CHAPTER 8

# Rotational Motion



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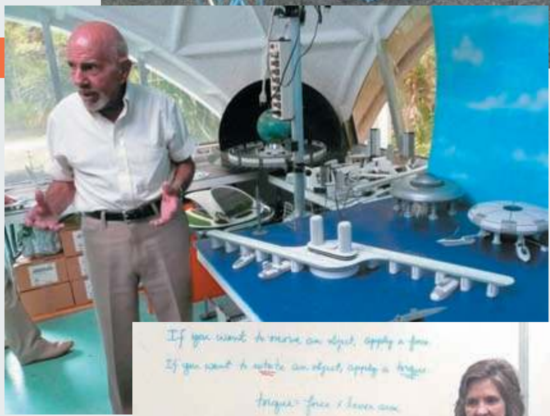

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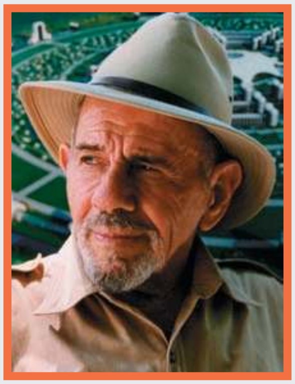
1 From what vertical distance above the top of the loop should Paul Stokstad, founder of Pasco, release the car for a successful loop-the-loop motion? 2 Futurist Jacque Fresco, the most passionate teacher I have ever met, inspired my love of teaching. 3 Mary Beth Monroe demonstrates a "torque feeler" before she passes it around for her students to try. 4 Diana Lininger Markham places an open container of water on the green dish, 5 which when swung in a circular path doesn't spill! Why?

Futurist thinker Jacque Fresco was the foremost influence in my transition from being a sign painter to pursuing a life in physics. I met Fresco through my sign-painting partner, Burl Grey, in Miami, Florida. My wife and I, along with cartoonist friend Ernie Brown, attended Fresco's dynamic weekly lectures in Miami Beach and sometimes at his home in Coral Gables. Charismatic Jacque has always been a visionary, believing that the best path to a better future is via science and technology and that a community with more engineers than lawyers is likely to be a better one. His topics revolved around the importance of expanded technology to better living, locally and globally. In my experience, Jacque was and is the very best of teachers—an enormous influence on my own teaching. He taught me to introduce concepts new to a student by first comparing them to familiar ideas—teaching by analogy. He felt that little or nothing would be learned if it was not tied to something similar, familiar, and already understood. He had a built-in "crap detector" that ensured an emphasis on the central parts of an idea. His students left every lecture with knowledge that was valued. The experience convinced me to take advantage of the GI Bill (I was a noncombat Korean War vet), get a college education, and pursue a career in science.



Jacque Fresco, with his associate, Roxanne Meadows, founded *The Venus Project* and the nonprofit organization *Future by Design* that reflect the culmination of Jacque's life work: the integration of the best of science and technology into a comprehensive plan for a new society based on human and environmental concerns—a global vision of hope for the future of humankind in our technological age. His vision is well stated in his many books and publications, on the Web, and in the movie *Zeitgeist Addendum*. Now in his late 90s, Jacque continues to inspire both young and old worldwide.

In typical lecture lessons, Jacque treated the distinctions between closely related ideas as well as their similarity. I recall one of his lessons that distinguished linear motion from rotational motion. Where does a child move faster on a merry-go-round—near the outside rail or near the inside rail—or is the speed the same in both places? Because the distinction between linear speed and rotational speed is poorly understood, Jacque said that asking this question to different people results in different answers. Just as "tail-end Charlie" at the end of a line of skaters making a turn moves faster than skaters near the center of the curve, so it is that railroad-train wheels on the outside track of a curve travel faster than wheels on the inside track. Jacque explained how the slight tapering of the wheel rims make this possible. This and other similarities and distinctions are treated in this chapter.

## 8.1 Circular Motion

Linear speed, which we simply called *speed* in earlier chapters, is the distance traveled per unit of time. A point on the outside edge of a merry-go-round or turntable travels a greater distance in one complete rotation than does a point nearer the center. Traveling a greater distance in the same time means a greater speed. Linear speed is greater on the outer edge of a rotating object than it is closer to the axis. The linear speed of something moving along a circular path can be called **tangential speed** because the direction of motion is tangent to the circumference of the circle. For circular motion, we can use the terms *linear speed* and *tangential speed* interchangeably. Units of linear or tangential speed are usually m/s or km/h.

**Rotational speed** (sometimes called *angular speed*) is the number of rotations or revolutions per unit of time. All parts of the rigid merry-go-round and turntable turn about the axis of rotation in the same amount of time. Thus, all parts share the same rate of rotation, or the same *number of rotations or revolutions per unit of time.* It is common to express rotational rates in revolutions per minute (RPM).[1] For example, most phonograph turntables, which were common in Mom

**fyi**

- When an object turns about an internal axis, the motion is a *rotation,* or spin. A merry-go-round, a turntable, and Earth rotate about a central internal axis. The term *revolution*, however, refers to an object moving about an external axis. Earth *revolves* once each year about the Sun, while it *rotates* once each day about its polar axis.

MasteringPhysics®

**TUTORIAL: Rotational Motion**

[1]Physics types usually describe rotational speed, $\omega$, in terms of the number of "radians" turned in a unit of time. There are a little more than 6 radians in a full rotation ($2\pi$ radians, to be exact). When a direction is assigned to rotational speed, we call it *rotational velocity* (often called *angular velocity*). Rotational velocity is a vector whose magnitude is the rotational speed. By convention, the rotational velocity vector lies along the axis of rotation and points in the direction of advance of a conventional right-handed screw.

**VIDEO: Rotational Speed**

**SCREENCAST: Circular Motion**

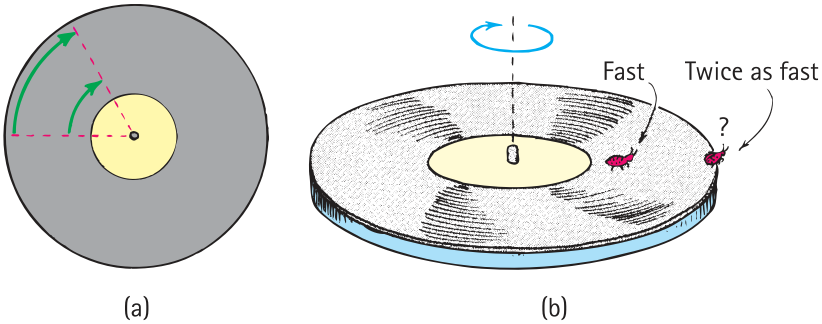



**FIGURE 8.1**
INTERACTIVE FIGURE MP

(a) When the turntable rotates, a point farther from the center travels a longer path in the same time and has a greater tangential speed. (b) A ladybug twice as far from the center moves twice as fast.

When a row of people, locked arm in arm at the skating rink, makes a turn, the motion of "tail-end Charlie" is evidence of a greater tangential speed.

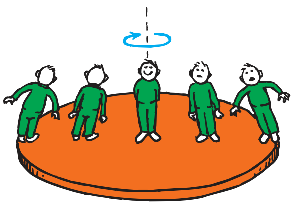


**FIGURE 8.2**
INTERACTIVE FIGURE MP

The tangential speed of each person is proportional to the rotational speed of the platform multiplied by the distance from the central axis.

**SCREENCAST: RR Wheels**

Why will a person with one leg shorter than the other tend to walk in circles when lost?

and Dad's time, rotate at $33\frac{1}{3}$ RPM. A ladybug sitting anywhere on the surface of the turntable revolves at $33\frac{1}{3}$ RPM.

Tangential speed and rotational speed are related. Have you ever ridden on a big, round, rotating platform in an amusement park? The faster it turns, the faster your tangential speed. This makes sense; the more RPMs, the faster your speed in meters per second. We say that tangential speed is *directly proportional* to rotational speed at any fixed distance from the axis of rotation.

Tangential speed, unlike rotational speed, depends on radial distance (the distance from the axis). At the very center of the rotating platform, you have no tangential speed at all; you merely rotate. But, as you approach the edge of the platform, you find yourself moving faster and faster. Tangential speed is directly proportional to distance from the axis for any given rotational speed.

So we see that tangential speed is proportional to both radial distance and rotational speed:[2]

$$\text{Tangential speed} \sim \text{radial distance} \times \text{rotational speed}$$

In symbolic form,

$$v \sim r\omega$$

where $v$ is tangential speed and $\omega$ (the Greek letter omega) is rotational speed. You move faster if the rate of rotation increases (bigger $\omega$). You also move faster if you move farther from the axis (bigger $r$). Move out twice as far from the rotational axis at the center, and you move twice as fast. Move out three times as far, and you have three times as much tangential speed. If you find yourself in any kind of rotating system, your tangential speed depends on how far you are from the axis of rotation.

When tangential speed undergoes change, we speak of a *tangential acceleration.* Any change in tangential speed indicates an acceleration parallel to tangential motion. For example, a person on a rotating platform that speeds up or slows down undergoes a tangential acceleration. We'll soon see that anything moving in a curved path undergoes another kind of acceleration—one directed to the center of curvature. This is *centripetal acceleration.* In the interest of "information overload," we'll not go into the details of tangential or centripetal acceleration.

[2]If you take a follow-up physics course, you will learn that when the proper units are used for tangential speed $v$, rotational speed $\omega$, and radial distance $r$, the direct proportion of $v$ to both $r$ and $\omega$ becomes the exact equation $v = r\omega$. So the tangential speed will be directly proportional to $r$ when all parts of a system simultaneously have the same $\omega$, as for a wheel or disk (or a flyswatter!).

## WHEELS ON RAILROAD TRAINS

Why does a moving railroad train stay on the tracks? Most people assume that the wheel flanges keep the wheels from rolling off. But, if you look at these flanges, you'll likely notice that they are rusty. They seldom touch the track, except when they follow slots that switch the train from one set of tracks to another. So how do the wheels of a train stay on the tracks? They stay on the track because their rims are slightly tapered.

If you roll a tapered cup across a surface, it makes a curved path (Figure 8.3). The wider part of the cup has a larger radius, rolls a greater distance per revolution, and therefore has a greater tangential speed than the narrow part. If you fasten a pair of cups together at their wide ends (simply taping them together) and roll the pair along a pair of parallel tracks, the cups will remain on the track and center themselves whenever they roll off center (Figure 8.4). This occurs because when the pair rolls to the left of center, say, the wide part of the left cup rides on the left track while the narrow part of the right cup rides on the right track. This steers the pair toward the center. If the pair "overshoots" toward the right, the process repeats, this time toward the left, as the wheels tend to center themselves. Likewise for a railroad train, where passengers feel the train swaying as these corrective actions occur.

This tapered shape is essential on the curves of railroad tracks. On any curve, the distance along the outer part is greater than the distance along the inner part (as in Figure 8.1a). So, whenever a vehicle follows a curve, its outer wheels travel faster than its inner wheels. For an automobile, this is not a problem because the wheels are freewheeling and roll independently of each other. For a train, however, like the pair of fastened cups, pairs of wheels are firmly connected and rotate together. Opposite wheels have the same RPM at any time. But, due to the slightly tapered rim of the wheel, its tangential speed along the track depends on whether it rides on the narrow part of the rim or the wide part. On the wide part, it travels faster. So, when a train rounds a curve, wheels on the outer track ride on the wide part of the tapered rims, while opposite wheels ride on their narrow parts. In this way, the wheels have different tangential speeds for the same rotational speed. This is $v \sim r\omega$ in action! Can you see that if the wheels were not tapered, scraping would occur and the wheels would squeal when a train rounds a curve?

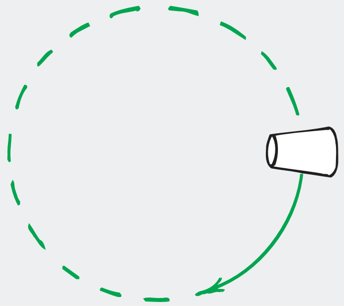


**FIGURE 8.3**
Because the wide part of the cup rolls faster than the narrow part, the cup rolls in a curve.

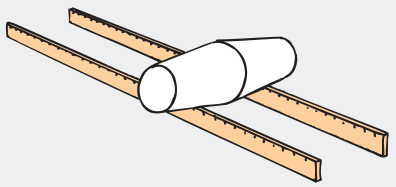


**FIGURE 8.4**
Two fastened cups stay on the tracks as they roll because, when they roll off center, the different tangential speeds due to the taper cause them to self-correct toward the center of the track.

