

# SDI LAB #5. ANGULAR MOMENTUM

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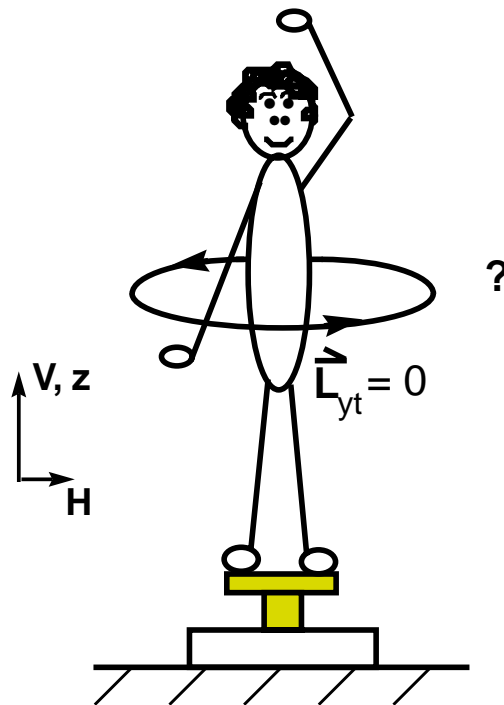
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A stationary student about to perform a "cat twist" on a low-friction turntable.

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## I. INTRODUCTION

In this lab you enhance your physical understanding of Newtonian mechanics by performing experiments (more advanced than those of SDI Lab #4, "Rotational Dynamics") which can be understood on the basis of Newton's Second Law of Rotational Motion (N2R) in the form

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_o)/(\Delta t) = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t) = I_o \vec{\alpha}_o \dots\dots\dots (1)$$

Here subscript "o" refers to the point about which the torque  $\vec{\tau}$ , angular velocity  $\vec{\omega}$ , angular acceleration  $\vec{\alpha}$ , and angular momentum  $\vec{L}$  are taken, and, *for the relatively simple conditions of motion studied in this lab*, the moment of inertia I is a scalar and  $\vec{L}$  is along  $\vec{\omega}$ .

### A. OBJECTIVES

1. To understand the relationship of torque and change of angular momentum (or angular velocity) in accord with Newton's second law of rotational motion, Eq. (1), for:
  - a. a spinning bicycle wheel,
  - b. the Old Spinning-Wheel-in-the-Suitcase trick.
  
2. To understand the requirements for and the manifestation of conservation of angular momentum (treated in parallel with conservation of energy) for:
  - a. holding weights while standing on a turntable,
  - b. holding a spinning bicycle while standing on a turntable,
  - c. the cat twist,
  - d. the contortions of astronauts in Sky Lab.

### B. HOW TO PREPARE FOR THIS LAB

#### 1. Review the Sec. I-B "Ground Rules for SDI Labs," of SDI Lab #0.1.

2. Before you begin, it will also help to study the following sections in the course text *Physics*, 4th ed. by Douglas Giancoli (or similar material in whatever text you are using) – Chap. 5, "Circular Motion, Gravitation"; Chapter 8, "Rotational Motion." In addition please review ALL the material we have covered in lectures, labs, and homework on frictional forces, circular motion, and rotational dynamics. Please also review the material of SDI Labs #3, "Circular Motion and Frictional Forces," and #4, "Rotational Dynamics."

3. At the start of this lab, your instructor, along with a few volunteer students, may demonstrate the Old Spinning-Wheel-in-the-Suitcase trick. If s(he) does, then later on you should return to this section and perform it for yourselves.

C. CONDITIONS OF MOTION IN THIS LAB

A complete treatment of the rotating rigid bodies studied in this lab would require the use of Newton's second law for rotational motion (N2R) in its most general form

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_o) / (\Delta t) \dots\dots\dots (2)$$

where  $\vec{L} = I\vec{\omega}$  is the angular momentum;  $\vec{\omega}$  is the angular velocity; and  $\vec{L}$ ,  $\vec{\omega}$ , and  $\vec{\tau}$  must be taken with respect to a *point* "o" which must either (a) be fixed in an inertial reference system or, (b) coincide with the center of mass (cm) of the body, or (c) have an acceleration which is either zero or is directed through (toward or away) from the cm. [See, e.g., R. Weinstock, *Am. J. Phys.* **57**, 1063 (1989)]. In general, I is not a scalar and  $\vec{L}$  is not in the same direction as  $\vec{\omega}$ . Fortunately, *for the relatively simple conditions of motion studied in this lab*, I is a scalar and  $\vec{L}$  is along  $\vec{\omega}$ . Thus we can use the abbreviation N2R to mean

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_o) / (\Delta t) = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t) = I_o \vec{\alpha}_o \dots\dots\dots (1)$$

where "o" is as previously described. The latter form will be used in Sec. II in order to emphasize the similarity of:

$$\vec{\tau}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t) \dots\dots\dots (N2R)$$

and

$$\vec{F} = m \lim_{\Delta t \rightarrow 0} (\Delta \vec{v} / \Delta t) \dots\dots\dots (N2)$$

and thus the similarity of the "levitations" of (1) the spinning (high- $\vec{\omega}$ ) bicycle wheel or gyroscope, and (2) the moving (high- $\vec{v}$ ) water (in the bucket rotated around your head) in SDI Lab #3, "Circular Motion and Frictional Forces."

