an approximately rigid arc with the top of the racket 1.5 m from your shoulder joint. The top accelerates from rest to a speed of 20 m/s in a time interval of 0.10 s. Determine (a) the magnitude of the average tangential acceleration of the top of the racket and (b) the magnitude of the rotational acceleration of your arm and racket.
10. * An ant clings to the outside edge of the tire of an exercise bicycle. When you start pedaling, the ant's speed increases from zero to 10 m/s in 2.5 s. The wheel's rotational acceleration is 13 rad/s^2. Determine everything you can about the motion of the wheel and the ant.
11. * The speedometer on a bicycle indicates that you travel 60 m while your speed increases from 0 to 10 m/s. The radius of the wheel is 0.30 m. Determine three physical quantities relevant to this motion.
12. * You pedal your bicycle so that its wheel's rotational speed changes from 5.0 rad/s to 8.0 rad/s in 2.0 s. Determine (a) the wheel's average rotational acceleration, (b) the angle through which it turns during the 2.0 s, and (c) the distance that a point 0.60 m from the axle travels.
13. **Mileage gauge** The odometer on an automobile actually counts axle turns and converts the number of turns to miles based on knowledge that the diameter of the tires is 0.62 m. How many turns does the axle make when traveling 10 miles?
14. * **Speedometer** The speedometer on an automobile measures the rotational speed of the axle and converts that to a linear speed of the car, assuming the car has 0.62-m-diameter tires. What is the rotational speed of the axle when the car is traveling at 20 m/s (45 mph)?
15. * **Ferris wheel** A Ferris wheel starts at rest, acquires a rotational velocity of ω rad/s after completing one revolution and continues to accelerate. Write an expression for (a) the magnitude of the wheel's rotational acceleration (assumed constant), (b) the time interval needed for the first revolution, (c) the time interval required for the second revolution, and (d) the distance a person travels in two revolutions if he is seated a distance l from the axis of rotation.
16. * You push a disk-shaped platform tangentially on its edge 2.0 m from the axle. The platform starts at rest and has a rotational acceleration of 0.30 rad/s^2. Determine the distance you must run while pushing the platform to increase its speed at the edge to 7.0 m/s.
17. ** EST Estimate what Earth's rotational acceleration would be in rad/s^2 if the length of a day increased from 24 h to 48 h during the next 100 years.

9.3 Newton's second law for rotational motion

18. A 0.30-kg ball is attached at the end of a 0.90-m-long stick. The ball and stick rotate in a horizontal circle. Because of air resistance and to keep the ball moving at constant speed, a continual push must be exerted on the stick, causing a 0.036-N · m torque. Determine the magnitude of the resistive force that the air exerts on the ball opposing its motion. What assumptions did you make?
19. **Centrifuge** A centrifuge with a 0.40-kg · m^2 rotational inertia has a rotational acceleration of 100 rad/s^2 when the power is turned on. (a) Determine the minimum torque that the motor supplies. (b) What time interval is needed for the centrifuge's rotational velocity to increase from zero to 5000 rad/s?
20. **Airplane turbine** What is the average torque needed to accelerate the turbine of a jet engine from rest to a rotational velocity of 160 rad/s in 25 s? The turbine's rotating parts have a 32-kg · m^2 rotational inertia.
21. * A turntable turning at rotational speed 33 rpm stops in 50 s when turned off. The turntable's rotational inertia is 1.0×10^{-2} kg · m^2. How large is the resistive torque that slows the turntable?
22. * The solid pulley in Figure P9.22 consists of a two-part disk, which initially rotates counterclockwise. Two ropes pull on the pulley as shown. The inner part has a radius of 1.5a, and the outer part has a radius of 2.0a. (a) Construct a force diagram for the pulley with the origin of the coordinate system at the center of the pulley. (b) Determine the torque produced by each force (including the sign) and the resultant torque exerted on the pulley. (c) Based on the results of part (b), decide on the signs of the rotational velocity and the rotational acceleration.

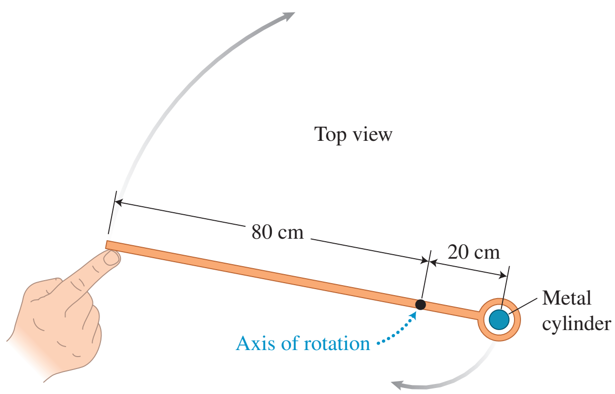
FIGURE P9.22



23. * The pulley shown in Figure P9.22 is initially rotating clockwise. Compare the forces exerted by the ropes on the disk in order for the wheel's rotational velocity to (a) remain constant, (b) increase in magnitude, and (c) decrease in magnitude. The outer radius is 2.0a compared to 1.5a for the inner radius.
24. The pulley shown in Figure P9.22 is initially rotating in the clockwise direction. The force that the rope on the right exerts on it is 1.5F and the force that the rope on the left exerts on it is F. Determine the ratio of the maximum radius of the inner circle compared to that of the outer circle in order for the wheel's rotational speed to decrease.