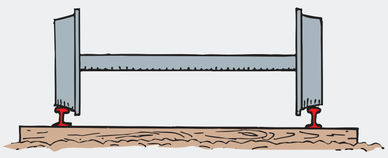


**FIGURE 8.5**
Wheels of a railroad train are slightly tapered (shown exaggerated here).

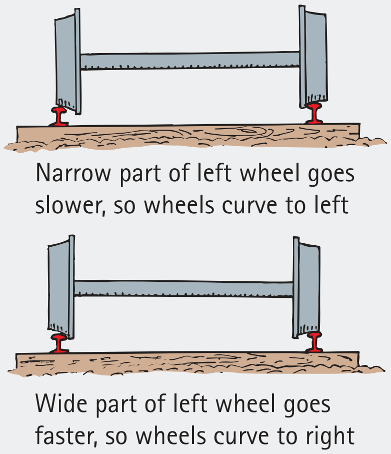



**FIGURE 8.6**
(Top) Along a track that curves to the left, the right wheel rides on its wide part and goes faster, while the left wheel rides on its narrow part and goes slower. (Bottom) The opposite is true when the track curves to the right.

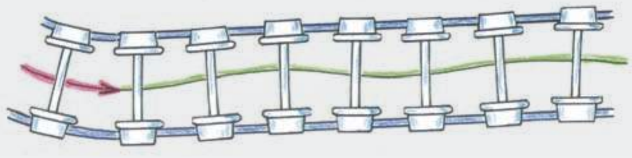


**FIGURE 8.7**
After rounding a curve, a train often oscillates on the straightaway as the wheels self-correct.

**FIGURE 8.8**
Cathy Candler asks her class which set of cups will self-correct when she rolls them along a pair of "meterstick tracks."

**fyi**

- An idea put forward in past years for boosting efficiency in electric rail travel was massive rotor disks—flywheels—beneath the flooring of railroad cars. When brakes were applied, rather than slowing the cars by converting braking energy to heat via friction, the braking energy would be diverted to revving the flywheels, which then could operate generators to supply electric energy for operating the train. The massiveness of the rotors turned out to make the scheme impractical, but the idea hasn't been lost. Today's hybrid automobiles do much the same thing—not mechanically, but electrically. Braking energy is diverted to electric batteries, which are then used for operating the automobile.

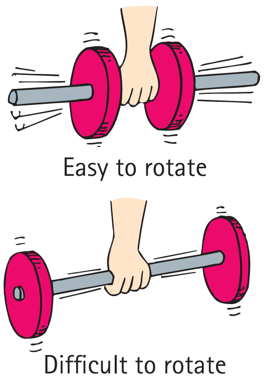



**FIGURE 8.9**
Rotational inertia depends on the distribution of mass relative to the axis of rotation.

**CHECK POINT**

1. **Imagine a ladybug sitting halfway between the rotational axis and the outer edge of the turntable in Figure 8.1b. When the turntable has a rotational speed of 20 RPM and the bug has a tangential speed of 2 cm/s, what will be the rotational and tangential speeds of her friend who sits at the outer edge?**
2. **Trains ride on a pair of tracks. For straight-line motion, both tracks are the same length. Not so for tracks along a curve, however. Which track is longer: the one on the outside of the curve or the one on the inside?**

**CHECK YOUR ANSWERS**

1. Since all parts of the turntable have the same rotational speed, her friend also rotates at 20 RPM. Tangential speed is a different story: Since the friend is twice as far from the axis of rotation, she moves twice as fast—4 cm/s.
2. Similar to Figure 8.1a, the track on the outside of the curve is longer—just as the circumference of a circle of greater radius is longer.

## 8.2 Rotational Inertia

Just as an object at rest tends to stay at rest and an object in motion tends to remain moving in a straight line, *an object rotating about an axis tends to remain rotating about the same axis unless interfered with by some external influence.* (We shall see shortly that this external influence is properly called a *torque.*) The property of an object to resist changes in its rotational state of motion is called **rotational inertia**.[3] Bodies that are rotating tend to remain rotating, while nonrotating bodies tend to remain nonrotating. In the absence of outside influences, a rotating top keeps rotating, while a top at rest remains at rest.

Like inertia for linear motion, rotational inertia depends on mass. The thick stone disk that rotates beneath a potter's wheel is very massive, and, once it is spinning, it tends to remain spinning. But, unlike linear motion, rotational inertia depends on the distribution of the mass about the axis of rotation. The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia. This is evident in industrial flywheels that are constructed so that most of their mass is concentrated far from the axis, along the rim. Once rotating, they have a greater tendency to remain rotating. When at rest, they are more difficult to get rotating.

Industrial flywheels provide a practical means of storing energy in electric power plants. When the plants continuously generate electricity, energy that is not needed when power demand is low is diverted to massive flywheels, which are the counterpart of electric batteries—but environmentally sound with no toxic metals or hazardous waste. The whirling wheels are then connected to generators to release the power when it's needed. When flywheels are combined in banks of ten or more connected to power grids, they offset fluctuations between supply and demand and run more smoothly. Cheers for rotational inertia!

The greater the rotational inertia of an object, the greater the difficulty in changing its rotational state. This fact is employed by a circus tightrope walker who carries a long pole to aid balance. Much of the mass of the pole is far from the axis of rotation, its midpoint. The pole, therefore, has considerable rotational inertia. If the tightrope walker starts to topple over, a tight grip on the pole rotates

[3] Often called *moment of inertia.*

the pole. But the rotational inertia of the pole resists, giving the tightrope walker time to readjust his or her balance. The longer the pole, the better for balance. And better still if massive objects are attached to the ends. But a tightrope walker with no pole can at least extend his or her arms full length to increase the body's rotational inertia.

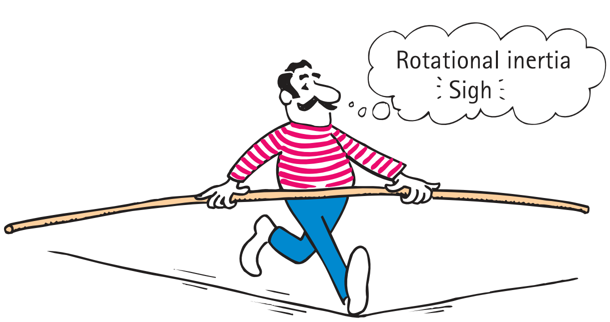



**FIGURE 8.10**
The tendency of the pole to resist rotation aids the tightrope walker.

The rotational inertia of the pole, or of any object, depends on the axis about which it rotates.[4] Compare the different rotations of a pencil in Figure 8.11. Consider three axes—one about its central core parallel to the length of the pencil, where the lead is; the second about the perpendicular midpoint axis; and the third about an axis perpendicular to one end. Rotational inertia is very small about the first position (about the lead); it's easy to rotate the pencil between your fingertips because almost all the mass is very close to the axis. About the second axis, like that used by the tightrope walker in the preceding illustration, rotational inertia is greater. About the third axis, at the end of the pencil so that it swings like a pendulum, rotational inertia is greater still.

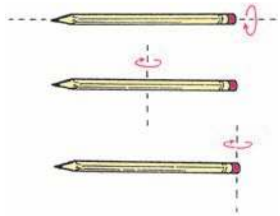


**FIGURE 8.11**
The pencil has different rotational inertias about different rotational axes.

A long baseball bat held near its narrow end has more rotational inertia than a short bat. Once the long bat is swinging, it has a greater tendency to keep swinging, but it is harder to bring it up to speed. A short bat, with less rotational inertia, is easier to swing—which explains why baseball players sometimes "choke up" on a bat by grasping it closer to the more massive end. Similarly, when you run with your legs bent, you reduce their rotational inertia so that you can rotate them back and forth more quickly. A long-legged person tends to walk with slower strides than a person with short legs. The different strides of creatures with different leg lengths are especially evident in animals. Giraffes, horses, and ostriches run with a slower gait than dachshunds, mice, and bugs.

**FIGURE 8.12**
You bend your legs when you run to reduce rotational inertia.

Because of rotational inertia, a solid cylinder starting from rest will roll down an incline faster than a hoop. Both rotate about a central axis, and the shape that has most of its mass far from its axis is the hoop. So, for its weight, a hoop has greater rotational inertia and is harder to start rolling. Any solid cylinder will outroll any

**VIDEO: Rotational Inertia Using Weighted Pipes**

**FIGURE 8.13**
Short legs have less rotational inertia than long legs. An animal with short legs has a quicker stride than people with long legs, just as a baseball batter can swing a short bat more quickly than a long one.

[4]When the mass of an object is concentrated at the radius $r$ from the axis of rotation (as for a simple pendulum bob or a thin ring), rotational inertia $I$ is equal to the mass $m$ multiplied by the square of the radial distance. For this special case, $I = mr^2$.

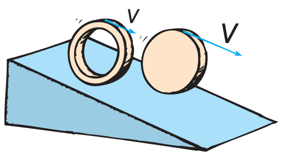



**FIGURE 8.14**
A solid cylinder rolls down an incline faster than a hoop, whether or not they have the same mass or outer diameter. A hoop has greater rotational inertia relative to its mass than a cylinder does.

hoop on the same incline. This doesn't seem plausible at first, but remember that any two objects, regardless of mass, will fall together when dropped. They will also slide together when released on an inclined plane. When rotation is introduced, the object with the greater rotational inertia *relative to its own mass* has the greater resistance to a change in its motion. Hence, any solid cylinder will roll down any incline with greater acceleration than any hollow cylinder, regardless of mass or radius. A hollow cylinder has more "laziness per mass" than a solid cylinder. Try it and see!

Figure 8.15 compares the rotational inertias of objects with various shapes and axes. It is not important for you to learn the equations shown in the figure, but can you see how they vary with the shape and axis?

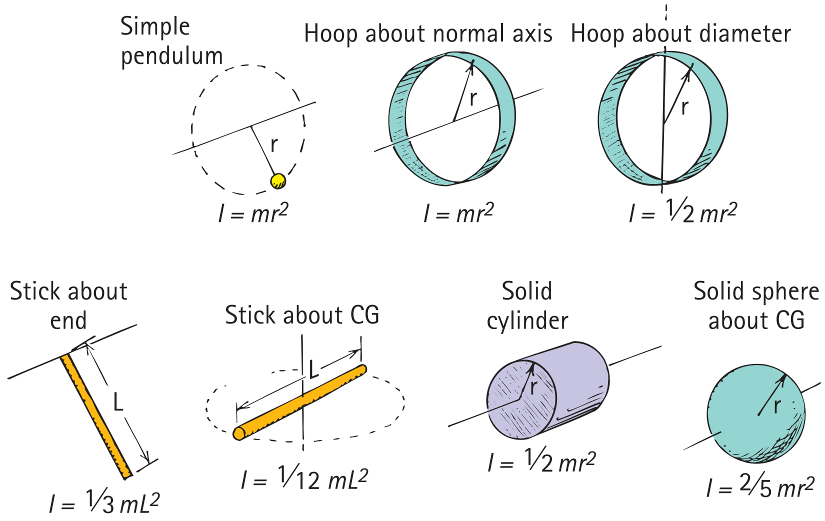



**FIGURE 8.15**
Rotational inertias of various objects, each of mass $m$, about indicated axes.

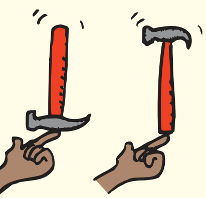
**VIDEO: Rotational Inertia Using a Hammer**

**FIGURE 8.16**
Sanjay Rebello adds to Figure 8.15 with the rotational inertia $I = \frac{2}{3}mr^2$ for a thin-walled hollow sphere.

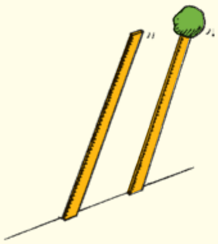
**VIDEO: Rotational Inertia with a Weighted Rod**

Note how rotational inertia very much depends on the location of the axis of rotation. A stick rotated about one end, for example, has four times greater rotational inertia than a stick rotated about its center.

**CHECK POINT**



1. **Consider balancing a hammer upright on the tip of your finger. The head is likely heavier than the handle. Is it easier to balance with the end of the handle on your fingertip, with the head at the top, or the other way around?**



2. **Consider a pair of metersticks standing nearly upright against a wall. If you release them, they'll rotate to the floor at the same time. But what if one meterstick has a massive hunk of clay stuck to its top end? Will it rotate to the floor in a longer or shorter time?**
3. **Just for fun, and since we're discussing round things, why are manhole covers circular in shape?**

**CHECK YOUR ANSWERS**

1. Stand the hammer with the handle on your fingertip and the head at the top. Why? Because it will have greater rotational inertia this way and be more resistant to a rotational change. (Try this yourself by balancing a spoon both ways on your fingertip.) Acrobats who balance a long vertical pole have an easier task when their friends are at the top of the pole. A pole empty at the top has less rotational inertia and is more difficult to balance!

2. Try it and see! (If you don't have clay, fashion something equivalent.)
3. Not so fast on this one. Give it some thought. If you don't come up with an answer, *then* look at the end of the chapter for an answer.

## 8.3 Torque

Hold the end of a meterstick horizontally with your hand. Dangle a weight from it near your hand and you can feel the stick twist. Now slide the weight farther from your hand and you can feel greater twist, even though the weight is the same. The force acting on your hand is the same. What's different is the *torque*.