## II. NEWTON'S SECOND LAW OF ROTATIONAL MOTION

### A. BICYCLE WHEEL

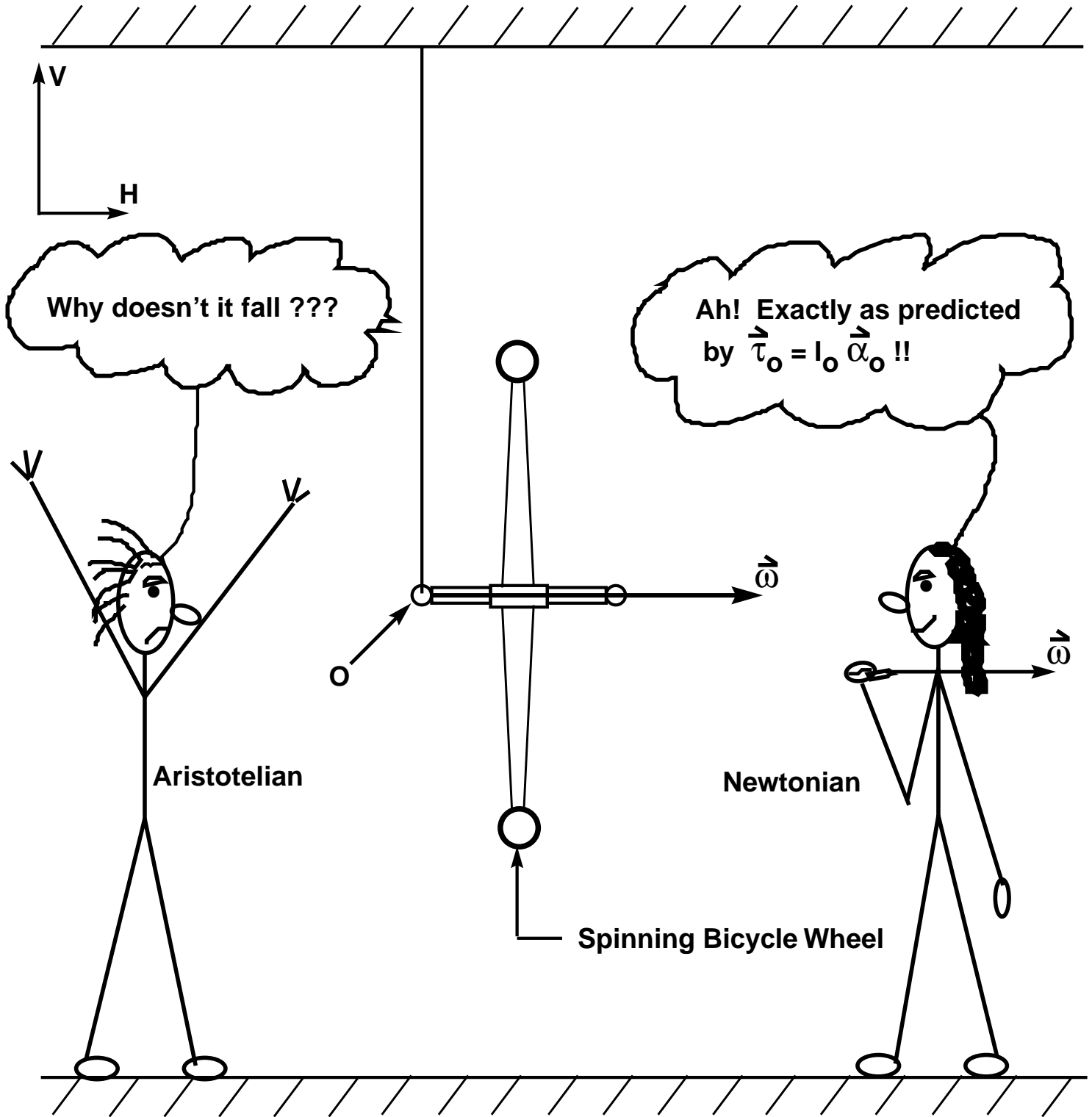


Fig. 1. An Aristotelian and a Newtonian confront a *spinning* bicycle wheel suspended by a rope from the ceiling.

1. A rim-loaded bicycle wheel (the tire is solid rubber so as to increase the wheel's moment of inertia) is suspended by a rope which runs from an end of the axle to the ceiling. Spin the wheel with your palm or an electric motor so as to obtain a high  $\vec{\omega}$  in the direction shown in Fig. 1. Use the right-hand rule (as does the Newtonian) to determine the direction of  $\vec{\omega}$ . To reduce the conical pendulum motion hold the rope rigidly with your hand about 1/2 meter above the axle. Release the wheel *with its axis horizontal*.

a. Carefully observe the motion as the *spinning* bicycle wheel *precesses* about a vertical axis through the suspension point O.

b. In Fig. 1 show all the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist). Are these vectors in qualitative accord with N2R,

$$\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta\vec{\omega}_o)/(\Delta t)? \quad \{Y, N, U, NOT\}$$

[HINT #1: The torque  $\vec{\tau}$ , moment of inertia I, and angular acceleration  $\vec{\alpha}$  must all be taken with respect to the point O where the rope attaches to the axle.]

[HINT #2: Study the vector model on the cart near the suspended bicycle wheel.]

c. Do you understand why the *spinning* bicycle wheel **does NOT** fall, i.e., rotate downwards? {Y,N,U, NOT}

[HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta\vec{\omega}_o)/(\Delta t)$  .]

Compare with the question of SDI Lab #3 regarding water in a bucket rotated rapidly over your head: "Do you understand why the water **does NOT** fall out of the bucket?"

2. Support the axle of the *non-spinning* bicycle wheel and then *release* it as in Fig. 2.

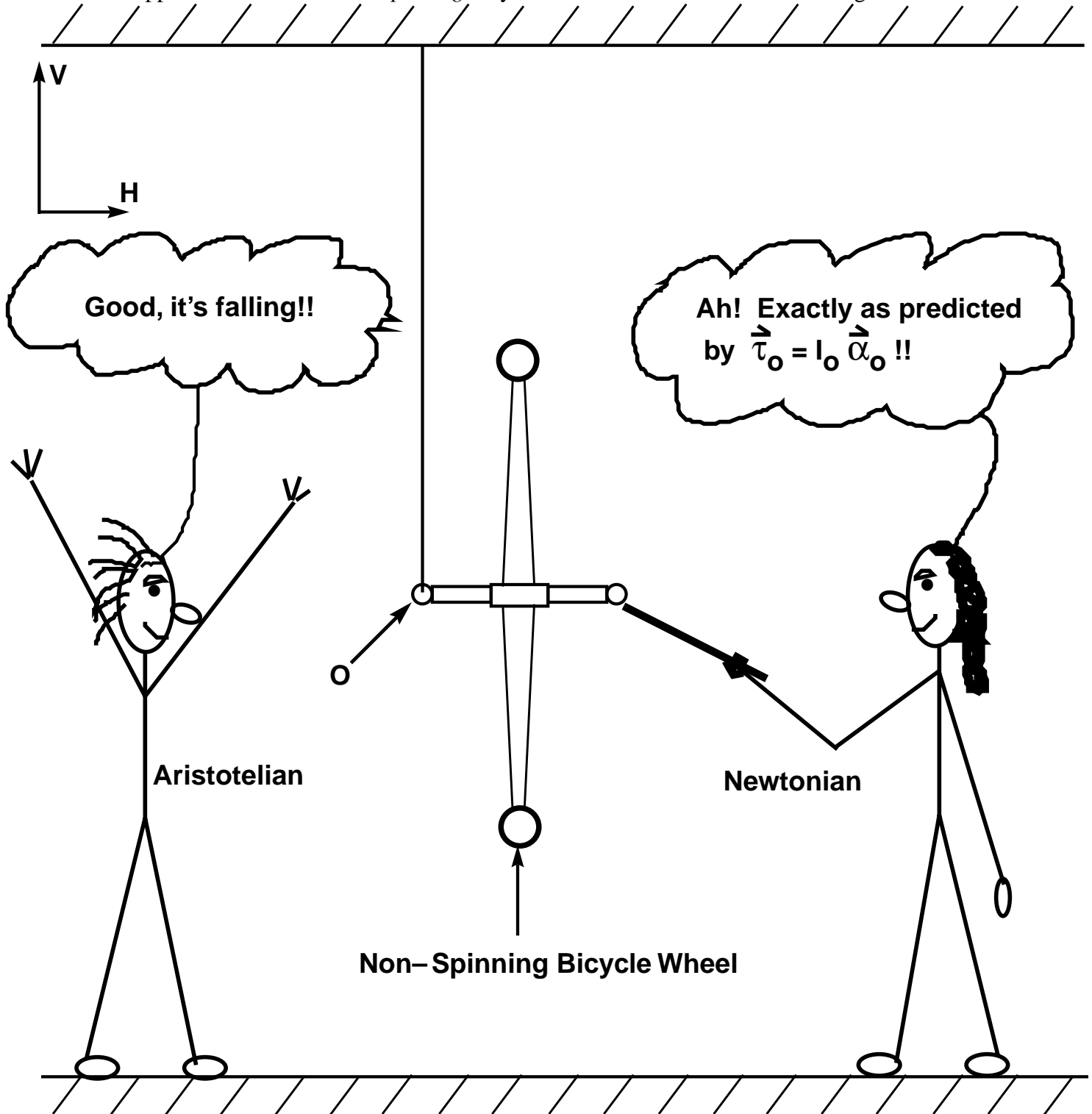


Fig. 2. An Aristotelian and a Newtonian confront a *non-spinning* bicycle wheel suspended by a rope from the ceiling at an instant *just after the wheel has been released* (note that the stick is NOT in contact with the axle.)