25. A 2.0-kg metal cylinder on a table is placed inside a hoop that is fixed at the end of a meter stick. The stick can rotate around a vertical axis that is located 20 cm from the cylinder (see **Figure P9.25**). By exerting a force on the stick, you can make the cylinder rotate in a horizontal plane, with the bottom surface of the cylinder sliding on the rough table. The coefficient of kinetic friction between the cylinder and the table is 0.3. (a) Determine the magnitude of the force that you need to exert on the end of the stick (perpendicular to it) so that the cylinder moves with a constant speed. (b) Determine the work done by your hand on the stick while your hand makes one full turn around the axis. Then determine the work done by the stick on the cylinder during the same time. Compare the two values and comment on the result. Does the result surprise you?

FIGURE P9.25



26. * **Equation Jeopardy 1** The equation below describes a rotational dynamics situation. Draw a sketch of a situation that is consistent with the equation and construct a word problem for which the equation might be a solution. There are many possibilities.

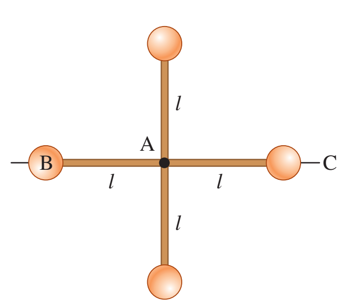
$$-(2.2\ \text{N})(0.12\ \text{m}) = [(1.0\ \text{kg})(0.12\ \text{m})^2]\alpha$$

27. * **Equation Jeopardy 2** The equation below describes a rotational dynamics situation. Draw a sketch of a situation that is consistent with the equation and construct a word problem for which the equation might be a solution. There are many possibilities.

$$-(2.0\ \text{N})(0.12\ \text{m}) + (6.0\ \text{N})(0.06\ \text{m}) = [(1.0\ \text{kg})(0.12\ \text{m})^2]\alpha$$

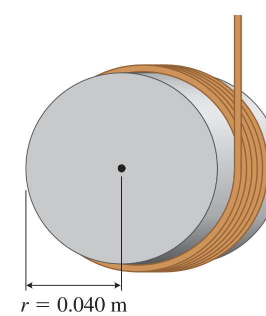
28. Derive an expression for the rotational inertia of the four balls shown in **Figure P9.28** about an axis perpendicular to the paper and passing through point A. The mass of each ball is m. Ignore the mass of the rods to which the balls are attached.

FIGURE P9.28



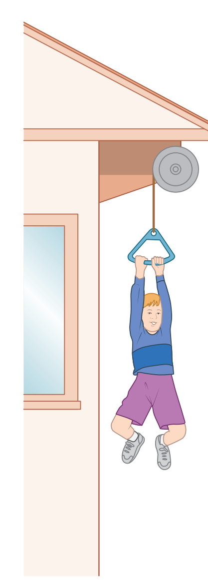
29. * Repeat the previous problem for an axis perpendicular to the paper through point B.
30. Repeat the previous problem for axis BC, which passes through two of the balls.
31. * **Merry-go-round** A mechanic needs to replace the motor for a merry-go-round. What torque specifications must the new motor satisfy if the merry-go-round should accelerate from rest to 1.5 rad/s in 8.0 s? You can consider the merry-go-round to be a uniform disk of radius 5.0 m and mass 25,000 kg.
32. * A small 0.80-kg train propelled by a fan engine starts at rest and goes around a circular track with a 0.80-m radius. The air exerts a 2.0-N force on the train. Determine (a) the rotational acceleration of the train and (b) the time interval needed for it to acquire a speed of 3.0 m/s. Indicate any assumptions you made.
33. * **Motor** You wish to buy a motor that will be used to lift a 20-kg bundle of shingles from the ground to the roof of a house. The shingles are to have a 1.5-m/s^2 upward acceleration at the start of the lift. The very light pulley on the motor has a radius of 0.12 m. Determine the minimum torque that the motor must be able to provide.
34. ** A string wraps around a 6.0-kg wheel of radius 0.20 m. The wheel is mounted on a frictionless horizontal axle at the top of an inclined plane tilted 37° below the horizontal. The free end of the string is attached to a 2.0-kg block that slides down the incline without friction. The block's acceleration while sliding down the incline is 2.0 m/s^2. (a) Draw separate force diagrams for the wheel and for the block. (b) Apply Newton's second law (either the translational form or the rotational form) for the wheel and for the block. (c) Determine the rotational inertia for the wheel about its axis of rotation.
35. * Elena, a black belt in tae kwon do, is experienced in breaking boards with her fist. A high-speed video indicates that her forearm is moving with a rotational speed of 40 rad/s when it reaches the board. The board breaks in 0.0040 s and her arm is moving at 20 rad/s just after breaking the board. Her fist is 0.32 m from her elbow joint and the rotational inertia of her forearm is 0.050 kg · m^2. Determine the average force that the board exerts on her fist while breaking the board (equal in magnitude to the force that her fist exerts on the board). Ignore the gravitational force that Earth exerts on her arm and the force that her triceps muscle exerts on her arm during the break.
36. ** **Like a yo-yo** Sam wraps a string around the outside of a 0.040-m-radius 0.20-kg solid cylinder and uses it like a yo-yo (**Figure P9.36**). When released, the cylinder accelerates downward at $(2/3)g$. (a) Draw a force diagram for the cylinder and apply the translational form of Newton's second law to the cylinder in order to determine the force that the string exerts on the cylinder. (b) Determine the rotational inertia of the solid cylinder. (c) Apply the rotational form of Newton's second law and determine the cylinder's rotational acceleration. (d) Is your answer to part (c) consistent with the application of $a = r\alpha$, which relates the cylinder's linear acceleration and its rotational acceleration? Explain.