Torque (rhymes with *dork*) is the rotational counterpart of force. Forces tend to change the motion of things; torques tend to twist or change the rotational motion of things. If you want to make a stationary object move or a moving object change its velocity, apply force. If you want to make a stationary object rotate or a rotating object change its rotational velocity, apply torque.

Just as rotational inertia differs from regular inertia, torque differs from force. Both rotational inertia and torque involve distance from the axis of rotation. In the case of torque, this distance, which provides leverage, is called the *lever arm*. It is the shortest distance between the applied force and the rotational axis. We define **torque** as the product of this lever arm and the force that tends to produce rotation:

$$\text{Torque} = \text{lever arm} \times \text{force}$$

Torques are intuitively familiar to youngsters playing on a seesaw. Kids can balance a seesaw even when their weights are unequal. Weight alone doesn't produce rotation—torque does also—and children soon learn that the distance they sit from the pivot point is every bit as important as their weight. The torque produced by the boy on the right in Figure 8.18 tends to produce clockwise rotation, while torque produced by the girl on the left tends to produce counterclockwise rotation. If the torques are equal, which makes the net torque zero, no rotation is produced.

Suppose that the seesaw is arranged so that the half-as-heavy girl is suspended from a 4-m rope hanging from her end of the seesaw (Figure 8.19). She is now 5 m from the fulcrum, and the seesaw is still balanced. We see that the lever-arm distance is still 3 m and not 5 m. The lever arm about any axis of rotation is the perpendicular distance from the axis to the line along which the force acts. This will always be the shortest distance between the axis of rotation and the line along which the force acts.

This is why the stubborn bolt shown in Figure 8.20 is more likely to turn when the applied force is perpendicular to the handle, rather than at an oblique angle as shown in the first figure. In the first figure, the lever arm is shown by the dashed line and it is shorter than the length of the wrench handle. In the second figure, the lever arm is the same length as the wrench handle. In the third figure, the lever arm is extended with a piece of pipe to provide more leverage and a greater torque.

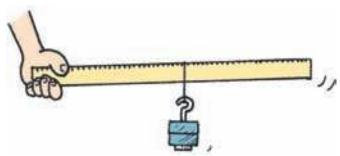


**FIGURE 8.17**
Move the weight farther from your hand and feel the difference between force and torque.

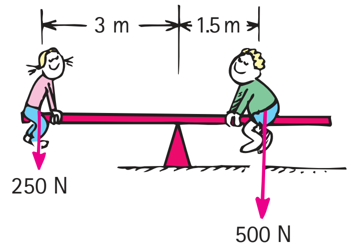



**FIGURE 8.18**
INTERACTIVE FIGURE MP
No rotation is produced when the torques balance each other.

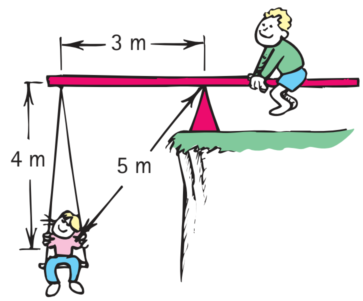



**FIGURE 8.19**
The lever arm is still 3 m.

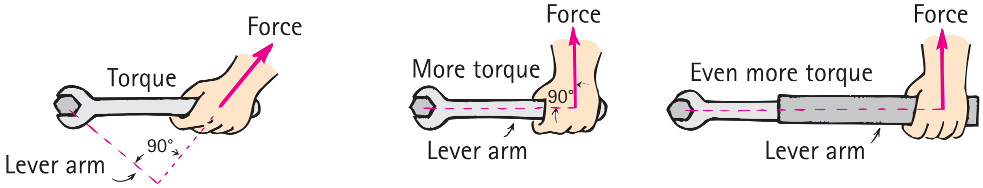



**FIGURE 8.20**
INTERACTIVE FIGURE MP
Although the magnitudes of the forces are the same in each case, the torques are different.

**SCREENCAST: Torque**

If and when all clocks are digital, will the terms *clockwise* and *counterclockwise* have meaning?

**VIDEO:** Difference Between Torque and Weight

**VIDEO:** Why a Ball Rolls Down a Hill

**SCREENCAST:** Balanced Torques

**SCREENCAST:** Torques on a Plank

**SCREENCAST:** Skateboard Torques

Recall the equilibrium rule in Chapter 2—that the sum of the forces acting on a body or any system must equal zero for mechanical equilibrium; that is, $\Sigma F = 0$. We now see an additional condition. The *net torque* on a body or on a system must also be zero for mechanical equilibrium: $\Sigma\tau = 0$, where $\tau$ (the Greek letter tau) stands for torque. Anything that is in mechanical equilibrium doesn't accelerate linearly or rotationally.

**CHECK POINT**

1. **If a pipe effectively extends a wrench handle to three times its length, by how much will the torque increase for the same applied force?**
2. **Consider the balanced seesaw in Figure 8.18. Suppose the girl on the left suddenly gains 50 N, such as by being handed a bag of apples. Where should she sit in order to be in balance, assuming the heavier boy does not move?**

**CHECK YOUR ANSWERS**

1. Three times more leverage for the same force produces three times more torque. (Caution: This method of increasing torque sometimes results in shearing off the bolt!)
2. She should sit ½ m closer to the center. Then her lever arm is 2.5 m. This checks: 300 N × 2.5 m = 500 N × 1.5 m.

## 8.4 Center of Mass and Center of Gravity

Toss a baseball into the air, and it will follow a smooth parabolic path. Toss a baseball bat spinning into the air, and its path is not smooth; its motion is wobbly, and it seems to wobble all over the place. But, in fact, it wobbles about a very special place, a point called the **center of mass (CM)** .

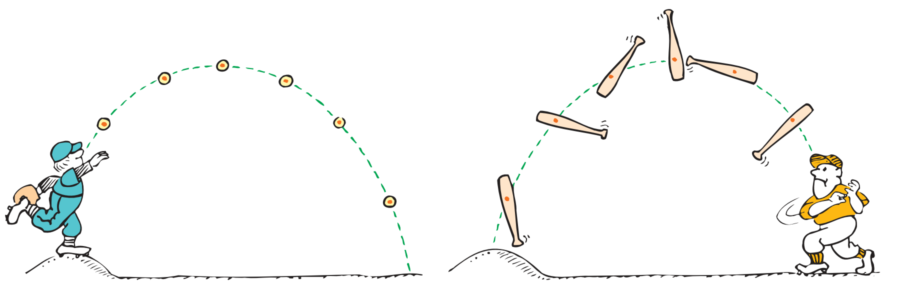


**FIGURE 8.21**
The center of mass of the baseball and that of the bat follow parabolic trajectories.

For a given body, the center of mass is the average position of all the mass that makes up the object. For example, a symmetrical object, such as a ball, has its center of mass at its geometrical center; by contrast, an irregularly shaped body, such as a baseball bat, has more of its mass toward one end. The center of mass of a baseball bat, therefore, is toward the thicker end. A solid cone has its center of mass exactly one-fourth of the way up from its base.

**Center of gravity (CG)** is a term popularly used to express center of mass. The center of gravity is simply the average position of the weight distribution. Since weight and mass are proportional, center of gravity and center

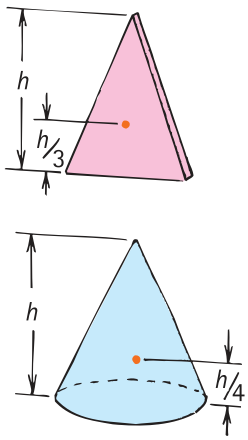



**FIGURE 8.22**
The center of mass for each object is shown by the red dot.

of mass refer to the same point of an object.[5] The physicist prefers to use the term *center of mass* because an object has a center of mass whether or not it is under the influence of gravity. However, we shall use either term for this concept, and we shall favor the term *center of gravity* when weight is part of the picture.

The multiple-flash photograph in Figure 8.23 shows a top view of a wrench sliding across a smooth horizontal surface. Note that its center of mass, indicated by the white dot, follows a straight-line path, while other parts of the wrench wobble as they move across the surface. Since there is no external force acting on the wrench, its center of mass moves equal distances in equal time intervals. The motion of the spinning wrench is the combination of the straight-line motion of its center of mass and the rotational motion about its center of mass.

**FIGURE 8.23**
The center of mass of the spinning wrench follows a straight-line path.

If the wrench were instead tossed into the air, no matter how it rotates, its center of mass (or center of gravity) would follow a smooth parabolic arc. The same is true for an exploding cannonball (Figure 8.24). The internal forces that act in the explosion do not change the center of gravity of the projectile. Interestingly enough, if air resistance can be ignored, the center of gravity of the dispersed fragments as they fly through the air will be in the same location as the center of gravity of the cannonball if the explosion hadn't occurred.

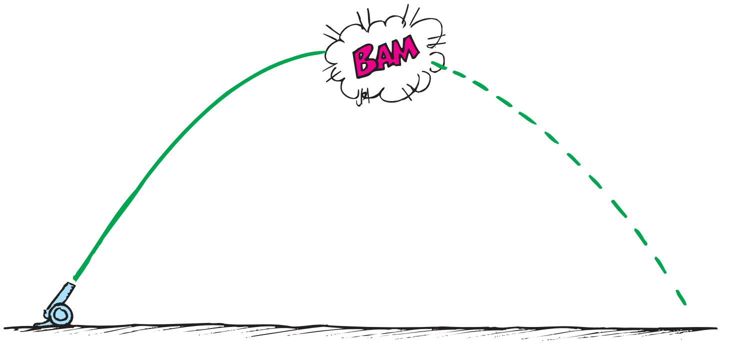



**FIGURE 8.24**
The center of mass of the cannonball and its fragments moves along the same path before and after the explosion.

**CHECK POINT**

1. **Where is the CG of a donut?**
2. **Can an object have more than one CG?**

**CHECK YOUR ANSWERS**

1. In the center of the hole!
2. No. A rigid object has one CG. If an object is nonrigid, such as a piece of clay or putty, and is distorted into different shapes, then its CG may change as its shape changes. Even then, it has one CG for any given shape.

[5]For almost all objects on and near Earth's surface, these terms are interchangeable. A small difference between center of gravity and center of mass can occur for an object large enough so that gravity varies from one part to another. For example, the center of gravity of the Empire State Building is about 1 millimeter below its center of mass. This is due to the lower stories being pulled a little more strongly by Earth's gravity than the upper stories. For everyday objects (including tall buildings), we can use the terms *center of gravity* and *center of mass* interchangeably.

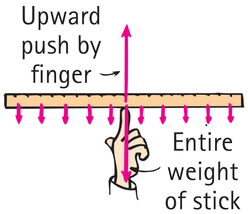



**FIGURE 8.25**
The weight of the entire stick behaves as if it were concentrated at the stick's center.

## Locating the Center of Gravity

The center of gravity of a uniform object, such as a meterstick, is at its midpoint because the stick acts as if its entire weight were concentrated there. If you support that single point, you support the entire stick. Balancing an object provides a simple method of locating its center of gravity. In Figure 8.25, the many small arrows represent the pull of gravity all along the meterstick. All of the arrows can be combined into a resultant force acting through the center of gravity. The entire weight of the stick may be thought of as acting at this single point. Hence, we can balance the stick by applying a single upward force in a direction that passes through this point.

The center of gravity of any freely suspended object lies directly beneath (or at) the point of suspension (Figure 8.26). If a vertical line is drawn through the point of suspension, the center of gravity lies somewhere along that line. To determine exactly where it lies along the line, we have only to suspend the object from some other point and draw a second vertical line through that point of suspension. The center of gravity lies where the two lines intersect.

The center of mass of an object may be a point where no mass exists. For example, the center of mass of a ring or a hollow sphere is at the geometrical center where no matter exists. Similarly, the center of mass of a boomerang is outside the physical structure, not within the material of the boomerang (Figure 8.28).

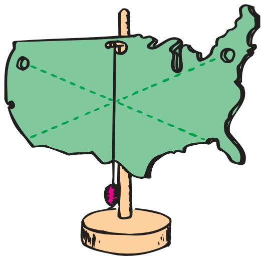


**FIGURE 8.26**
Finding the center of gravity of an irregularly shaped object.

**VIDEO: Learning the Center of Gravity**

**FIGURE 8.27**
The athlete executes a "Fosbury flop" to clear the bar while his center of gravity passes beneath the bar.