a. Carefully observe the motion as the *non-spinning* bicycle wheel rotates *clockwise* about OO'', an axis through O and perpendicular to the paper in Fig. 2. Fig. 2 shows the bicycle wheel at an instant just after having been released.

b. In Fig. 2 show ALL the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta \vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist). Are these vectors in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t)$ ? {Y, N, U, NOT}

c. Do you understand why the spinning bicycle wheel **does** fall, i.e., rotate downwards? {Y, N, U, NOT}

HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t)$

Compare with the question of SDI Lab #3 regarding water in a bucket held stationary over your head: "Do you understand why the water **does** fall out of the bucket?"

3. Returning to the case of the *spinning* bicycle wheel, repeat "1" above but now spin the wheel so that  $\vec{\omega}$  has about the same magnitude as in Fig. 1 but is *opposite in direction*. In Fig. 3 show all the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist).

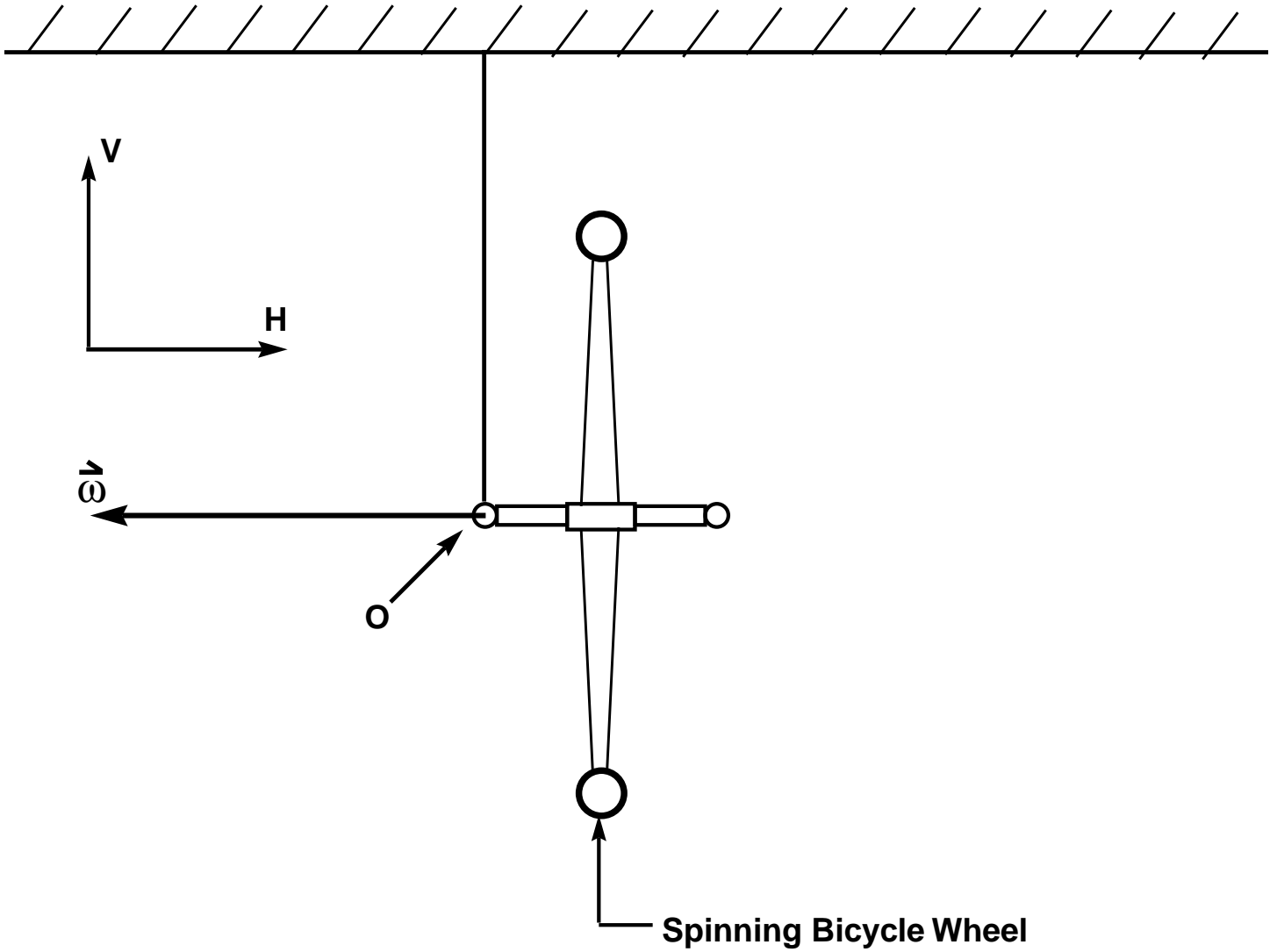


Fig. 3. The spinning bicycle wheel now has an angular velocity  $\vec{\omega}$  of about the same magnitude as in Fig. 1 but opposite in direction.

- a. Are these vectors in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t)$ ?  
 {Y, N, U, NOT}



b. Comparing Figs. 1 and 3 can you give a qualitative relationship between the directions of  $\vec{\omega}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

c. Is your relation in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t)_o$ ?  
{Y, N, U, NOT}

4. Repeat "3" above but with a much reduced angular velocity  $\vec{\omega}$  as shown below in Fig. 4. In Fig. 4 show all the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist)