FIGURE P9.36



37. * **Fire escape** A unique fire escape for a three-story house is shown in **Figure P9.37**. A 30-kg child grabs a rope wrapped around a heavy flywheel outside a bedroom window. The flywheel is a 0.40-m-radius uniform disk with a mass of 120 kg. (a) Make a force diagram for the child as he moves downward at increasing speed and another for the flywheel as it turns faster and faster. (b) Use Newton's second law for translational motion and the child force diagram to obtain an expression relating the force that the rope exerts on him and his acceleration. (c) Use Newton's second law for rotational motion and the flywheel force diagram to obtain an expression relating the force the rope exerts on the flywheel and the rotational acceleration of the flywheel. (d) The child's acceleration a and the flywheel's rotational acceleration α are related by the equation $a = r\alpha$, where r is the flywheel's radius. Combine this with your equations in parts (b) and (c) to determine the child's acceleration and the force that the rope exerts on the wheel and on the child.

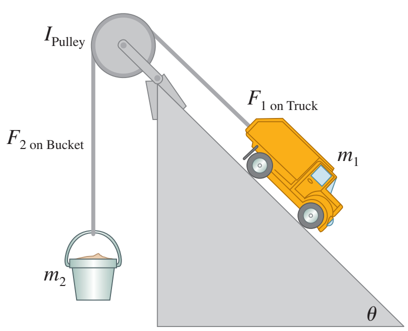
FIGURE P9.37



38. ** An Atwood machine is shown in Example 9.4. Use $m_1 = 0.20$ kg, $m_2 = 0.16$ kg, $M = 0.50$ kg, and $R = 0.10$ m. (a) Construct separate force diagrams for block 1, for block 2, and for the solid cylindrical pulley. (b) Determine the rotational inertia of the pulley. (c) Use the force diagrams for blocks 1 and 2 and Newton's second law to write expressions relating the unknown accelerations of the blocks. (d) Use the pulley force diagram and the rotational form of Newton's second law to write an expression for the rotational acceleration of the pulley. (e) Noting that $a = R\alpha$ for the pulley, use the three equations from parts (c) and (d) to determine the magnitude of the acceleration of the hanging blocks.

39. ** A physics problem involves a massive pulley, a bucket filled with sand, a toy truck, and an incline (see **Figure P9.39**). You push lightly on the truck so it moves down the incline. When you stop pushing, it moves down the incline at constant speed and the bucket moves up at constant speed. (a) Construct separate force diagrams for the pulley, the bucket, and the truck. (b) Use the truck force diagram and the bucket force diagram to help write expressions in terms of quantities shown in the figure for the forces $F_{1\text{ on Truck}}$ and $F_{2\text{ on Bucket}}$ that the rope exerts on the truck and that the rope exerts on the bucket. (c) Use the rotational form of Newton's second law to determine if the tension force $F_{1\text{ on Pulley}}$ that the rope on the right side exerts on the pulley is the same, greater than, or less than the force $F_{2\text{ on Pulley}}$ that the rope exerts on the left side.

FIGURE P9.39



40. * A thin rod of length L and mass m rotates around an axis perpendicular to the rod, passing through the rod's left end. Treat the rod as an object made up of five rods of length $L/5$. Derive an expression for the rotational inertia of this five-piece object around the same axis, assuming each piece is a point-like object with mass $m/5$. Compare your result with the expression for the rotational inertia of the one-piece rod ($I = \frac{1}{3}mL^2$). Discuss the similarities and the differences.

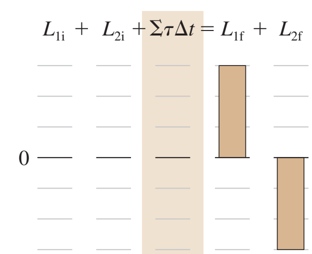
9.4 Rotational momentum

41. * (a) Determine the rotational momentum of a 10-kg disk-shaped flywheel of radius 9.0 cm rotating with a rotational speed of 320 rad/s. (b) With what magnitude rotational speed must a 10-kg solid sphere of 9.0 cm radius rotate to have the same rotational momentum as the flywheel?
42. **Ballet** A ballet student with her arms and a leg extended spins with an initial rotational speed of 1.0 rev/s. As she draws her arms and leg in toward her body, her rotational inertia becomes 0.80 kg · m^2 and her rotational velocity is 4.0 rev/s. Determine her initial rotational inertia.
43. * A 0.20-kg block moves at the end of a 0.50-m string along a circular path on a frictionless air table. The block's initial rotational speed is 2.0 rad/s. As the block moves in the circle, the string is pulled down through a hole in the air table at the axis of rotation. Determine the rotational speed and tangential speed of the block when the string is 0.20 m from the axis.
44. * **Puck on a string** You attach a 100-g puck to a string and let the puck glide in a circle on a horizontal, frictionless air table. The other end of the string passes through a hole at the center of the table. You pull down on the string so that the puck moves along a circular path of radius 0.40 m. It completes one revolution in 4.0 s. If you pull harder on the string so the radius of the circle slowly decreases to 0.20 m, what is the new period of revolution?
45. * **Equation Jeopardy 3** The equation below describes a process. Draw a sketch representing the initial and final states of the process and construct a word problem for which the equation could be a solution.