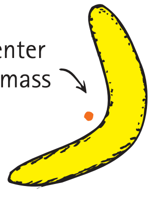
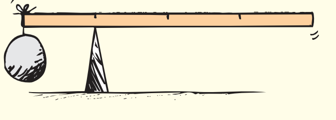



**FIGURE 8.28**
The center of mass can be outside the mass of a body.

**CHECK POINT**

1. **Where is the center of mass of Earth's crust?**
2. **A uniform meterstick supported at the 25-cm mark balances when a 1-kg rock is suspended at the 0-cm end. What is the mass of the meterstick?**



**CHECK YOUR ANSWERS**

1. Like a giant basketball, Earth's crust is a spherical shell with its center of mass at Earth's center.
2. The mass of the meterstick is 1 kg. Why? The system is in equilibrium, so any torques must be balanced: The torque produced by the weight of the rock is balanced by the equal but oppositely directed torque produced by the weight of the stick *applied at its CG, the 50-cm mark.* The support force at the 25-cm mark is applied midway between the rock and the stick's CG, so the lever arms about the support point are equal (25 cm). This means that the weights (and hence the masses) of the rock and stick are also equal. (Note that we don't have to go through the laborious task of considering the fractional parts of the stick's weight on either side of the fulcrum because the CG of the whole stick really is at one point—the 50-cm mark!) Interestingly, the CG of the rock + stick system is at the 25-cm mark—directly above the fulcrum.

## Stability

The location of the center of gravity is important for stability, as Figures 8.29–8.36 show. If we draw a line straight down from the center of gravity of an object of any shape and it falls inside the base of the object, then the object is in stable **equilibrium**; it will balance. If the line falls outside the base, the object is unstable. Why doesn't the famous Leaning Tower of Pisa topple over? As we can see in Figure 8.29, a line from the center of gravity of the tower to the level of the ground falls inside its base, so the Leaning Tower has stood for several centuries. If the tower leaned far enough so that the center of gravity extended beyond the base, then an unbalanced torque would topple the tower.

When you stand erect (or lie flat), your center of gravity is within your body. Why is the center of gravity lower in an average woman than it is in an average man of the same height? Is your center of gravity always at the same point in your body? Is it always inside your body? What happens to it when you bend over?

If you are fairly flexible, you can bend over and touch your toes without bending your knees. Ordinarily, when you bend over and touch your toes, you extend your lower extremity, as shown on the left side of Figure 8.31, so that your center of gravity is above a base of support, your feet. If you attempt to touch your toes when standing against a wall, however, you cannot counterbalance yourself, and your center of gravity soon protrudes beyond your feet, as shown on the right side of Figure 8.31.

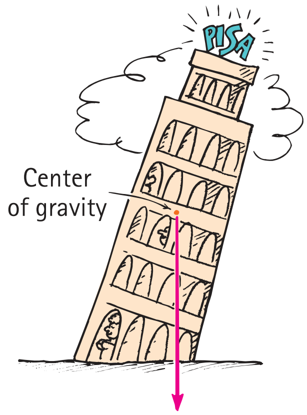



**FIGURE 8.29**
The center of gravity of the Leaning Tower of Pisa lies above its base of support, so the tower is in stable equilibrium.

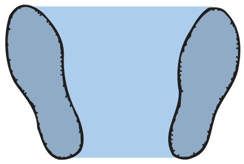


**FIGURE 8.30**
When you stand, your center of gravity is somewhere above the base area bounded by your feet. Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

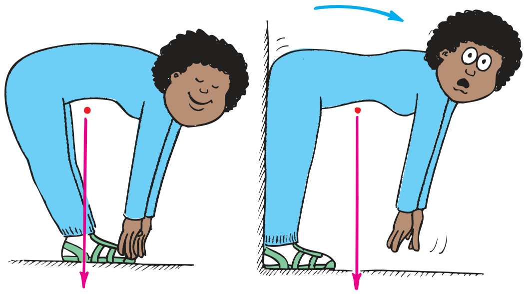


**FIGURE 8.31**
You can lean over and touch your toes without falling over only if your center of gravity is above the area bounded by your feet.

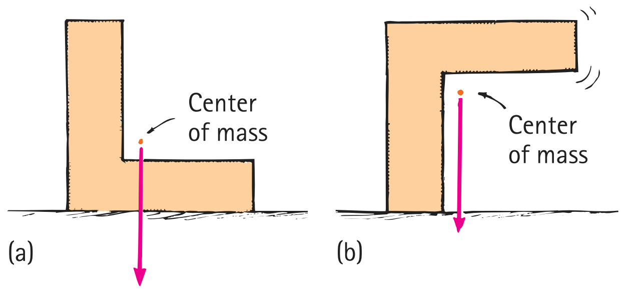
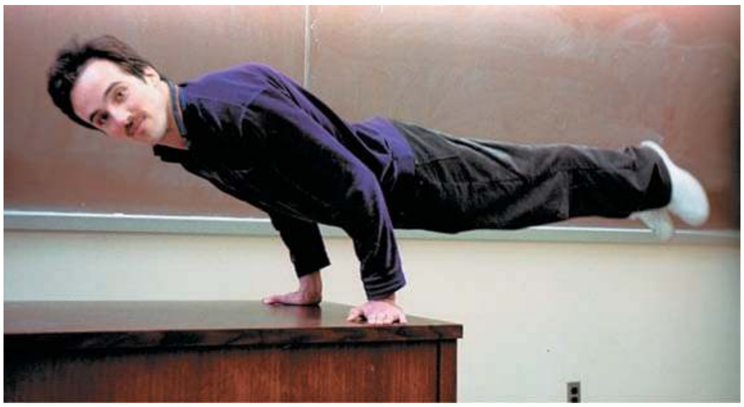



**FIGURE 8.32**
The center of mass of the L-shaped object is located where no mass exists. (a) The center of mass is above the base of support, so the object is stable. (b) The center of mass is not above the base of support, so the object is unstable and will topple over.



**VIDEO: Toppling**

**FIGURE 8.33**
Where is Alexei's center of gravity relative to his hands?

You rotate because of an unbalanced torque. This is evident in the two L-shaped objects shown in Figure 8.34. Both are unstable and will topple unless fastened to the level surface. It is easy to see that even if both shapes have the same weight, the one on the right is more unstable. This is because of the longer lever-arm distance and, hence, the greater torque.

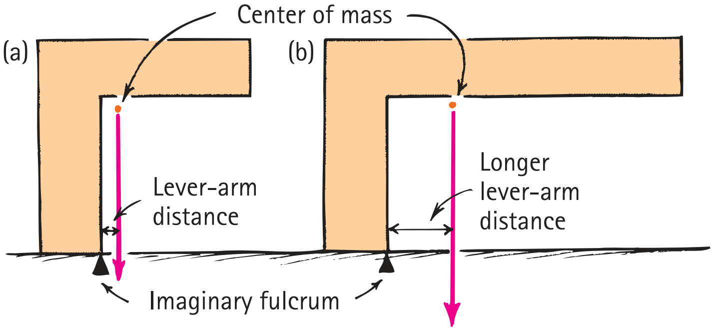



**FIGURE 8.34**
The greater torque acts on the object in (b) for two reasons. What are they?

**FIGURE 8.35**
Gyroscopes and computer-assisted motors make continuous adjustments in the self-balancing electric vehicles to keep the combined CGs of Mark, Tenny, and the vehicles above the wheelbase.

Try balancing the pole end of a broom upright on the palm of your hand. The support base is quite small and relatively far beneath the center of gravity, so it's difficult to maintain balance for very long. After some practice, you can balance the broom if you learn to make slight movements of your hand to respond exactly to variations in balance. You learn to avoid underresponding or overresponding to the slightest variations in balance. The intriguing Segway Human Transporter (Figure 8.35) does much the same. Variations in balance are quickly sensed by gyroscopes, and an internal computer regulates a motor to keep the vehicle upright. The computer regulates corrective adjustments of the wheel speed in a way quite similar to the way in which your brain coordinates your adjustive action when balancing a long pole on the palm of your hand. Both feats are truly amazing.

To reduce the likelihood of tipping, it is usually advisable to design objects with a wide base and low center of gravity. The wider the base, the higher the center of gravity must be raised before the object tips over.

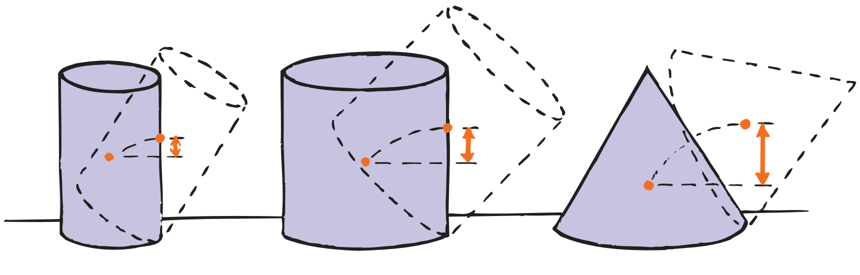
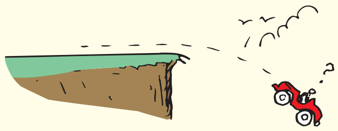


**FIGURE 8.36**
Stability is determined by the vertical distance that the center of gravity is raised in tipping. An object with a wide base and a low center of gravity is more stable.

**CHECK POINT**

1. **Why is it dangerous to slide open the top drawers of a fully loaded file cabinet that is not secured to the floor?**
2. **When a car drives off a cliff, why does it rotate forward as it falls?**



**CHECK YOUR ANSWERS**

1. The file cabinet is in danger of tipping because the CG may extend beyond the support base. If it does, then torque due to gravity tips the cabinet.
2. When all the wheels are on the ground, the car's CG is above a support base and no tipping occurs. But when the car drives off a cliff, the front wheels are first to leave the ground and the car is supported only by the rear wheels. The CG then extends beyond the support base, and rotation occurs. Interestingly, the speed of the car is related to how much time the CG is not supported and, hence, the amount the car rotates while it falls.

## 8.5 Centripetal Force

Any force directed toward a fixed center is called a **centripetal force**. *Centripetal* means "center-seeking" or "toward the center." When we whirl a tin can on the end of a string, we find that we must keep pulling on the string—exerting a centripetal force (Figure 8.37). The string transmits the centripetal force, which pulls the can into a circular path. Gravitational and electrical forces can produce centripetal forces. The Moon, for example, is held in an almost circular orbit by gravitational force directed toward the center of Earth. The orbiting electrons in atoms experience an electrical force toward the central nuclei. Anything that moves in a circular path does so because it's acted upon by a centripetal force.

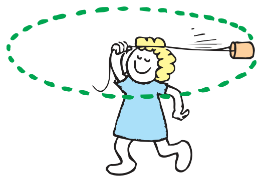


**FIGURE 8.37**
The force exerted on the whirling can is toward the center.

Centripetal force depends on the mass $m$, tangential speed $v$, and radius of curvature $r$ of the circularly moving object. In lab, you'll likely use the exact relationship

$$F = mv^2/r$$

Notice that speed is squared, so twice the speed needs four times the force. The inverse relationship with the radius of curvature tells us that half the radial distance requires twice the force.

Centripetal force is not a basic force of nature; it is simply the name given to any force, whether string tension, gravitational, electrical, or whatever, that is directed toward a fixed center. If the motion is circular and executed at constant speed, this force is at right angles to the path of the moving object.

When an automobile rounds a corner, the friction between the tires and the road provides the centripetal force that holds the car in a curved path (Figure 8.39).

**VIDEO: Centripetal Force**

**SCREENCAST: Centripetal Force**

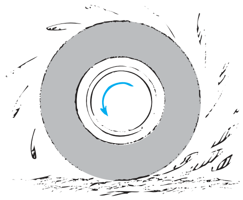


**FIGURE 8.38**
The centripetal force (adhesion of mud on the spinning tire) is not great enough to hold the mud on the tire, so it flies off in straight-line tangents.

If this friction is insufficient (due to oil or gravel on the pavement, for example), the tires slide sideways and the car fails to make the curve; the car tends to skid tangentially off the road.

Centripetal force plays the main role in the operation of a centrifuge. A familiar example is the spinning tub in an automatic washing machine (Figure 8.41). In its spin cycle, the tub rotates at high speed and produces a centripetal force on the wet clothes, which are forced into a circular path by the inner wall of the tub. The tub exerts great force on the clothes, but the holes in the tub prevent the exertion of the same force on the water in the clothes. The water escapes tangentially out the holes. Strictly speaking, the clothes are forced away from the water; the water is not forced away from the clothes. Think about that.

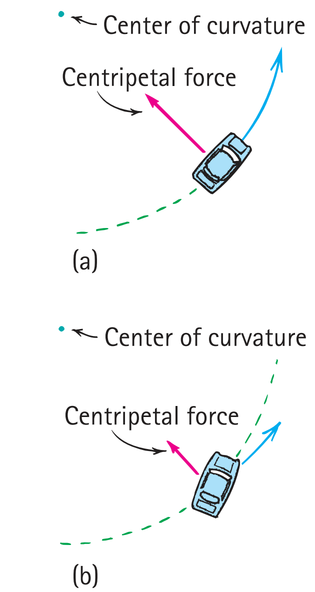



**FIGURE 8.39**
(a) When a car goes around a curve, there must be a force pushing the car toward the center of the curve. (b) A car skids on a curve when the centripetal force (friction of road on tires) is not great enough.