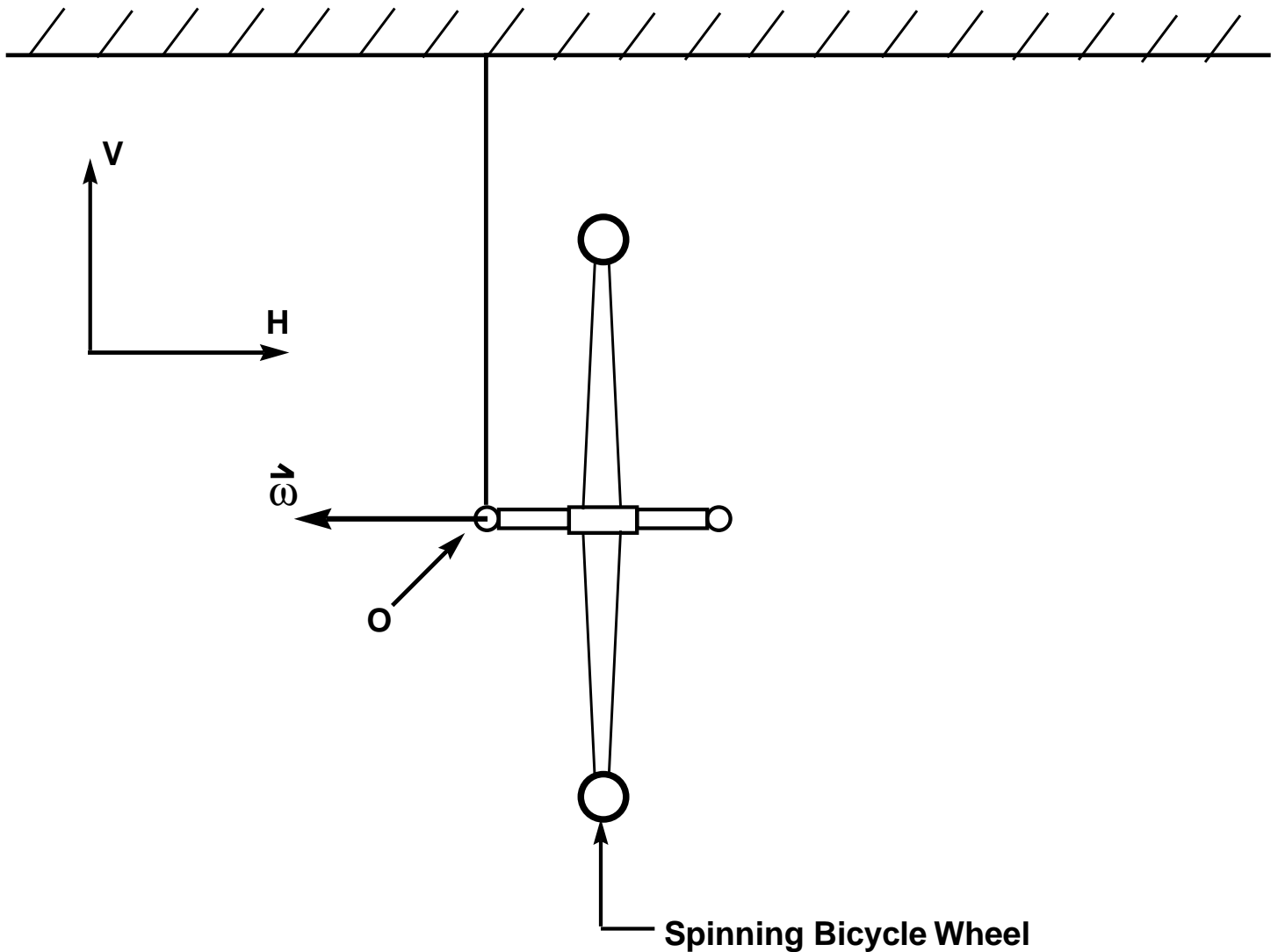


Fig. 4. The spinning bicycle wheel has an  $\vec{\omega}$  in the same direction as in Fig. 3, but the  $\vec{\omega}$  is much reduced in magnitude.

- a. Are these vectors in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t)_o$ ?  
 {Y, N, U, NOT}

b. Comparing Figs. 3 and 4 can you give a qualitative relationship between the magnitudes of  $\vec{\omega}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

c. Is your relationship in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t)$ ?  
{Y, N, U, NOT}

5. Torque Test on the Trapdoor to Death!

Remove the bicycle wheel from the suspension.\* Stand on the trapdoor marked "A" (Alligator Pit). Set the wheel spinning. Hold the axle with one hand at each end and with the axle **horizontal** as in Fig. 5. Have a lab partner *suddenly* and *simultaneously* (a) indicate with the green vector arrow  $\vec{\omega}_f$  at your table a final direction (for simplicity either *vertically up* or *vertically down*) for the vector  $\vec{\omega}$  of the wheel, starting from the wheel's horizontal initial  $\vec{\omega}_i$ , (b) push the trapdoor button. The trapdoor will open unless you apply a torque such as to direct  $\vec{\omega}$  along  $\vec{\omega}_f$  *within 20 seconds!* If you pass this then in the drawing below, show  $\vec{\omega}_f$  (final),  $\vec{\tau}_i$  (the *initial* torque applied by you), and the forces(s)  $\vec{F}_i$  *initially* applied by you to the axle. If you do not pass the test and are consumed by the alligators you may omit this and subsequent sections.

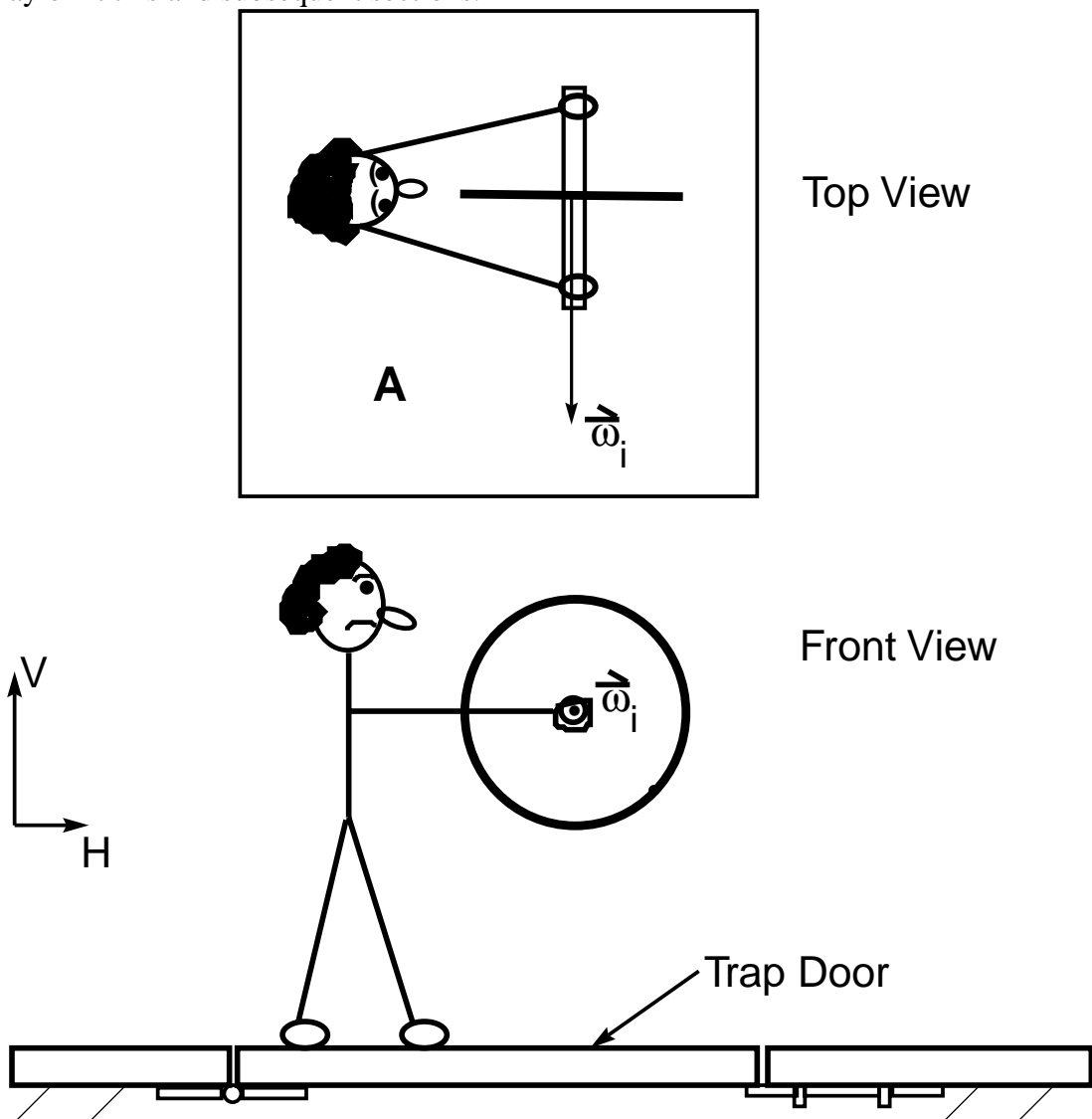


Fig. 5. A student (you) stands on the trap door to the alligator pit holding a rotating bicycle wheel.