$$\left(\frac{2}{5}mR^2\right)\left(\frac{2\pi}{30\text{ days}}\right) = \left[\frac{2}{5}m\left(\frac{R}{100}\right)^2\right]\left(\frac{2\pi}{T_f}\right)$$

46. ** A student sits motionless on a stool that can turn friction-free about its vertical axis (total rotational inertia I). The student is handed a spinning bicycle wheel, with rotational inertia I_{wheel}, that is spinning about a vertical axis with a counterclockwise rotational velocity ω_0. The student then turns the bicycle wheel over (that is, through 180°). Estimate, in terms of ω_0, the final rotational velocity acquired by the student.
47. * **Neutron star** An extremely dense neutron star with mass equal to that of the Sun has a radius of about 10 km—about the size of Manhattan Island. These stars are thought to rotate once about their axis every 0.03 to 4 s, depending on their size and mass. Suppose that the neutron star described in the first sentence rotates once every 0.040 s. If its volume then expanded to occupy a uniform sphere of radius 1.4×10^8 m (most of the Sun's mass is in a sphere of this size) with no change in mass or rotational momentum, what time interval would be required for one rotation? By comparison, the Sun rotates once about its axis each month.
48. * A boy of mass m is standing on the edge of a merry-go-round platform, which is initially rotating with a constant rotational velocity ω_i (initial state). The rotational inertia of the merry-go-round is I, and the radius of the platform is r. After the boy steps down from the platform, tangentially to its edge, the merry-go-round continues rotating with a smaller rotational velocity ω_f, and the boy continues moving away from the platform with constant speed v (final state). (a) Draw three rotational momentum bar charts, choosing the boy, the merry-go-round, and both of them as a system. (b) Derive the expression for ω_f in terms of the relevant quantities.
49. * **Bar chart jeopardy** The rotational momentum bar chart in **Figure P9.49** describes a rotational dynamics situation. Draw a sketch of a situation that is consistent with the equation and write a word problem to which the bar chart could apply.

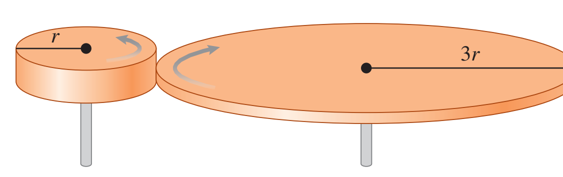
FIGURE P9.49



9.5 Rotational kinetic energy

50. A grinding wheel with rotational inertia I gains rotational kinetic energy K after starting from rest. Determine an expression for the wheel's final rotational speed.
51. * The rotational speed of a flywheel increases by 40%. By what percent does its rotational kinetic energy increase? Explain your answer.
52. ** Two uniform disks are made from the same material and have equal masses. The radius of disk B is three times larger than the radius of disk A. The disks touch each other and rotate at constant angular speed without skidding (**Figure P9.52**). Determine (a) the ratio of the disks' thicknesses d_B/d_A, (b) the ratio of the disks' angular velocities ω_B/ω_A, (c) the ratio of the disks' rotational inertias I_B/I_A, (d) the ratio of the disks' rotational momenta L_B/L_A, and (e) the ratio of the disks' rotational kinetic energies $K_{\text{rot B}}/K_{\text{rot A}}$.

FIGURE P9.52



53. * **Flywheel energy for car** The U.S. Department of Energy had plans for a 1500-kg automobile to be powered completely by the rotational kinetic energy of a flywheel. (a) If the 300-kg flywheel (included in the 1500-kg mass of the automobile) had a 6.0-kg · m^2 rotational inertia and could turn at a maximum rotational speed of 3600 rad/s, determine the energy stored in the flywheel. (b) How many accelerations from a speed of zero to 15 m/s could the car make before the flywheel's energy was dissipated, assuming 100% energy transfer and no flywheel regeneration during braking? Ignore rotational kinetic energy of car wheels.

54. * **Flywheel energy** Engineers at the University of Texas at Austin are developing an Advanced Locomotive Propulsion System that uses a gas turbine and perhaps the largest high-speed flywheel in the world in terms of the energy it can store. The flywheel can store 4.8×10^8 J of energy when operating at its maximum rotational speed of 15,000 rpm. At that rate, the perimeter of the rotor moves at approximately 1,000 m/s. Determine the radius of the flywheel and its rotational inertia.