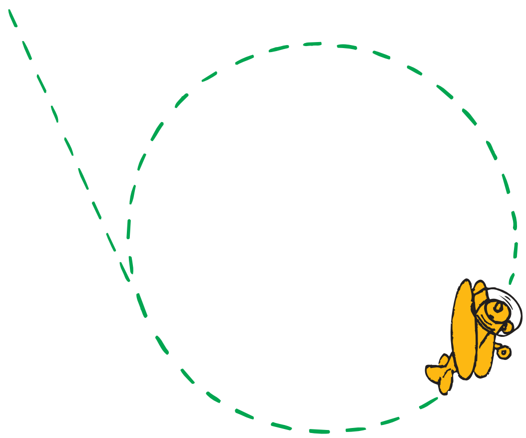


**FIGURE 8.40**
Large centripetal forces on the wings of an aircraft enable it to fly in circular loops. The acceleration away from the straight-line path the aircraft would follow in the absence of centripetal force is often several times greater than the acceleration due to gravity, *g*. For example, if the centripetal acceleration is 50 m/s$^2$ (5 times as great as 10 m/s$^2$), we say that the aircraft is undergoing 5 *g*'s. For a pilot, the number of *g*'s is defined by the force on the seat of his or her pants. So at the bottom of the loop where the pilot's weight lines up with the centripetal force, the pilot experiences 6 *g*'s. Typical fighter aircraft are designed to withstand accelerations up to 8 or 9 *g*'s. The pilot as well as the aircraft must withstand this amount of acceleration. Pilots of fighter aircraft wear pressure suits to prevent their blood from flowing away from the head toward the legs, which could cause a blackout.

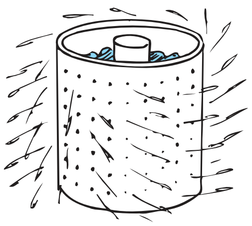


**FIGURE 8.41**
The clothes are forced into a circular path, but the water is not.

## PRACTICING PHYSICS: WATER-BUCKET SWING

Half fill a bucket of water and swing it in a vertical circle, as Marshall Ellenstein demonstrates. The bucket and water accelerate toward the center of your swing. If you swing the bucket fast enough, the water won't fall out at the top. Interestingly, although the water won't fall *out,* it still falls. The trick is to swing the bucket fast enough so that the bucket falls as fast as the water inside the bucket falls. Can you see that because the bucket is circling, the water moves tangentially—and stays in the bucket? In Chapter 10, we'll learn that an orbiting space shuttle similarly falls while in orbit. The trick is to impart sufficient tangential velocity to the shuttle so that it falls around the curved Earth rather than into it.

## 8.6 Centrifugal Force

Although centripetal force is center directed, an occupant inside a revolving or rotating system seems to experience an outward force. This apparent outward force is called **centrifugal force**. *Centrifugal* means "center-fleeing" or "away from the center." In the case of a can whirling at the end of a string, it is a common misconception that a centrifugal force pulls outward on the can. If the string holding the whirling can breaks (Figure 8.42), the can doesn't move radially outward, but goes off in a tangent straight-line path—because *no* force acts on it. We illustrate this further with another example.

Suppose you are a passenger in a car that stops suddenly. You pitch forward toward the dashboard. When this occurs, you don't say that something forced you forward. In accord with the law of inertia, you pitched forward because of the absence of a force, which seat belts would have provided. Similarly, if you are in a car that rounds a sharp corner to the left, you tend to pitch outward to the right—not because of some outward or centrifugal force, but because there is no centripetal force holding you in circular motion (again, the purpose of seat belts). The idea that a centrifugal force bangs you against the car door is a misconception. (Sure, you push out against the door, but only because the door pushes in on you—Newton's third law.)

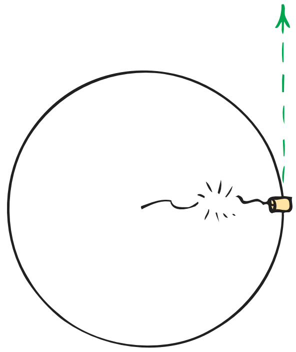


**FIGURE 8.42**
When the string breaks, the whirling can moves in a straight line, tangent to—not outward from—the center of its circular path.

**SCREENCAST: Centrifugal Force**

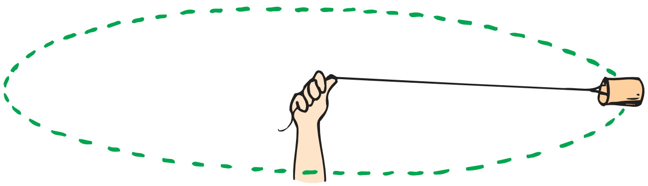


**FIGURE 8.43**
The only force that is exerted on the whirling can (if we ignore gravity) is directed *toward* the center of circular motion. This is a centripetal force. No *outward* force acts on the can.

Likewise, when you swing a can in a circular path, no force pulls the can outward—the only force on the can is the string pulling it inward. There is no outward force on the can. Now suppose there is a ladybug inside the whirling can (Figure 8.44). The can presses against the bug's feet and provides the centripetal force that holds it in a circular path. From our outside stationary frame of reference, we see no centrifugal force exerted on the ladybug, just as no centrifugal force banged us against the car door. The centrifugal force effect is caused not by a real force, but by inertia—the tendency of the moving object to follow a straight-line path. But try telling that to the ladybug!

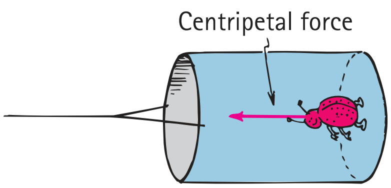



**FIGURE 8.44**
The can provides the centripetal force necessary to hold the ladybug in a circular path.

### Centrifugal Force in a Rotating Reference Frame

If we stand at rest and watch somebody whirling a can overhead in a horizontal circle, we see that the force on the can is centripetal, just as it is on a ladybug inside the can. The bottom of the can exerts a force on the ladybug's feet. If we ignore gravity, no other force acts on the ladybug. But, as viewed from inside the frame of reference of the revolving can, things appear very different.[6]

*In the rotating frame of reference of the ladybug,* in addition to the force of the can on the ladybug's feet, there is an apparent centrifugal force that is exerted on the ladybug. Centrifugal force *in a rotating reference frame* is a force in its own right, as real as the pull of gravity. However, there is a fundamental difference. Gravitational force is an interaction between one mass and another. The gravity we experience

[6]A frame of reference in which a free body exhibits no acceleration is called an *inertial* frame of reference. Newton's laws are seen to hold exactly in an inertial frame. A rotating frame of reference, in contrast, is an accelerating frame of reference. Newton's laws are not valid in an accelerating frame of reference.

is our interaction with Earth. But for centrifugal force in the rotating frame, no such agent exists—there is no interaction counterpart. Centrifugal force *feels* like gravity, but there's no *something* pulling. Nothing produces the pulling; the force we feel is instead a result of rotation. No pulling counterpart exists. For this reason, physicists call it an "inertial" force (or sometimes a *fictitious* force)—an *apparent* force—and not a real force like gravity, electromagnetic forces, and nuclear forces. Nevertheless, to observers who are in a rotating system, centrifugal force feels like, and is interpreted to be, a very real force. Just as gravity is ever present at Earth's surface, centrifugal force is ever present in a rotating system.

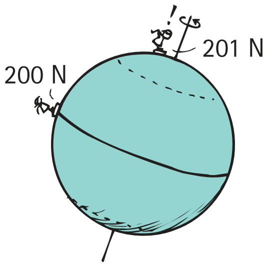
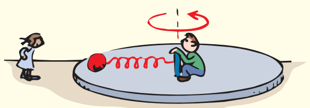



**FIGURE 8.45**
In the frame of reference of the spinning Earth, we experience a centrifugal force that slightly decreases our weight. Like an outside horse on a merry-go-round, we have the greatest tangential speed farthest from Earth's axis, at the equator. Centrifugal force is therefore greatest for us when we are at the equator and zero for us at the poles, where we have no tangential speed. So, strictly speaking, if you want to lose weight, walk toward the equator!

**CHECK POINT**

**A heavy iron ball is attached by a spring to a rotating platform, as shown in the sketch. Two observers, one in the rotating frame and one on the ground at rest, observe its motion. Which observer sees the ball being pulled outward, stretching the spring? Which sees the spring pulling the ball into circular motion?**



**CHECK YOUR ANSWERS**

The observer in the reference frame of the rotating platform states that a centrifugal force pulls radially outward on the ball, which stretches the spring. The observer in the rest frame states that a centripetal force supplied by the stretched spring pulls the ball into circular motion. Only the observer in the rest frame can identify an action–reaction pair of forces; where action is spring on ball, reaction is ball on spring. The rotating observer can't identify a reaction counterpart to the centrifugal force because there isn't any!

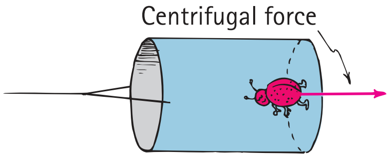



**FIGURE 8.46**
From the reference frame of the ladybug inside the whirling can, it is being held to the bottom of the can by a force that is directed away from the center of circular motion. The ladybug calls this outward force a *centrifugal* force, which is as real to it as gravity.

## Simulated Gravity

Consider a colony of ladybugs living inside a bicycle tire—a balloon tire with plenty of room inside. If we toss the bicycle wheel through the air or drop it from an airplane high in the sky, the ladybugs will be in a weightless condition. They will float freely while the wheel is in free fall. Now spin the wheel. The ladybugs will feel themselves pressed to the outer part of the tire's inner surface. If the wheel is spun at just the right speed, the ladybugs will experience *simulated gravity* that will feel like the gravity they are accustomed to. Gravity is simulated by centrifugal force. The "down" direction to the ladybugs will be what we would call radially outward, away from the center of the wheel.

Humans today live on the outer surface of this spherical planet and are held here by gravity. The planet has been the cradle of humankind. But we will not stay

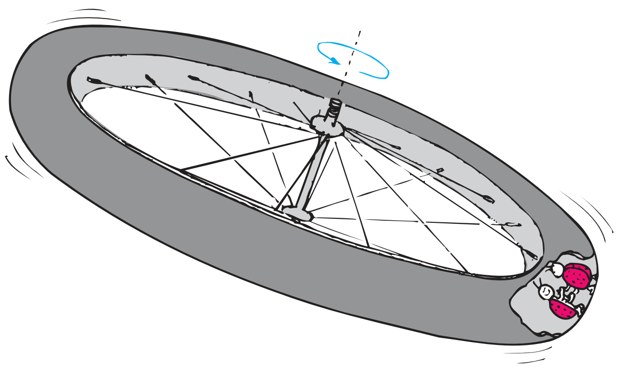


**FIGURE 8.47**
If the spinning wheel freely falls, the ladybugs inside will experience a centrifugal force that feels like gravity when the wheel spins at the appropriate rate. To the occupants, the direction "up" is toward the center of the wheel and "down" is radially outward.

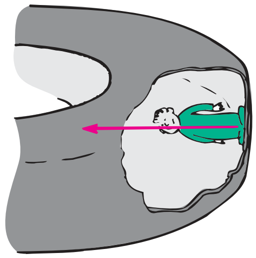


**FIGURE 8.48**
The interaction between the man and the floor, as seen from a stationary frame of reference outside the rotating system. The floor presses against the man (action), and the man presses back on the floor (reaction). The only force exerted on the man is by the floor. It is directed toward the center and is a centripetal force.

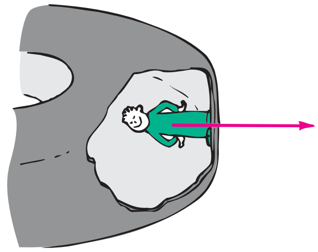


**FIGURE 8.49**
As seen from inside the rotating system, in addition to the man–floor interaction, there is a centrifugal force exerted on the man at his center of mass. It seems as real as gravity. Yet, unlike gravity, it has no reaction counterpart—there is nothing out there that he can pull back on. Centrifugal force is not part of an interaction, but it is a consequence of rotation. It is therefore called an apparent force, or a *fictitious force*.

in the cradle forever. We are becoming a spacefaring people. Occupants of today's space vehicles feel weightless because they experience no *support force.* They're not pressed against a supporting floor by gravity, nor do they experience a centrifugal force due to spinning. Over extended periods of time, this can cause loss of muscle strength and detrimental changes in the body, such as loss of calcium from the bones. But future space travelers need not be subject to weightlessness. Their space habitats will probably spin, like the ladybug's spinning wheel, effectively supplying a support force and nicely simulating gravity.