\*Some students may prefer to use the smaller wheel and axle available at the lab tables.

## B. THE OLD SPINNING-WHEEL-IN-THE-SUITCASE TRICK

R.W. Wood [famous American physicist, pioneer in physical optics, boomerang expert, legendary trickster, and author of the invaluable guide book *How to Tell the Birds from the Flowers* (Dover, New York, 1959)] sometimes carried a suitcase containing a spinning bicycle wheel. He would hand his suitcase to a porter with the instructions "Follow Me!" He would then walk rapidly through a door and make a sharp  $90^\circ$  turn, Fig. 6.

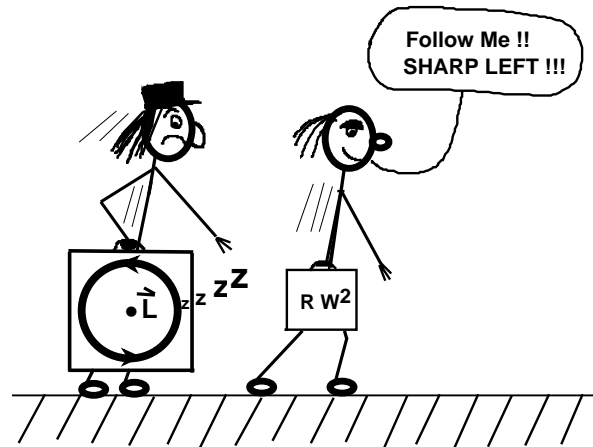


Fig. 6. The Old Spinning-Wheel-In-The-Suitcase Trick. R.W. Wood ( $RW^2$ ) leads a porter around a sharp turn.

1. *THOUGHT EXPERIMENT*: Can you predict what will happen to the the suitcase when the porter makes the left turn ordered by Wood? {Y, N, U, NOT}

### 2. *REAL EXPERIMENT*:

Use the electric motor to rev up the wheel in the suitcase so that the  $\vec{\omega}$  (and hence  $\vec{L}$ ) vectors are in the same direction as in Fig. 6 (use the right-hand rule for the direction of  $\vec{\omega}$ ). Place an arrow marked  $\vec{L}$  on the suitcase to indicate its direction.) Notice the behavior of the suitcase when you make sharp turns first to the left as in Fig. 6 and then to the right. Is your prediction in "1" verified? {Y, N, U, NOT}

3. In Fig. 7 the porter (you) applies a torque  $\vec{\tau}$  on the suitcase grip so as to initiate a sharp turn to the porter's left as in Fig. 6. Show (a) the  $\vec{\tau}$  and  $\vec{F}$  vectors applied to the handle by the porter, (b) the  $\Delta\vec{L}$  vector. [HINT:  $\vec{\tau}$  applied by the porter is due to a "couple," i.e., forces at the two ends of the handle which are equal in magnitude but opposite in direction.]

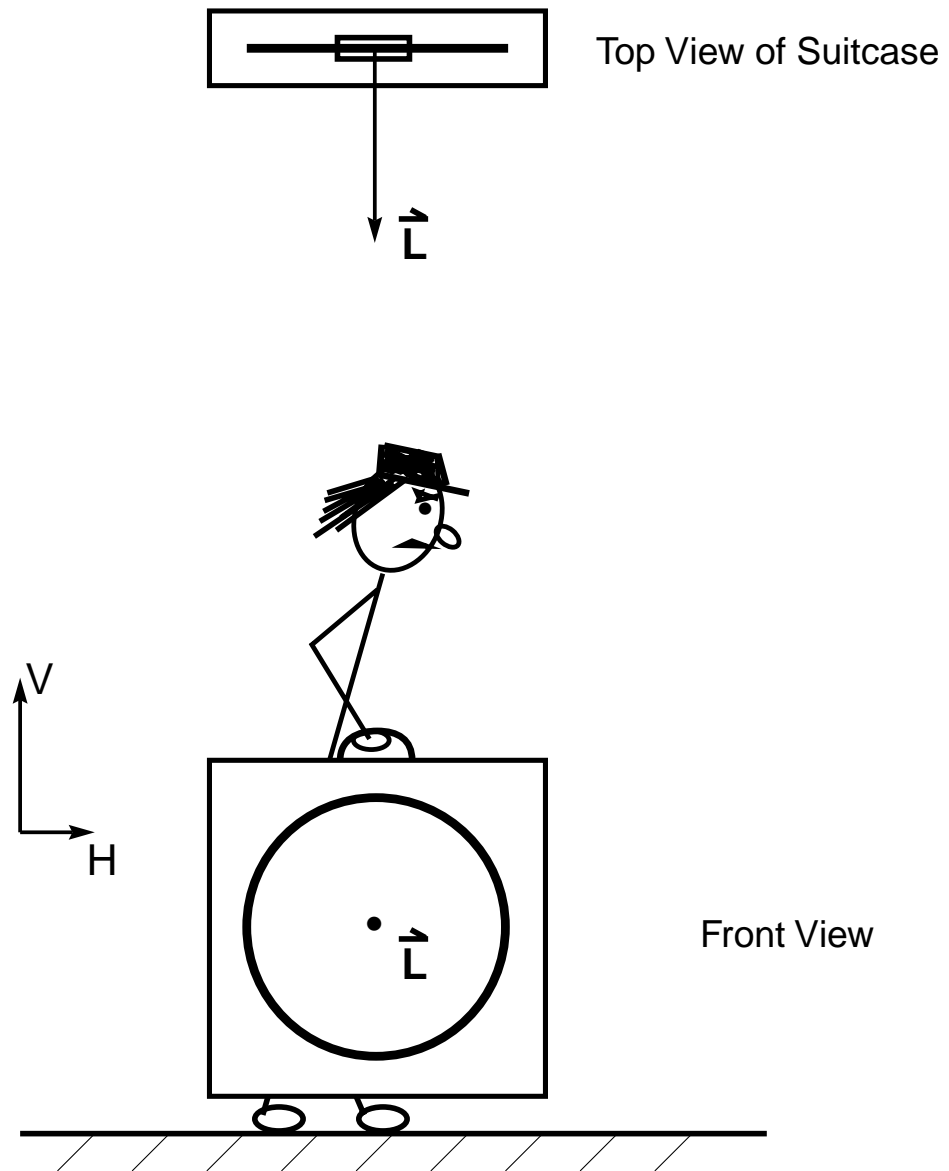


Fig. 7. The porter (you) applies a torque to the suitcase handle so as to initiate a sharp turn to the porter's left.

a. Are the vectors you have drawn in qualitative accord with N2R:

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_o) / (\Delta t) = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o) / (\Delta t) = I_o \vec{\alpha}_o ? \quad \{Y, N, U, NOT\}$$

4. Sam Smart tried to out-torque the notorious R.W. Wood by disguising himself as a porter. When Wood handed Smart (played by you) the spinning-wheel suitcase, Smart applied a smart torque  $\vec{\tau}_s$  so as to initiate a sharp turn to Sam's left while at the same time keeping the suitcase vertical. Fig. 8 shows Sam Smart and the suitcase. Show (a) the position of Smart's hands on the suitcase, (b) the smart torque  $\vec{\tau}_s$  and the  $\vec{F}_s$  vectors applied by Smart to the suitcase, (c) the  $\Delta\vec{L}$  vector. [HINT:  $\vec{\tau}_s$  applied by the Smart is due to a "couple," i.e., forces  $\vec{F}_s$  equal in magnitude but opposite in direction.]