55. * **Equation Jeopardy 4** The equations below represent the initial and final states of a process (plus some ancillary information). Construct a sketch of a process that is consistent with the equations and write a word problem for which the equations could be a solution.

$$(80\text{ kg})(9.8\text{ N/kg})(16\text{ m}) = \tfrac{1}{2}(80\text{ kg})v_f^2 + \tfrac{1}{2}(240\text{ kg}\cdot\text{m}^2)\omega_f^2$$
$$v_f = (0.40\text{ m})\omega_f$$

56. ** **Rotating student** A student sitting on a chair on a circular platform of negligible mass rotates freely on an air table at initial rotational speed 2.0 rad/s. The student's arms are initially extended with 6.0-kg dumbbells in each hand. As the student pulls her arms in toward her body, the dumbbells move from a distance of 0.80 m to 0.10 m from the axis of rotation. The initial rotational inertia of the student's body (not including the dumbbells) with arms extended is $6.0\text{ kg}\cdot\text{m}^2$, and her final rotational inertia is $5.0\text{ kg}\cdot\text{m}^2$. (a) Determine the student's final rotational speed. (b) Determine the change of kinetic energy of the system consisting of the student together with the two dumbbells. (c) Determine the change in the kinetic energy of the system consisting of the two dumbbells alone without the student. (d) Determine the change of kinetic energy of the system consisting of the student alone without the dumbbells. (e) Compare the kinetic energy changes in parts (b) through (d).

57. * A turntable whose rotational inertia is $1.0 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ rotates on a frictionless air cushion at a rotational speed of 2.0 rev/s. A 1.0-g beetle falls to the center of the turntable and then walks 0.15 m to its edge. (a) Determine the rotational speed of the turntable with the beetle at the edge. (b) Determine the kinetic energy change of the system consisting of the turntable and the beetle. (c) Account for this energy change.

58. ** Repeat the previous problem, only assume that the beetle initially falls on the edge of the turntable and stays there.

59. * A bug of a known mass m stands at a distance d cm from the axis of a spinning disk (mass m_d and radius r_d) that is rotating at f_i revolutions per second. After the bug walks out to the edge of the disk and stands there, the disk rotates at f_f revolutions per second. (a) Use the information above to write an expression for the rotational inertia of the disk. (b) Determine the change of kinetic energy in going from the initial to the final situation for the total bug-disk system.

60. * **Merry-go-round** A carnival merry-go-round has a large disk-shaped platform of mass 120 kg that can rotate about a center axle. A 60-kg student stands at rest at the edge of the platform 4.0 m from its center. The platform is also at rest. The student starts running clockwise around the edge of the platform and attains a speed of 2.0 m/s relative to the ground. (a) Determine the rotational velocity of the platform. (b) Determine the change of kinetic energy of the system consisting of the platform and the student.

61. * EST You hold an apple by its stem between your thumb and index finger and spin it so that the apple is rotating at approximately constant speed. Estimate the rotational kinetic energy and the rotational momentum of the apple. Indicate any assumptions that you made.

General Problems

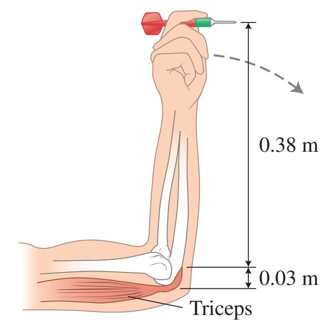
62. * **Stopping Earth's rotation** Suppose that Superman wants to stop Earth so it does not rotate. He exerts a force on Earth $\vec{F}_{\text{S on E}}$ at Earth's equator tangent to its surface for a time interval of 1 year. What magnitude force must he exert to stop Earth's rotation? Indicate any assumptions you make when completing your estimate.

63. * BIO EST **Punting a football** Estimate the tangential acceleration of the foot and the rotational acceleration of the leg of a football punter during the time interval that the leg starts to swing forward in an arc until the instant just before the foot hits the ball. Indicate any assumptions that you make and be sure that your method is clear.

64. * EST Estimate the average rotational acceleration of a car tire as you leave an intersection after a light turns green. Discuss the choice of numbers used in your estimate.

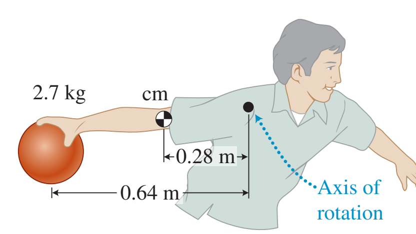
65. * BIO **Triceps and darts** Your upper arm is horizontal and your forearm is vertical with a 0.010-kg dart in your hand (**Figure P9.65**). When your triceps muscle contracts, your forearm initially swings forward with a rotational acceleration of 35 rad/s^2. Determine the force that your triceps muscle exerts on your forearm during this initial part of the throw. The rotational inertia of your forearm is $0.12\text{ kg}\cdot\text{m}^2$ and the dart is 0.38 m from your elbow joint. Your triceps muscle attaches 0.03 m from your elbow joint.