The significantly smaller International Space Station doesn't rotate. Therefore, its crew members have to adjust to living in a weightless environment. Rotating habitats may follow later, perhaps in huge, lazily rotating structures where occupants will be held to the inner surfaces by centrifugal force. Such a rotating habitat supplies a simulated gravity so that the human body can function normally. Structures of small diameter would have to rotate at high speeds to provide a simulated gravitational acceleration of 1*g.* Sensitive and delicate organs in our inner ears sense rotation. Although there appears to be no difficulty at a single revolution per minute (RPM) or so, many people find it difficult to adjust to speeds greater than 2 or 3 RPM (although some easily adapt to 10 or so RPM). To simulate normal Earth gravity at 1 RPM requires a large structure—one about 2 km in diameter. Centrifugal acceleration is directly proportional to the radial distance from the hub, so a variety of *g* states is possible. If the structure rotates so that inhabitants on the inside of the outer edge experience 1 *g,* then halfway to the axis they would experience 0.5 *g.* At the axis itself they would experience weightlessness (0 *g*). The variety of fractions of *g* possible from the rim of a rotating space habitat holds promise for a different and (at this writing) as yet unexperienced environment. In this still very hypothetical structure, we would be able to perform ballet at 0.5 *g,* diving and acrobatics at 0.2 *g* and lower-*g* states, and three-dimensional soccer (and new sports not yet conceived) in very low *g* states.

A rotating habitat need not be a huge wheel. Gravity could be simulated in a pair of circling pods connected by a long cable.

**VIDEO: Simulated Gravity**

**CHECK POINT**

**If Earth were to spin faster about its axis, your weight would be less. If you were in a rotating space habitat that increased its spin rate, you'd "weigh" more. Explain why greater spin rates produce opposite effects in these cases.**

**CHECK YOUR ANSWER**

You're on the *outside* of the spinning Earth, but you'd be on the *inside* of a spinning space habitat. A greater spin rate on the outside of the Earth tends to throw you *off* a weighing scale, causing it to show a decrease in weight, but *against* a weighing scale *inside* the space habitat to show an increase in weight.

## 8.7 Angular Momentum

**SCREENCAST: Angular Momentum**

Things that rotate, whether a colony in space, a cylinder rolling down an incline, or an acrobat doing a somersault, remain rotating until something stops them. A rotating object has an "inertia of rotation." Recall from Chapter 6 that all moving objects have "inertia of motion" or *momentum*—the product of mass and velocity. This kind of momentum is **linear momentum**. Similarly, the "inertia of rotation" of rotating objects is called **angular momentum**. A planet orbiting the Sun, a rock whirling at the end of a string, and the tiny electrons whirling about atomic nuclei all have angular momentum.

Angular momentum is defined as the product of rotational inertia and rotational velocity:

$$\text{Angular momentum} = \text{rotational inertia} \times \text{rotational velocity}$$

It is the counterpart of linear momentum:

$$\text{Linear momentum} = \text{mass} \times \text{velocity}$$

Like linear momentum, angular momentum is a vector quantity and has direction as well as magnitude. In this book, we won't treat the vector nature of angular momentum (or even of torque, which also is a vector), except to acknowledge the remarkable action of the gyroscope. The rotating bicycle wheel in Figure 8.50 shows what happens when a torque caused by Earth's gravity acts to change the direction of its angular momentum (which is along the wheel's axle). The pull of gravity that normally acts to topple the wheel over and change its rotational axis causes it instead to *precess* about a vertical axis. You must do this yourself while standing on a turntable to fully believe it. You probably won't fully understand it unless you do follow-up study sometime in the future.


**FIGURE 8.50**

Angular momentum keeps the wheel axle nearly horizontal when a torque supplied by Earth's gravity acts on it. Instead of causing the wheel to topple, the torque causes the wheel's axle to turn slowly around the circle of students. This is called *precession*.

For the case of an object that is small compared with the radial distance to its axis of rotation, such as a tin can swinging from a long string or a planet orbiting in a circle around the Sun, the angular momentum can be expressed as the magnitude of its linear momentum, $mv$, multiplied by the radial distance, $r$ (Figure 8.51). In shorthand notation,

$$\text{Angular momentum} = mvr$$

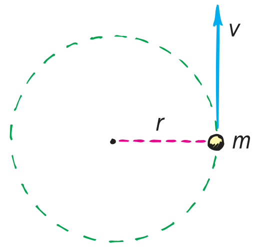



**FIGURE 8.51**
A small object of mass $m$ whirling in a circular path of radius $r$ with speed $v$ has angular momentum $mvr$.

Just as an external net force is required to change the linear momentum of an object, an external net torque is required to change the angular momentum of an object. We can state a rotational version of Newton's first law (the law of inertia):

**An object or system of objects will maintain its angular momentum unless acted upon by an external net torque.**

Our solar system has angular momentum that includes the Sun, the spinning and orbiting planets, and myriad other smaller bodies. The angular momentum of the solar system today will be its angular momentum for eons to come. Only an external torque from outside the solar system can change it. In the absence of such a torque, we say the angular momentum of the solar system is conserved.

## 8.8 Conservation of Angular Momentum

Just as the linear momentum of any system is conserved if no net force acts on the system, angular momentum is conserved if no net torque acts. The **law of conservation of angular momentum** states:

**If no external net torque acts on a rotating system, the angular momentum of that system remains constant.**

**VIDEO: Conservation of Angular Motion Using a Rotating Platform**

This means that, with no external torque, the product of rotational inertia and rotational velocity at one time will be the same as at any other time.

An interesting example illustrating the conservation of angular momentum is shown in Figure 8.52. The man stands on a low-friction turntable with weights extended. His rotational inertia, $I$, with the help of the extended weights, is relatively large in this position. As he slowly turns, his angular momentum is the product of his rotational inertia and rotational velocity, $\omega$. When he pulls the weights inward, the rotational inertia of his body and the weights is considerably reduced. What is the result? His rotational speed increases! This example is best appreciated by the turning person who feels changes in rotational speed that seem to be mysterious. But it's straightforward physics! This procedure is used by a figure skater who starts to whirl with her arms and perhaps a leg extended and then draws her arms and leg in to obtain a greater rotational speed. Whenever a rotating body contracts, its rotational speed increases.

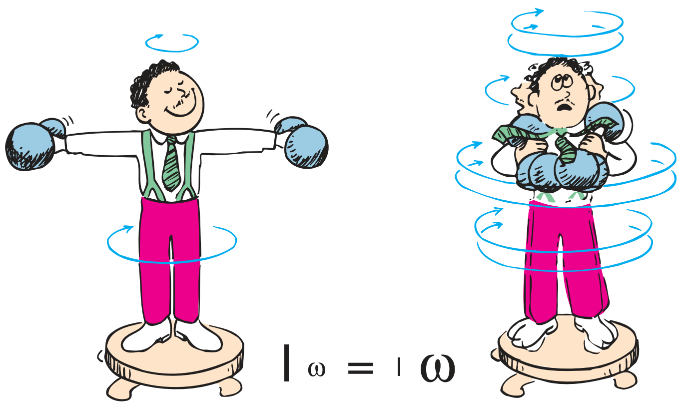



**FIGURE 8.52**
INTERACTIVE FIGURE MP®
Conservation of angular momentum. When the man pulls his arms and the whirling weights inward, he decreases his rotational inertia $I$, and his rotational speed correspondingly increases.

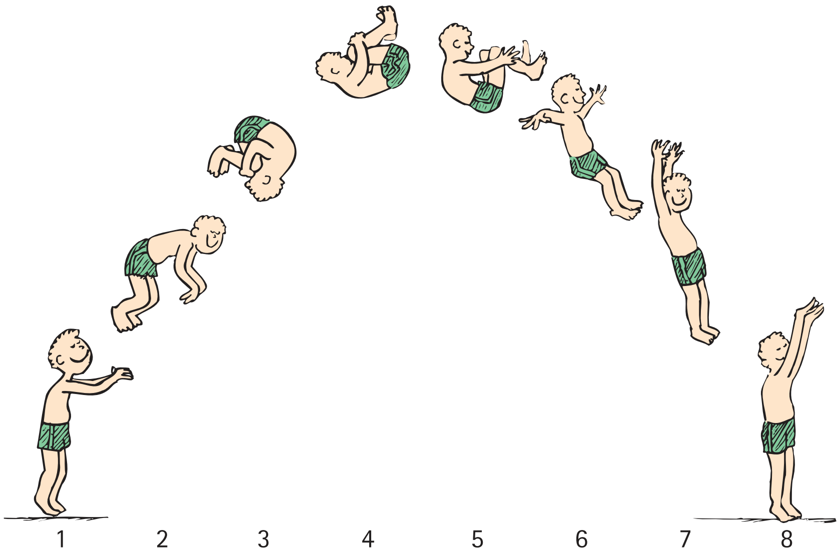



**FIGURE 8.53**
Rotational speed is controlled by variations in the body's rotational inertia as angular momentum is conserved during a forward somersault.

Why do short acrobats have an advantage in tumbling and in other end-over-end rotational motions?

Similarly, when a gymnast is spinning freely in the absence of unbalanced torques on his or her body, angular momentum does not change. However, rotational speed can be changed by simply making variations in rotational inertia. This is done by moving some part of the body toward or away from the axis of rotation.

If a cat is held upside down and dropped a short distance to the floor below, it is able to execute a twist and land upright, even if it has no initial angular momentum. Zero-angular-momentum twists and turns are performed by turning one part of the body against the other. While falling, the cat bends its spine and swings it around to twist in the opposite direction. During this maneuver the total angular momentum remains zero (Figure 8.54). When it is over, the cat is not turning. This maneuver rotates the body through an angle, but it does not create continuing rotation. To do so would violate angular momentum conservation.

Humans can perform similar twists without difficulty, though not as fast as a cat. Astronauts have learned to make zero-angular-momentum rotations as they orient their bodies in preferred directions when floating freely in space.

The law of conservation of angular momentum is seen in the motions of planets and the shapes of galaxies. It is fascinating to note that the conservation of angular momentum tells us that the Moon is getting farther away from Earth. This is because the Earth's daily rotation is slowly decreasing due to the friction of ocean waters on the ocean bottom, just as an automobile's wheels slow down when brakes are applied. This decrease in Earth's angular momentum is accompanied by an equal increase in the angular momentum of the Moon in its orbital motion about Earth, which results in the Moon's increasing distance from Earth and decreasing speed. This increase of distance amounts to one-quarter of a centimeter per rotation. Have you noticed that the Moon is getting farther away from us lately? Well, it is; each time we see another full Moon, it is one-quarter of a centimeter farther away!

Oh yes—before we end this chapter, we'll give an answer to Check Point Question 3 back on page 138. Manhole covers are circular because a circular cover is the only shape that can't fall into the hole. A square cover, for example, can be tilted vertically and turned so it can drop diagonally into the hole. Likewise for every other shape. If you're working in a manhole and some fresh kids are horsing around above, you'll be glad the cover is round!

**FIGURE 8.54**
Time-lapse photo of a falling cat.

For instructor-assigned homework, go to www.masteringphysics.com

## SUMMARY OF TERMS (KNOWLEDGE)

**Tangential speed** The linear speed tangent to a curved path, such as in circular motion.

**Rotational speed** The number of rotations or revolutions per unit of time; often measured in rotations or revolutions per second or per minute. (Scientists usually measure it in radians per second.)

**Rotational inertia** The property of an object to resist any change in its state of rotation: If at rest, the body tends to remain at rest; if rotating, it tends to remain rotating and will continue to do so unless acted upon by an external net torque.

**Torque** The product of force and lever-arm distance, which tends to produce or change rotation:

$$\text{Torque} = \text{lever arm} \times \text{force}$$

**Center of mass (CM)** The average position of the mass of an object. The CM moves as if all the external forces acted at this point.

**Center of gravity (CG)** The average position of weight or the single point associated with an object where the force of gravity can be considered to act.

**Equilibrium** The state of an object in which it is not acted upon by a net force or a net torque.

**Centripetal force** A force directed toward a fixed point, usually the cause of circular motion:

$$F = mv^2/r$$

**Centrifugal force** An outward force apparent in a rotating frame of reference. It is apparent (fictitious) in the sense that it is not part of an interaction but is a result of rotation—with no reaction-force counterpart.

**Linear momentum** The product of the mass of an object and its linear velocity.

**Angular momentum** The product of a body's rotational inertia and rotational velocity about a particular axis. For an object that is small compared with the radial distance, angular momentum can be expressed as the product of mass, speed, and radial distance of rotation.

**Conservation of angular momentum** When no external torque acts on an object or a system of objects, no change of angular momentum can occur. Hence, the angular momentum before an event involving only internal torques or no torques is equal to the angular momentum after the event.

## READING CHECK QUESTIONS (COMPREHENSION)

### 8.1 Circular Motion

1. An object is executing circular motion. What is the direction of instantaneous linear speed?
2. On a rotating turntable, does tangential speed or rotational speed vary with distance from the center?
3. A tapered cup rolled on a flat surface makes a circular path. What does this tell you about the tangential speed of the rim of the wide end of the cup compared with that of the rim of the narrow end?
4. How does the tapered rim of a wheel on a railroad train allow one part of the rim to have a greater tangential speed than another part when it is rolling on a track?