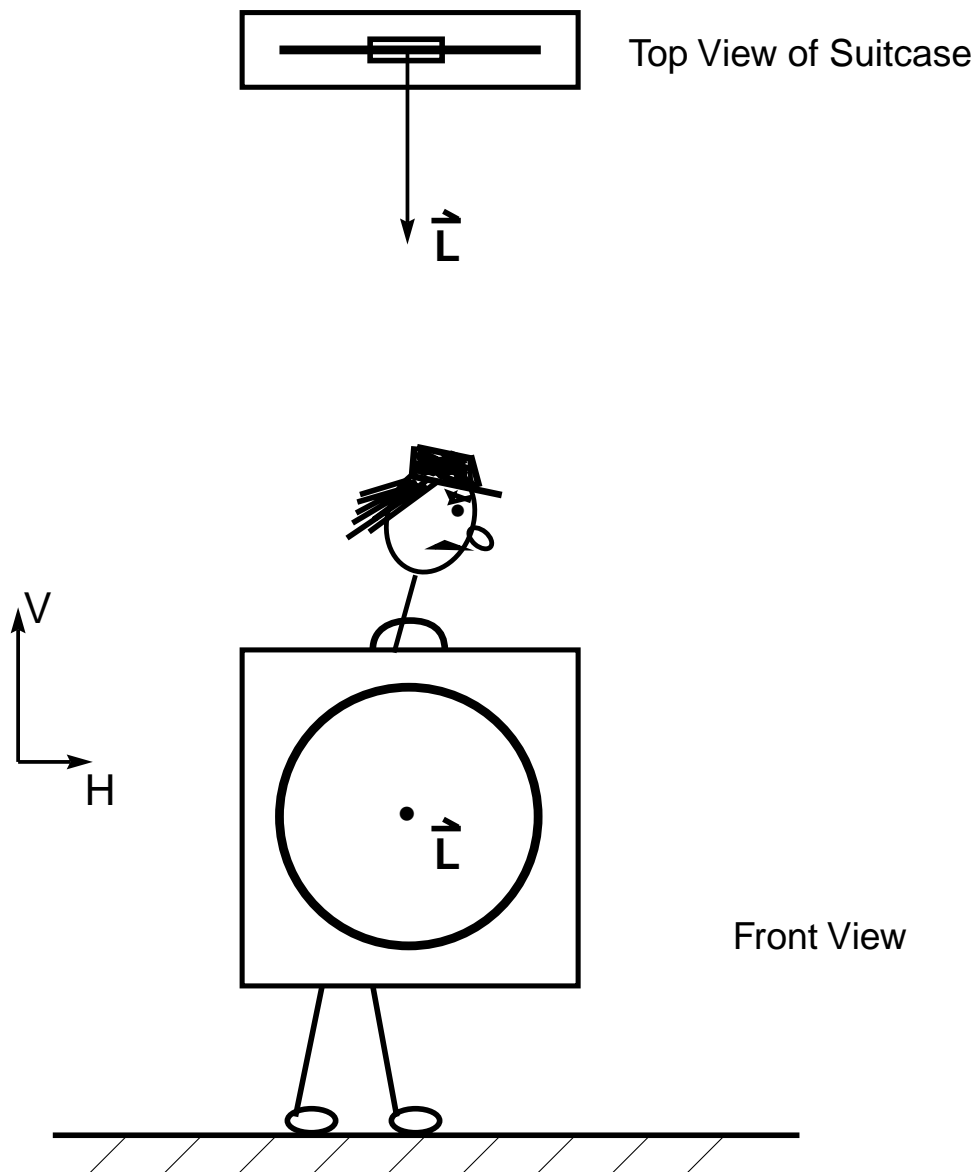


Fig. 8. Sam Smart (played by you) applies a torque to the suitcase so as to initiate a sharp turn to his left, while at the same time keeping the suitcase vertical.

### III. CONSERVATION OF ANGULAR MOMENTUM

According to N2R, for the relatively simple experiments of this lab

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{\mathbf{L}}_o)/(\Delta t) = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t) = I_o \vec{\alpha}_o \dots\dots\dots(1)$$

Consider the left hand side of this equation :

$$\vec{\tau}_o = \lim_{\Delta t \rightarrow 0} (\Delta \vec{\mathbf{L}}_o)/(\Delta t) \dots\dots\dots(2)$$

In the turntable experiments of this section we shall be concerned with the vertical (call them z) components of this **vector** equation, Eq. 3:

$$\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{\mathbf{L}}_z)/(\Delta t) \dots\dots\dots(3)$$

where  $\vec{\tau}_z$  is the *externally* applied torque along the z axis *on some system*, and  $\vec{\mathbf{L}}_z$  is the angular momentum on that *system* along the z axis. Note that the gravitational force  $\vec{\mathbf{F}}$  on system by Earth acts along the z-axis and therefore cannot exert a torque along that axis. (This is evident from the defining relation  $\vec{\tau} \equiv \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ . Here  $\vec{\tau}$  must be perpendicular to both  $\vec{\mathbf{r}}$  and  $\vec{\mathbf{F}}$ , so if  $\vec{\mathbf{F}}$  is along z then  $\vec{\tau}$  due to  $\vec{\mathbf{F}}$  must be perpendicular to z. (More physically, it's easy to see that a force along z cannot cause an angular acceleration  $\vec{\alpha}$  about z and since  $\vec{\tau}$  and  $\vec{\alpha}$  must be in the same direction there can be no  $\vec{\tau}$  along z.)

Suppose you stand on a low-friction turntable holding a monkey as in Fig. 9 (INITIAL STATE). Let the *system* be "monkey plus you plus turntable" ("myt"). If no *external* torque  $\vec{\tau}_z$  (along z) is applied to this *system* "myt" then Eq. (4) says that  $\Delta \vec{\mathbf{L}}_z$  for the *system* "myt" must be zero. For convenience we shall designate the z-component-of-the-total-system-angular-momentum as just " $\vec{\mathbf{L}}_{myt}$ " and the change in z-component as  $\Delta \vec{\mathbf{L}}_{myt} = \Delta \vec{\mathbf{L}}_z = 0$ . In the usual parlance of physics  $\vec{\mathbf{L}}_{myt}$  is "conserved." Since the friction in the bearing of the lab turntable is very low and at low angular velocities the air frictional resistance is negligible,  $\vec{\tau}_z$  will be indeed be negligible. Now suppose that in the INITIAL STATE you, the monkey, and the turntable are all stationary in the lab reference frame as in Fig. 9. Then  $\vec{\mathbf{L}}_{myt}$  for the system must remain zero for all time as long as  $\vec{\tau}_z = 0$ .