FIGURE P9.65



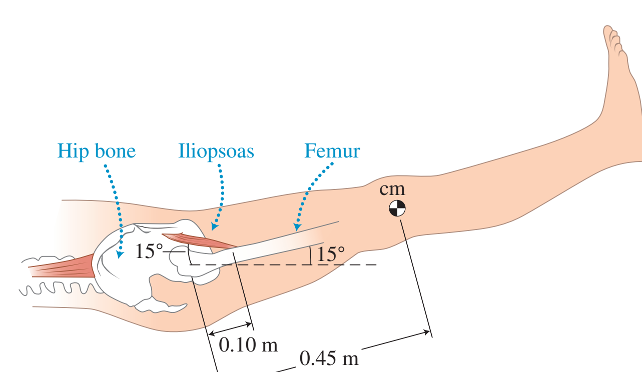
66. * BIO **Bowling** At the start of your throw of a 2.7-kg bowling ball, your arm is straight behind you and horizontal (**Figure P9.66**). Determine the rotational acceleration of your arm if the muscle is relaxed. Your arm is 0.64 m long, has a rotational inertia of $0.48\text{ kg}\cdot\text{m}^2$, and has a mass of 3.5 kg with its center of mass 0.28 m from your shoulder joint.

FIGURE P9.66



67. ** BIO **Leg lift** You are doing one-leg leg lifts (**Figure P9.67**) and decide to estimate the force that your iliopsoas muscle exerts on your upper leg bone (the femur) when being lifted (the lifting involves a variety of muscles). The mass of your entire leg is 15 kg, its center of mass is 0.45 m from the hip joint, and its rotational inertia is $4.0\text{ kg}\cdot\text{m}^2$, and you estimate that the rotational acceleration of the leg being lifted is 35 rad/s^2. For calculation purposes assume that the iliopsoas attaches to the femur 0.10 m from the hip joint. Also assume that the femur is oriented 15° above the horizontal and that the muscle is horizontal. Estimate the force that the muscle exerts on the femur.

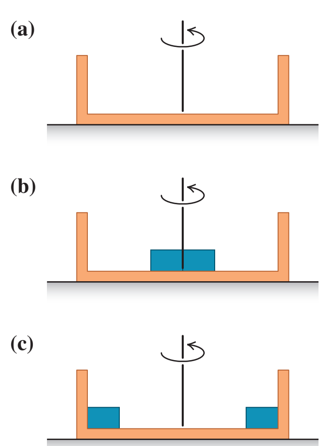
FIGURE P9.67



68. * A horizontal, circular platform can rotate around a vertical axis at its center with negligible friction. You decide to use the rotating platform to design a procedure that will allow you to determine the unknown rotational inertias of different objects, for example, a bowl. You know the rotational inertia of the platform I_P. Which of the following procedures would best serve your task? (More than one answer could be correct.) Comment on the procedures that are not suitable and explain what is wrong with them.
 (a) Exert a constant torque τ on the empty platform and measure its angular acceleration α_P. Then place the bowl on the platform at its center and repeat the previous experiment, obtaining α_{PB}. Calculate the rotational inertia of the bowl using the expression $I_B = \dfrac{\tau}{\alpha_{PB} - \alpha_P}$.
 (b) Perform the same experiment as in (a) but use the expression $I_B = I_P\left(\dfrac{\alpha_P}{\alpha_{PB}} - 1\right)$.
 (c) Spin the empty platform and measure its angular velocity ω_i. While the platform is rotating, place the bowl carefully on the platform at its center. Wait until the bowl rotates together with the platform. Measure the final angular velocity of the bowl-platform system ω_f. Calculate the rotational inertia of the bowl using the expression $I_B = I_P\left(\dfrac{\omega_i}{\omega_f} - 1\right)$.
 (d) Perform the same experiment as in (c) but use the expression $I_B = I_P\left(\dfrac{\omega_i^2}{\omega_f^2} - 1\right)$.
 (e) Measure the mass of the platform m_P, the mass of the bowl m_B, the radius of the platform r_P, and the radius of the bowl r_B. Calculate the rotational inertia of the bowl using the expression $I_B = I_P \cdot \dfrac{m_B r_B^2}{m_P r_P^2}$.

69. * You have an empty cylindrical metal can and two metal nuts. In your first experiment, you spin the can around its axis and place it on a rough table (Figure P9.69a). The can slows down due to friction, with the rotational acceleration $\alpha_1 = 30\ \text{rad/s}^2$ until it stops rotating. In your second experiment, you first fix two nuts near the center of the can (Figure P9.69b) and spin it again before placing it on the table. In this case, the can slows down with the rotational acceleration $\alpha_2 = 38\ \text{rad/s}^2$. In your third experiment, you spin the can with the nuts fixed at opposite sides of the can (Figure P9.69c). Now the can slows down with rotational acceleration $\alpha_3 = 26\ \text{rad/s}^2$. Explain (a) why $\alpha_2 > \alpha_1$, (b) why $\alpha_3 < \alpha_2$, and (c) why $\alpha_3 < \alpha_1$.

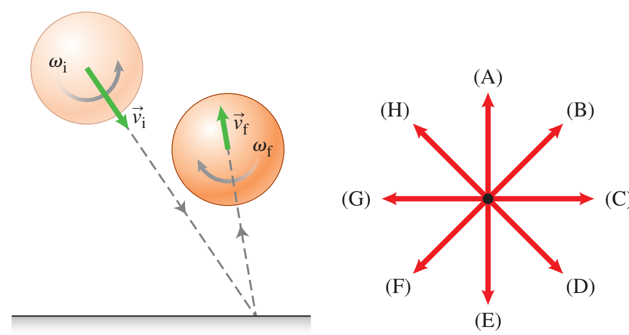
FIGURE P9.69



70. ** In the previous problem, each nut has a mass of 8.0 g. The distances between the centers of mass of the nuts and the axis of rotation are 0.8 cm in Experiment 2 and 3.5 cm in Experiment 3. Determine the rotational inertia of the can (without nuts). Indicate any assumptions that you made. (Hint: The torque due to the kinetic friction force in Experiments 2 and 3 is the same.)