### 8.2 Rotational Inertia

5. What is rotational inertia, and how is it similar to inertia as studied in earlier chapters?
6. Can two objects have the same inertia but a different moment of inertia?
7. As distance increases between most of the mass of an object and its center of rotation, does rotational inertia increase or decrease?
8. Consider three axes of rotation for a pencil: along the lead, at right angles to the lead at the middle, and at right angles to the lead at one end. Rate the rotational inertias about each axis from smallest to largest.
9. Which is easier to get swinging: a baseball bat held at the narrow end or a bat held closer to the massive end (choked up)?
10. Why does bending your legs when running enable you to swing your legs to and fro more rapidly?
11. An object is rolling down an incline, what is the relationship between its acceleration and its rotational inertia?

### 8.3 Torque

12. How is torque similar to force?
13. When does a force produce zero torque?
14. How do clockwise and counterclockwise torques compare when a system is balanced?

### 8.4 Center of Mass and Center of Gravity

15. If you toss a stick into the air, it appears to wobble all over the place. Specifically, about what place does it wobble?
16. Where is the center of mass of a baseball? Where is its center of gravity? Where are these centers for a baseball bat?
17. If you hang at rest by your hands from a vertical rope, where is your center of gravity with respect to the rope?
18. Is the center of mass of an irregularly shaped body its geometric center?
19. What is the relationship between the center of gravity and the support base for an object that is in stable equilibrium?
20. Why doesn't the Leaning Tower of Pisa topple over?
21. In terms of center of gravity, support base, and torque, why can't you stand with your heels and back to a wall and then bend over to touch your toes and return to your stand-up position?

## 8.5 Centripetal Force

22. When you whirl a can tied at the end of a string in a circular path, why do you need to keep pulling on the string? What is the role of the string?
23. Is it an inward force or an outward force that is exerted on the clothes during the spin cycle of an automatic washing machine?

## 8.6 Centrifugal Force

24. If the string that holds a whirling can in its circular path breaks, what kind of force causes it to move in a straight-line path: centripetal, centrifugal, or no force? What law of physics supports your answer?
25. If you are not wearing a seat belt in a car that rounds a curve, and you slide across your seat and slam against a car door, what kind of force is responsible for your slide: centripetal, centrifugal, or no force? Why is the correct answer "no force"?
26. Why is centrifugal force in a rotating frame called a "fictitious force"?
27. How can gravity be simulated in an orbiting space station?

## 8.7 Angular Momentum

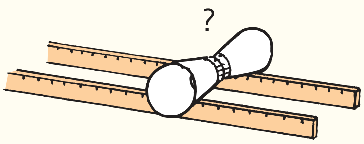
28. How is angular momentum related to inertia?
29. What is the law of inertia for rotating systems in terms of angular momentum?

## 8.8 Conservation of Angular Momentum

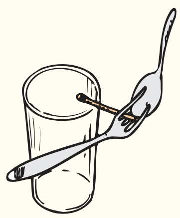
30. If a skater who is spinning pulls her arms in so as to reduce her rotational inertia by half, by how much will her angular momentum change? By how much will her rate of spin increase? (Why do your answers differ?)

## THINK AND DO (HANDS-ON APPLICATION)

31. Contact Grandpa and tell him how you're learning to distinguish between closely related concepts, using the examples of force and torque. Tell him how the two are similar and how they differ. Suggest where he can find "hands-on" things in his home to illustrate the difference between the two. Also cite an example that shows how the net force on an object can be zero while the net torque isn't, as well as an example of the other way around. (Now make your Grandpa's day and *send* an actual letter to him!)
32. Fasten a pair of foam cups together at their wide ends and roll them along a pair of metersticks that simulate railroad tracks. Note how they self-correct whenever their path departs from the center. Question: If you taped the cups together at their narrow ends, so they tapered oppositely as shown, would the pair of cups self-correct or self-destruct when rolling slightly off center?

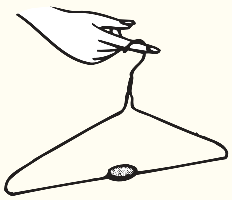



33. Fasten a fork, spoon, and wooden match together as shown in the sketch. The combination will balance nicely—on the edge of a glass, for example. This happens because the center of gravity actually "hangs" below the point of support.



34. Stand with your heels and back against a wall and try to bend over and touch your toes. You'll find that you have to stand away from the wall to do so without toppling over. Compare the minimum distance of your heels from the wall with the distance for a friend of the opposite sex. Who can touch their toes with their heels nearer to the wall: men or women? On the average and in proportion to height, which sex has the lower center of gravity?
35. First ask a friend to stand facing a wall with her toes touching the wall. Then ask her to stand on the balls of her feet without toppling backward. Your friend won't be able to do it. Now you explain why it can't be done.
36. Rest a meterstick on two extended forefingers as shown. Slowly bring your fingers together. At what part of the stick do your fingers meet? Can you explain why this always happens, no matter where you start your fingers?

37. Place the hook of a wire coat hanger over your finger. Carefully balance a coin on the straight wire on the bottom directly under the hook. You may have to flatten the wire with a hammer or fashion a tiny platform with tape. With a surprisingly small amount of practice you can swing the hanger and balanced coin back and forth and then in a circle. Centripetal force holds the coin in place.



## PLUG AND CHUG (EQUATION FAMILIARIZATION)

**Torque = lever arm × force**

38. Calculate the torque produced by a 100-N perpendicular force at the end of a 0.3-m-long wrench.
39. How does the torque produced change if the force reduces to 50 N while keeping the length of the wrench unchanged?

**Centripetal force: $F = mv^2/r$**

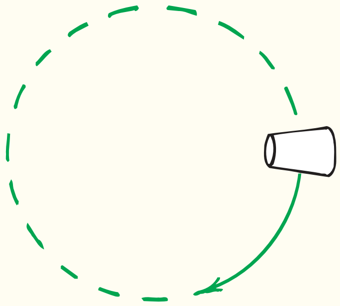
40. A horizontal string whirls a 1-kg can in a circle of radius 2 m with a speed of 2 m/s. Calculate the tension in the string.
41. Calculate the force of friction that keeps an 80-kg person sitting on the edge of a horizontal rotating platform when the person sits 2 m from the center of the platform and has a tangential speed of 3 m/s.

**Angular momentum = $mvr$**

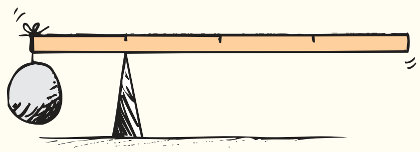
42. Calculate the angular momentum of the whirling can in problem 40.
43. If a person's speed doubles and all else remains the same, what will be the person's angular momentum?

## THINK AND SOLVE (MATHEMATICAL APPLICATION)

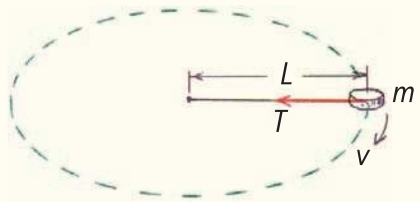
44. The diameter of the base of a tapered drinking cup is 6 cm. The diameter of its mouth is 9 cm. The path of the cup curves when you roll it on the top of a table. Which end, the base or the mouth, rolls faster? How much faster?



45. To tighten a bolt, you push with a force of 80 N at the end of a wrench handle that is 0.25 m from the axis of the bolt.
    a. What torque are you exerting?
    b. You move your hand inward to be only 0.10 m from the bolt. To achieve the same torque, show that you should exert 200 N of force.
    c. Do your answers depend on the direction of your push relative to the direction of the wrench handle?
46. The rock and meterstick balance at the 25-cm mark, as shown in the sketch. The meterstick has a mass of 1 kg. What must be the mass of the rock?



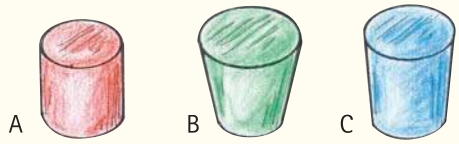
47. In one of the photos at the beginning of this chapter, Mary Beth uses a torque feeler that consists of a meterstick held at the 0-cm end with a weight dangling from various positions along the stick. When the stick is held horizontally, torque is produced when a 1-kg object hangs from the 50-cm mark. How much more torque is exerted when the object is hung from the 75-cm mark? The 100-cm mark?
48. An ice puck of mass $m$ revolves on an icy surface in a circle at speed $v$ at the end of a horizontal string of length $L$. The tension in the string is $T$.



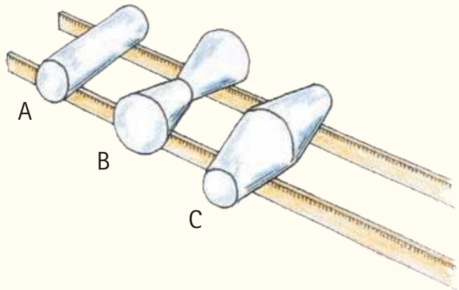

    a. Write the equation for centripetal force, and substitute the values $T$ and $L$ appropriately. Then, with a bit of elementary algebra, rearrange the equation so that it solves for mass.
    b. Show that the mass of the puck is 5 kg when the length of the string is 2 m, the string tension is 10 N, and the tangential speed of the puck is 2 m/s.
49. If a trapeze artist rotates once each second while sailing through the air and contracts to reduce her rotational inertia to one-third of what it was, how many rotations per second will result?
50. A small space telescope at the end of a tether line of length $L$ moves at linear speed $v$ about a central space station.
    a. What will be the linear speed of the telescope if the length of the line is reduced to $0.33L$?
    b. If the initial linear speed of the telescope is 1.0 m/s, what is its speed when pulled in to one-third its initial distance from the space station?

## THINK AND RANK (ANALYSIS)

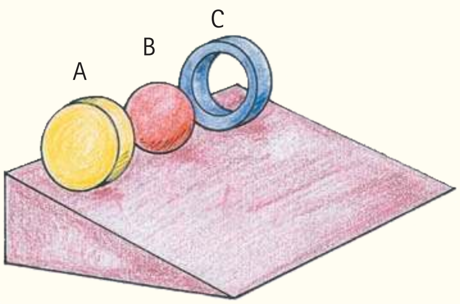
51. The three cups are rolled on a level surface. Rank the cups by how far they depart from a straight-line path (most curved to least curved).



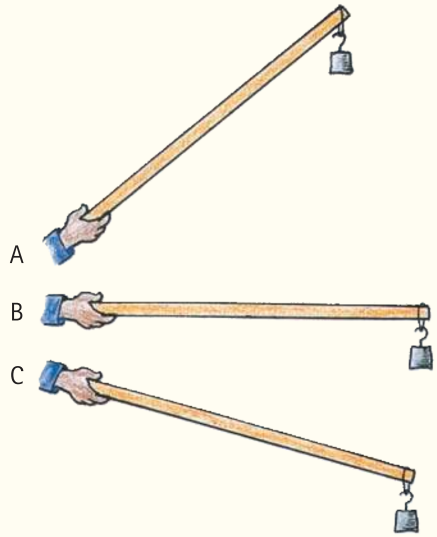

52. Three types of rollers are placed on slightly inclined parallel meterstick tracks as shown. From greatest to least, rank the rollers in terms of their ability to remain stable as they roll.



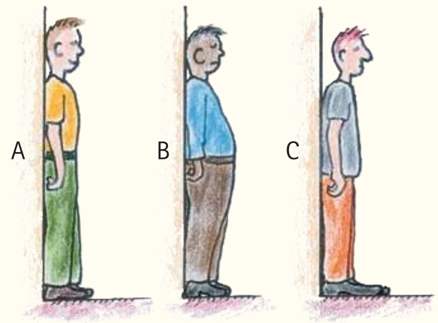

53. Beginning from a rest position, a solid disk A, a solid ball B, and a hoop C race down an inclined plane. Rank them in reaching the bottom: winner, second place, and third place.




54. You hold a meterstick at one end with the same mass suspended at the opposite end. Rank the torque needed to keep the stick steady, from largest to smallest.




55. Three physics majors in good physical shape stand with their backs against a wall. Their task is to lean over and touch their toes without toppling over. Rank their chances for success from highest to lowest.




## THINK AND EXPLAIN (SYNTHESIS)

56. While riding on a carnival Ferris wheel, Sam Nasty horses around and climbs out of his chair and along the spoke so that he is halfway to the axis. How does his rotational speed compare with that of his friends who remain in the chair? How does his tangential speed compare? Why are your answers different?

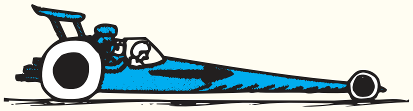
57. Dan and Sue cycle at the same speed. The tires on Dan's bike are larger in diameter than those on Sue's bike. Which wheels, if either, have the greater rotational speed?