Suppose that you do not lift your feet from the turntable and that there is no slipping between your feet and the turntable. Then  $\vec{\omega}_y = \vec{\omega}_t$ , so that it's convenient to lump you and the turntable together with  $\vec{\omega}_y = \vec{\omega}_t = \vec{\omega}_{yt}$ . No matter how the monkey dances about and gyrates, the **total system angular momentum**  $\vec{\mathbf{L}}_{myt} = \vec{\mathbf{L}}_m + \vec{\mathbf{L}}_{yt}$  must remain zero! Thus, if we consider an INITIAL STATE at some time t, and a FINAL STATE at some later time t + Δt, then  $(\vec{\mathbf{L}}_{myt})_{initial} = (\vec{\mathbf{L}}_{myt})_{final} = 0$ , as shown in Fig. 9.



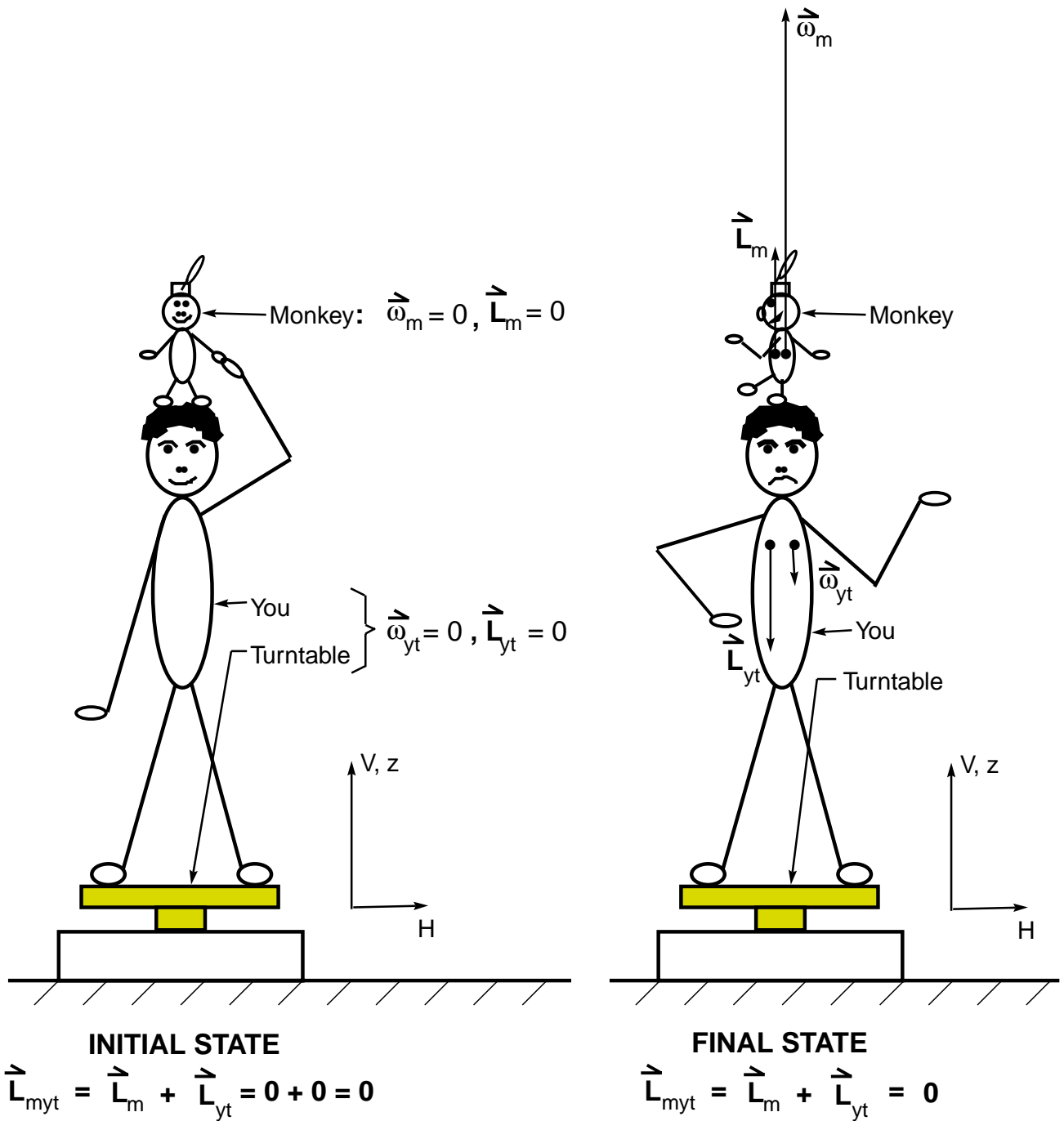


Fig. 9. Since  $\vec{\tau}_z = 0$ , the angular momentum  $\vec{L}_{myt}$  of the system "monkey plus you plus turntable" is conserved in accord with  $\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_z) / (\Delta t) = 0$ .

## A. HOLD WEIGHTS WHILE STANDING ON A TURNTABLE

### 1. THOUGHT EXPERIMENT

Suppose you were to (a) hold approximately 1.5 kg masses in each hand, (b) stand motionless with arms extended on a turntable, (c) have a partner exert a torque on you by pushing tangentially on one of your extended arms so as to give the weights, you, and the turntable a slow angular velocity  $\vec{\omega}_{\text{wyt}}$  about a vertical axis and hence an angular momentum  $\vec{L}_{\text{wyt}}$  about a vertical axis, (d) move the masses *horizontally* inward to a position close to your body. Consider the system to be "weights plus you plus turntable" (wyt). In Fig. 10 below sketch your qualitative *prediction* of the final states of  $\vec{\omega}_{\text{wyt}}$  and  $\vec{L}_{\text{wyt}}$ . (Note that in this simple case  $\vec{\omega}_{\text{wyt}}$  for system will be the same as the  $\vec{\omega}$ 's for its component parts, i.e.,  $\vec{\omega}_{\text{wyt}} = \vec{\omega}_w = \vec{\omega}_y = \vec{\omega}_t$ .)