71. * **Superball** If you give a superball backspin and throw it toward a horizontal floor, it is possible that the ball bounces backward, as shown in Figure P9.71. If the ball has a color pattern or stripes, you may also notice that during the collision with the ground, the direction of rotation of the ball changes, as indicated in the figure. Both changes (the change of the ball's translational velocity and the change of the ball's angular velocity) are the result of a force exerted on the ball by the ground during the collision. (a) In which approximate direction did the ground exert the force on the ball in the case shown in the figure? Choose the best answer from the options given in the figure and explain your answer. (b) The ball has initial and final velocity (with components v_{xi}, v_{yi} and v_{xf}, v_{yf}) and initial and final angular velocity (ω_i and ω_f). During the collision, the ground exerts a force (determined by F_x and F_y) on the ball. Complete the table below by drawing crosses in the cells that correctly connect changes of the quantities in the first column and the components of the force during the collision. Explain your answers.

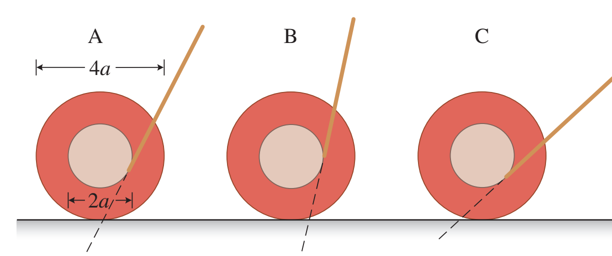
FIGURE P9.71



	F_x	F_y
$v_{xf} - v_{xi}$		
$v_{yf} - v_{yi}$		
$\omega_f - \omega_i$		

72. ** **Yo-yo trick** A yo-yo rests on a horizontal table. The yo-yo is free to roll but friction prevents it from sliding. When the string exerts one of the following tension forces on the yo-yo (shown in Figure P9.72), which way does the yo-yo roll? Try the problem for each force: (a) $\vec{T}_{\text{A S on Y}}$; (b) $\vec{T}_{\text{B S on Y}}$; and (c) $\vec{T}_{\text{C S on Y}}$. (Hint: Think about torques about a pivot point where the yo-yo touches the table.)

FIGURE P9.72

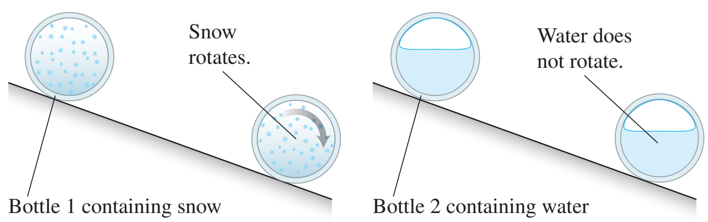


73. * EST **White dwarf** A star the size of our Sun runs out of nuclear fuel and, without losing mass, collapses to a white dwarf star the size of our Earth. If the star initially rotates at the same rate as our Sun, which is once every 25 days, determine the rotation rate of the white dwarf. Indicate any assumptions you make.

Reading Passage Problems

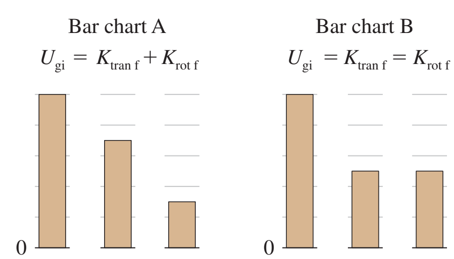
Rolling versus sliding Our knowledge of rotational kinetic energy helps explain a very simple but rather mysterious experiment that you can perform at home. For this experiment, you need two identical plastic water bottles (1 and 2). Fill bottle 1 with snow (if you do not have snow, you can use whipped cream). Fill bottle 2 with water so that the masses of filled bottles are the same. Place one bottle at the top of an inclined plane and let it roll down. Then repeat the experiment with the other bottle (Figure 9.18). You observe an interesting effect: when bottle 1 rolls down, it rotates and the solid snow inside rotates with the bottle. Bottle 2 rotates, too, but the water inside does not rotate much. Thus, in effect, the water slides down the incline and does not roll. When the snow-filled bottle rolls down, it rotates as a solid cylinder, acquiring rotational kinetic energy in addition to translational kinetic energy. In the case of the water-filled bottle, only the bottle rotates; the water inside just translates. Rolling is a combination of translation and rotation, whereas sliding involves only translation. The water bottle has almost no rotational kinetic energy and a larger translational kinetic energy at the end of the plane.

FIGURE 9.18



74. Which bar chart in **Figure P9.74** corresponds to the bottle filled with solid snow rolling from the top of the incline to the bottom?
    (a) Bar chart A
    (b) Bar chart B
    (c) Both charts could represent the trip.