58. Use the equation $v = r\omega$ to explain why the end of a flyswatter moves much faster than your wrist when you swat a fly.

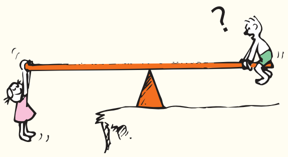
59. The wheels of railroad trains are tapered, a feature especially important on curves. How, if at all, does the amount of taper relate to how great the sharpness of curves can be?

60. Flamingos are frequently seen standing on one leg with the other leg lifted. Is rotational inertia enhanced with long legs? What can you say about the bird's center of mass with respect to the foot on which it stands?

61. The front wheels of a racing vehicle are located far out in front to help keep the vehicle from nosing upward when it accelerates. What physics concepts play a role here?

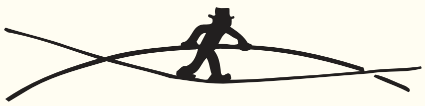


62. Which will have the greater acceleration rolling down an incline: a bowling ball or a volleyball? Defend your answer.
63. A softball and a basketball start from rest and roll down an incline. Which ball reaches the bottom first? Defend your answer.
64. Is the net torque changed when a partner on a seesaw stands or hangs from her end instead of sitting? (Does the weight or the lever arm change?)

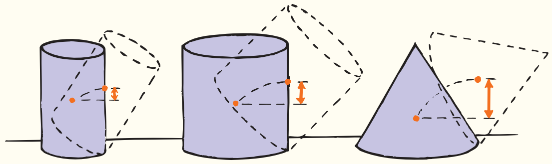


65. Can a force produce a torque when there is no lever arm?
66. When you pedal a bicycle, maximum torque is produced when the pedal sprocket arms are in the horizontal position, and no torque is produced when they are in the vertical position. Explain.

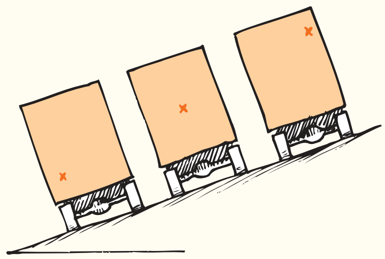
67. When the line of action of a force intersects the center of mass of an object, can that force produce a torque about the object's center of mass?
68. When a bowling ball leaves your hand, it may not spin. But farther along the alley, it does spin. What produces the spinning?
69. Why does sitting closest to the center of a vehicle provide the most comfortable ride in a bus traveling on a bumpy road, in a ship in a choppy sea, or in an airplane in turbulent air?
70. Assuming your feet are not held down, which is more difficult: doing sit-ups with your knees bent or with your legs straight out?
71. Explain why a long pole is more beneficial to a tightrope walker if the pole droops.



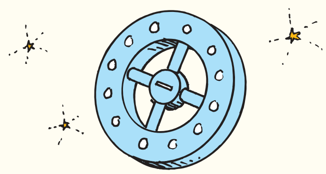
72. Why must you bend forward when carrying a heavy load on your back?
73. Why is the wobbly motion of a single star an indication that the star has one or more planets orbiting around it?
74. Why is it easier to carry the same amount of water in two buckets, one in each hand, than in a single bucket?
75. Where is the center of mass of Earth's atmosphere?
76. Using the ideas of torque and center of gravity, explain why a ball rolls down a hill.
77. Why is it important to secure file cabinets to the floor, especially cabinets with heavy loads in the top drawers?
78. Describe the comparative stabilities of the three objects shown in terms of work and potential energy.



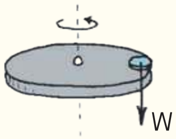
79. The centers of gravity of the three trucks parked on a hill are shown by the Xs. Which truck(s) will tip over?



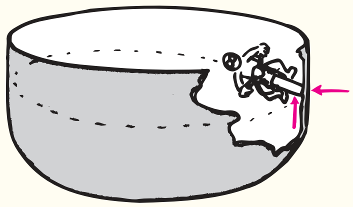
80. A racing car on a flat circular track needs friction between the tires and the track to maintain its circular motion. How much more friction is required for twice the speed?
81. An object is moving with constant speed in a circular path. Is a force acting on it?
82. As a car speeds up when rounding a curve, does the centripetal force on the car also increase? Defend your answer.
83. When you are in the front passenger seat of a car turning to the left, you may find yourself pressed against the right-side door. Why do you press against the door? Why does the door press on you? Does your explanation involve a centrifugal force or Newton's laws?
84. Friction is needed for a car rounding a curve. But, if the road is banked, friction may not be required at all. What, then, supplies the needed centripetal force? (*Hint:* Consider vector components of the normal force on the car.)
85. Under what conditions could a fast-moving car remain on a banked track covered with slippery ice?
86. Explain why a centripetal force does *not* do work on a circularly moving object.
87. The occupant inside a rotating space habitat of the future feels that she is being pulled by artificial gravity against the outer wall of the habitat (which becomes the "floor"). Explain what is going on in terms of Newton's laws and centripetal force.



88. The sketch shows a coin at the edge of a turntable. The weight of the coin is shown by the vector ***W***. Two other forces act on the coin—the normal force and a force of friction that prevents it from sliding off the edge. Draw force vectors for both of these.



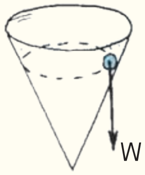

89. A motorcyclist is able to ride on the vertical wall of a bowl-shaped track as shown. Friction of the wall on the tires is shown by the vertical red vector. (a) How does the magnitude of this vertical vector compare with the weight of the motorcycle and rider? (b) Does the horizontal red vector represent the normal force acting on the bike and rider, the centripetal force, both, or neither? Defend your answer.



90. The sketch shows a conical pendulum. The bob swings in a circular path. The tension ***T*** and weight ***W*** are shown by vectors. Draw a parallelogram with these vectors, and show that their resultant lies in the plane of the circle. (See the parallelogram rule in Chapter 2.) What is the name of this resultant force?


91. You sit at the middle of a large turntable at an amusement park as it is set spinning and then allowed to spin freely. When you crawl toward the edge of the turntable, does the rate of the rotation increase, decrease, or remain unchanged? What physics principle supports your answer?

92. Consider a ball rolling around in a circular path on the inner surface of a cone. The weight of the ball is shown by the vector ***W***. Without friction, only one other force acts on the ball—a normal force. (a) Draw in the vector for the normal force. (The length of the vector depends on the next step, b.) (b) Using the parallelogram rule, show that the resultant of the two vectors is along the radial direction of the ball's circular path. (Yes, the normal is appreciably greater than the weight!)




93. A sizable quantity of soil is washed down the Mississippi River and deposited in the Gulf of Mexico each year. What effect does this tend to have on the length of a day? (*Hint:* Relate this to Figure 8.52.)

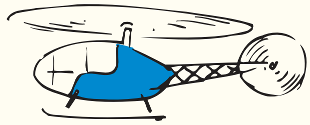
94. If all of Earth's inhabitants moved to the equator, how would this affect Earth's rotational inertia? How would it affect the length of a day?

95. Strictly speaking, as more and more skyscrapers are built on the surface of Earth, does the day tend to become longer or shorter? And, strictly speaking, does the falling of autumn leaves tend to lengthen or shorten the 24-hour day? What physics principle supports your answers?

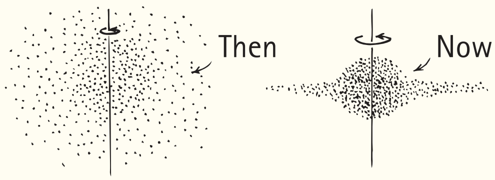
96. If the world's populations moved to the North Pole and the South Pole, would the 24-hour day become longer, shorter, or stay the same?

97. If the polar ice caps on Earth's solid surface were to melt, the oceans would be deeper. Strictly speaking, what effect would this have on Earth's rotation?

98. Why does a typical small helicopter with a single main rotor have a second small rotor on its tail? Describe the consequence if the small rotor fails in flight.



99. We believe that our galaxy was formed from a huge cloud of gas. The original cloud was far larger than the present size of the galaxy, was more or less spherical, and was rotating much more slowly than the galaxy is now. In this sketch, we see the original cloud and the galaxy as it is now (seen edgewise). Explain how the inward pull of gravity and the conservation of angular momentum contribute to the galaxy's present shape and why it rotates faster now than when it was a larger, spherical cloud.

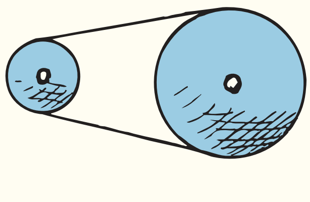



100. Earth is not spherical but bulges at the equator. Jupiter bulges more. What is the cause of these bulges?

## THINK AND DISCUSS (EVALUATION)

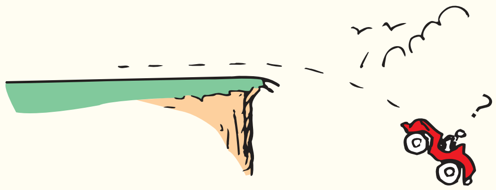
101. An automobile speedometer is configured to read speed proportional to the rotational speed of its wheels. If the oversize wheels of this red car were installed without making speedometer corrections, discuss whether the speedometer reading will be high, or low—or no change.

102. A large wheel is coupled to a wheel with half the diameter, as shown in the sketch. How does the rotational speed of the smaller wheel compare with that of the larger wheel? Discuss the comparisons of the tangential speeds at the rims (assuming the belt doesn't slip).

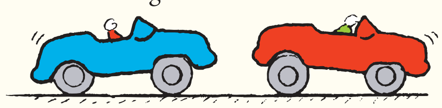


103. In this chapter, we learned that an object may *not* be in mechanical equilibrium even when $\Sigma F = 0$. Discuss what else defines mechanical equilibrium.

104. When a car drives off a cliff it rotates forward as it falls. For a higher speed off the cliff, discuss whether the car will rotate more or less. (Consider the time that the unbalanced torque acts.)



105. Discuss why a car noses up when accelerating and noses down when braking.



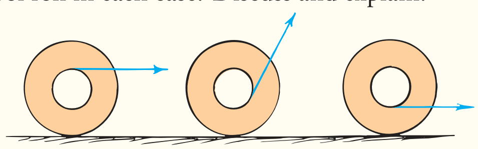
106. Discuss how a ramp would help you to distinguish between two identical-looking spheres of the same weight, one solid and the other hollow.

107. Which will roll down an incline faster: a can of water or a can of ice? Discuss and explain.

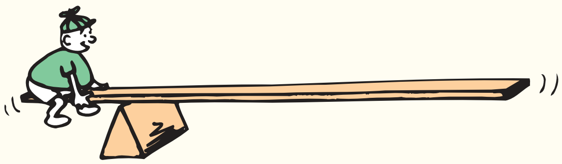
108. Why are lightweight tires preferred in bicycle racing? Discuss and explain.

109. A youngster who has entered a soapbox derby (in which four-wheel, unpowered vehicles roll from rest down a hill) asks if large massive wheels or lightweight ones should be used. Also, should the wheels have spokes or be solid? What advice do you offer?

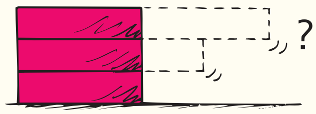
110. The spool is pulled in three ways, as shown. There is sufficient friction for rotation. In what direction will the spool roll in each case? Discuss and explain.



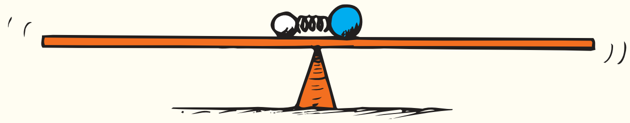
111. Nobody at the playground wants to play with the obnoxious boy, so he fashions a seesaw as shown so he can play without anyone else. Discuss how this is done.



112. How can the three bricks be stacked so that the top brick has maximum horizontal displacement from the bottom brick? For example, stacking them as suggested by the dashed lines would be unstable and the bricks would topple. (*Hint:* Start with the top brick and work down. At every interface the CG of the bricks above must not extend beyond the end of the supporting brick.)



113. A long track balanced like a seesaw supports a golf ball and a more massive billiard ball with a compressed spring between the two. When the spring is released, the balls move away from each other. Does the track tip clockwise, tip counterclockwise, or remain in balance as the balls roll outward? What principles do you use for your discussion and explanation?



114. With respect to Diana's finger, where is the center of mass of the plastic bird? Discuss how this balance can be accomplished during its manufacture.

115. When a long-range cannonball is fired toward the equator from a northern (or southern) latitude, it lands west of its "intended" longitude. Discuss why this is so. (*Hint:* Consider a flea jumping from partway out to the outer edge of a rotating turntable.)

116. Most often we say that force causes acceleration. But when Evan took a ride in a rocket sled at Bonneville Salt Flats, blood was forced to the back of his brain, nearly blacking him out. Discuss and identify the cause of this force.