FIGURE P9.74

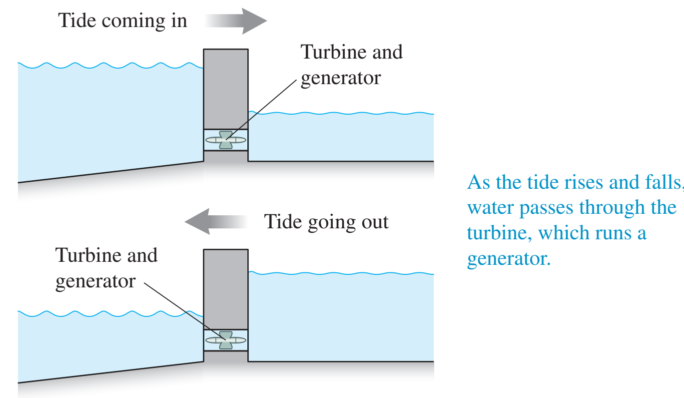


75. Which bar chart in **Figure P9.74** corresponds to the bottle with the shortest time interval from the top of the incline to the bottom?
    (a) Bar chart A
    (b) Bar chart B
    (c) The charts represent the same time interval for the trip.
76. Based on the results from the two previous questions, which bottle should take the shortest time interval to complete the race?
    (a) Bottle 1
    (b) Bottle 2
    (c) Both bottles take the same time interval.
77. Which statement best explains your answer to the previous problem?
    (a) Most of the initial gravitational potential energy is converted into translational kinetic energy.
    (b) Some of the initial gravitational potential energy is converted to rotational kinetic energy.
    (c) All initial gravitational potential energy is converted to kinetic energy.

Tidal energy Tides are now used to generate electric power in two ways. In the first, huge dams can be built across the mouth of a river where it exits to the ocean. As the ocean tide moves in and out of this tidal basin or estuary, the water flows through tunnels in the dam (see **Figure 9.19**). This flowing water turns turbines in the tunnels that run electric generators. Unfortunately, this technique works best with large increases in tides—a 5-m difference between high and low tide. Such differences are found at only a small number of places. Currently, France is the only country that successfully uses this power source. A tidal basin plant in France, the Rance Tidal Power Station, makes 240 megawatts of power—enough energy to power 240,000 homes. Damming tidal basins can have negative environmental effects because of reduced tidal flow and silt buildup. Another disadvantage is that they can only generate electricity when the tide is flowing in or out, for about 10 hours each day.

A second method for collecting energy from the tidal flow (as well as all water flow) is to place turbines directly in the water—like windmills in moving water instead of in moving air. These water turbines have the advantages that they are much cheaper to build, they do not have the environmental problems of a tidal basin, and there are many more suitable sites for such water flow energy farms. Also, the energy density of flowing water is about 800 times the energy density of dry air flow. Verdant Power is developing turbine prototypes in the East River near New York City and in the Saint Lawrence Seaway in Canada, and they are looking at other sites in the Puget Sound and all over the world. The worldwide potential for hydroelectric power is about 25 terawatts $= 25 \times 10^{12}$ J/s—enough to supply the world's energy needs.

FIGURE 9.19 Dams built across tidal basins can generate electric power.



78. If the Rance Tidal Power Station in France could produce power 24 hours a day, which answer below is closest to the daily amount of energy in joules that it could produce?
    (a) 240 J
    (b) 240×10^6 J
    (c) 6×10^9 J
    (d) 2.5×10^{10} J
    (e) 2×10^{13} J
79. Suppose a tidal basin is 5 m above the ocean at low tide and that the area of the basin is 4×10^7 m^2 (about 4 miles by 4 miles). Which answer below is closest to the gravitational potential energy change if the water is released from the tidal basin to the low-tide ocean level? The density of water is 1000 kg/m^3. (Hint: The level does not change by 5 m for all of the water.)
    (a) 5×10^8 J
    (b) 5×10^{11} J
    (c) 1×10^{12} J
    (d) 5×10^{12} J
    (e) 1×10^{13} J
80. The Rance tidal basin can only produce electricity when what is occurring?
    (a) Water is moving into the estuary from the ocean.
    (b) Water is moving into the ocean from the estuary.
    (c) Water is moving in either direction.
    (d) The Moon is full.
    (e) The Moon is full and directly overhead.
81. Why do water turbines seem more promising than tidal basins for producing electric energy?
    (a) Turbines are less expensive to build.
    (b) Turbines have less impact on the environment.
    (c) There are many more locations for turbines than for tidal basins.
    (d) Turbines can operate 24 hours/day versus only 10 hours/day for tidal basins.
    (e) All of the above
82. Why do water turbines have an advantage over air turbines (windmills)?
    (a) Air moves faster than water.
    (b) The energy density of moving water is much greater than that of moving air.
    (c) Water turbines can float from one place to another, whereas air turbines are fixed.
    (d) All of the above
    (e) None of the above
83. Which of the following is a correct statement about water turbines?
    (a) Water turbines can operate only in moving tidal water.
    (b) Water turbines can produce only a small amount of electricity.
    (c) Water turbines have not had a proof of concept.
    (d) Water turbines cause significant ocean warming.
    (e) None of the above are correct statements.