### PREDICTION

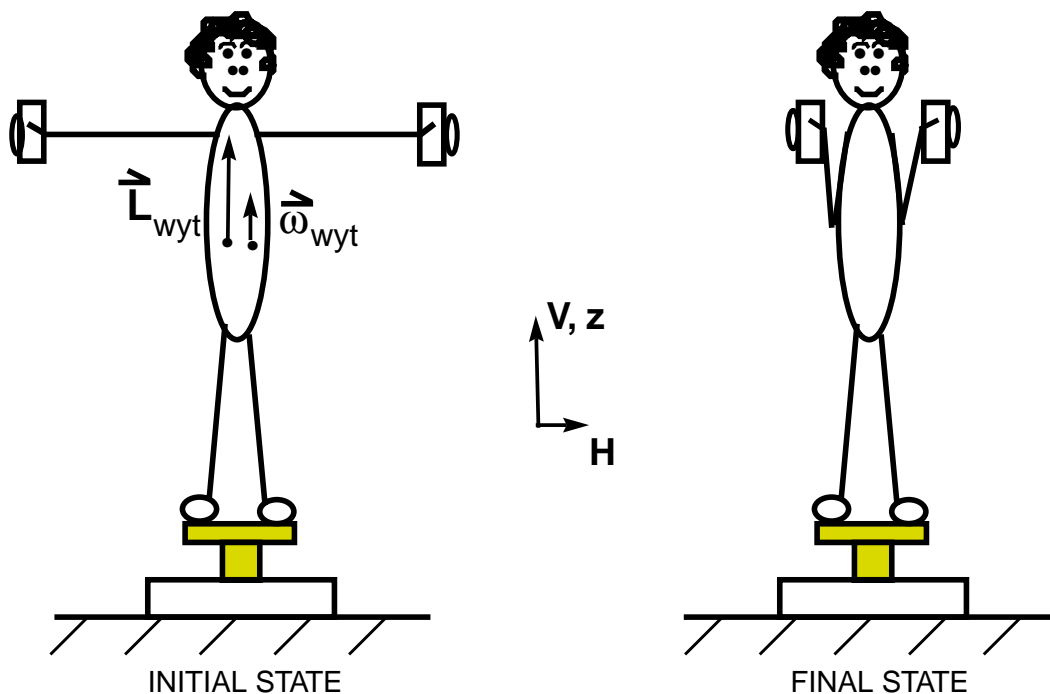


Fig. 10. You rotate on a turntable holding weights with arms extended. Then you bring the weights in close to your body. Show your qualitative prediction of the final state angular velocity  $\vec{\omega}_{\text{wyt}}$  and final state angular momentum  $\vec{L}_{\text{wyt}}$  of the weights, you, and the turntable.

2. REAL EXPERIMENT

Perform the experiment in "1" above. Show the result below.

EXPERIMENTAL RESULTS

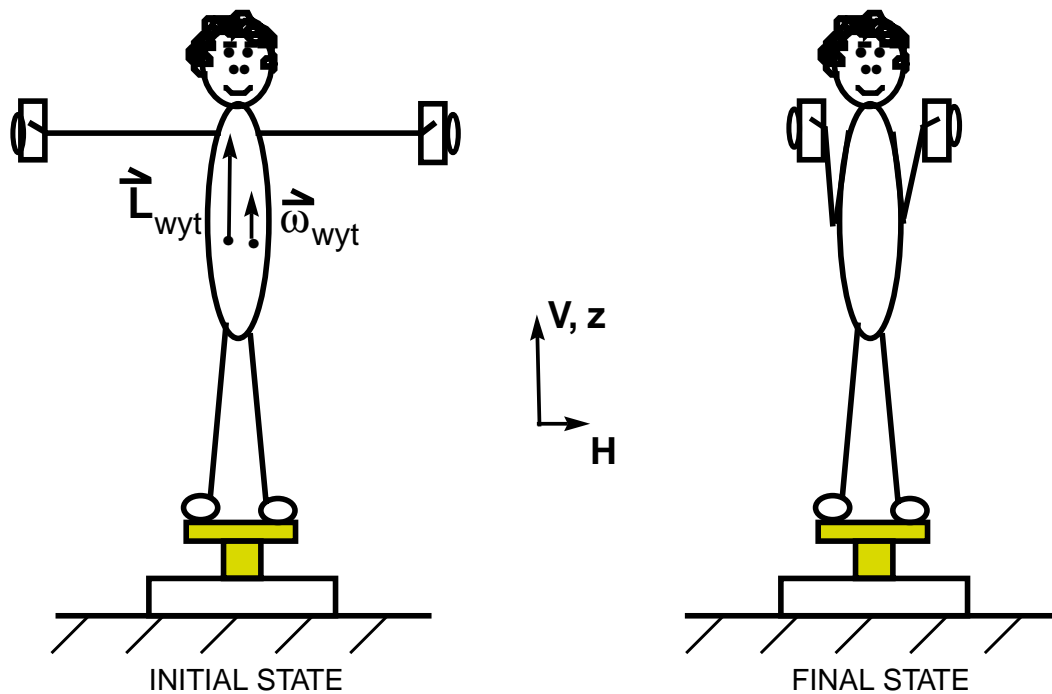


Fig. 11. You rotate on a turntable holding weights with arms extended. Then you bring the weights in close to your body. Show the qualitative results by diagramming the final state angular velocity  $\vec{\omega}_{wyt}$  and final state angular momentum  $\vec{L}_{wyt}$  of the weights, you, and the turntable.

a. Is your prediction verified? {Y, N, U, NOT}

b. Are your experimental results in qualitative accord with N2R in the form of Eq. (3),

$$\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_z) / (\Delta t) ? \{Y, N, U, NOT\}$$

3. In the above experiment, when you bring the weights in close to your body, do you do work on the weights? {Y, N, U, NOT} [HINT: Do you have to exert a *radial* force on the weights in order to bring them in close to your body (here *radial* means along radii between your body and your outstretched hands holding the weights) ? If so, do you exert the radial force over some radial distance? What is the definition of work?]

4. Does the *kinetic* energy  $K$  of the system composed of the weights, you, and the turntable remain constant as you bring the weights in close to your body? {Y, N, U, NOT}

5. Does the *mechanical* energy  $E$  of the system composed of the weights, you, and the turntable remain constant as you bring the weights in close to your body? {Y, N, U, NOT} [HINT: Recall that  $E \equiv U + K$ ]

6. Does the *total* energy of the system composed of the weights, you, and the turntable remain constant as you bring the weights in close to your body? {Y, N, U, NOT} [HINT: *Total* energy includes *all* forms of energy including mechanical energy(kinetic and potential); "chemical energy" such as that stored in your body, an electric battery, or gasoline; and thermal energy associated with the kinetic energy of disordered random motions of individual atoms.]

7. Is the Law of Conservation Of Energy obeyed in this experiment? {Y, N, U, NOT} [Hint: According to the Law of Conservation Of Energy "Total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant."]

B. STAND MOTIONLESS ON A TURNTABLE AND ACCEPT A SPINNING WHEEL

1. THOUGHT EXPERIMENT:

Suppose you were to (a) stand motionless on a turntable and have a partner steady you so that the initial angular momentum of you plus the turntable  $\vec{L}_{yt}$  is zero and (b) have another partner set a bicycle wheel into rotation so that it has a high  $\vec{L}_{w1}$  pointed vertically upward, all as shown as shown in Fig. 12 (state #1); then (c) take the spinning wheel from your partner by grasping one end of the axle with one hand and the other end of the axle with the other hand, all as shown in Fig. 12 (state #2). (Try for a torqueless transfer in which your partner does not exert a torque upon you in giving you the wheel.) After the spinning wheel is in your hand do NOT change the orientation of the wheel – keep  $\vec{L}_w$  vertically up as it was before the transfer. In Fig. 12 show your qualitative predictions for the state #2 (a) angular momentum  $\vec{L}_w$  of the wheel, and (b) the angular momentum of the system you plus the turntable  $\vec{L}_{yt}$ . What is the total angular momentum of the system (you+ turntable+wheel) in state #1? \_\_\_\_\_; in state #2? \_\_\_\_\_.

PREDICTIONS

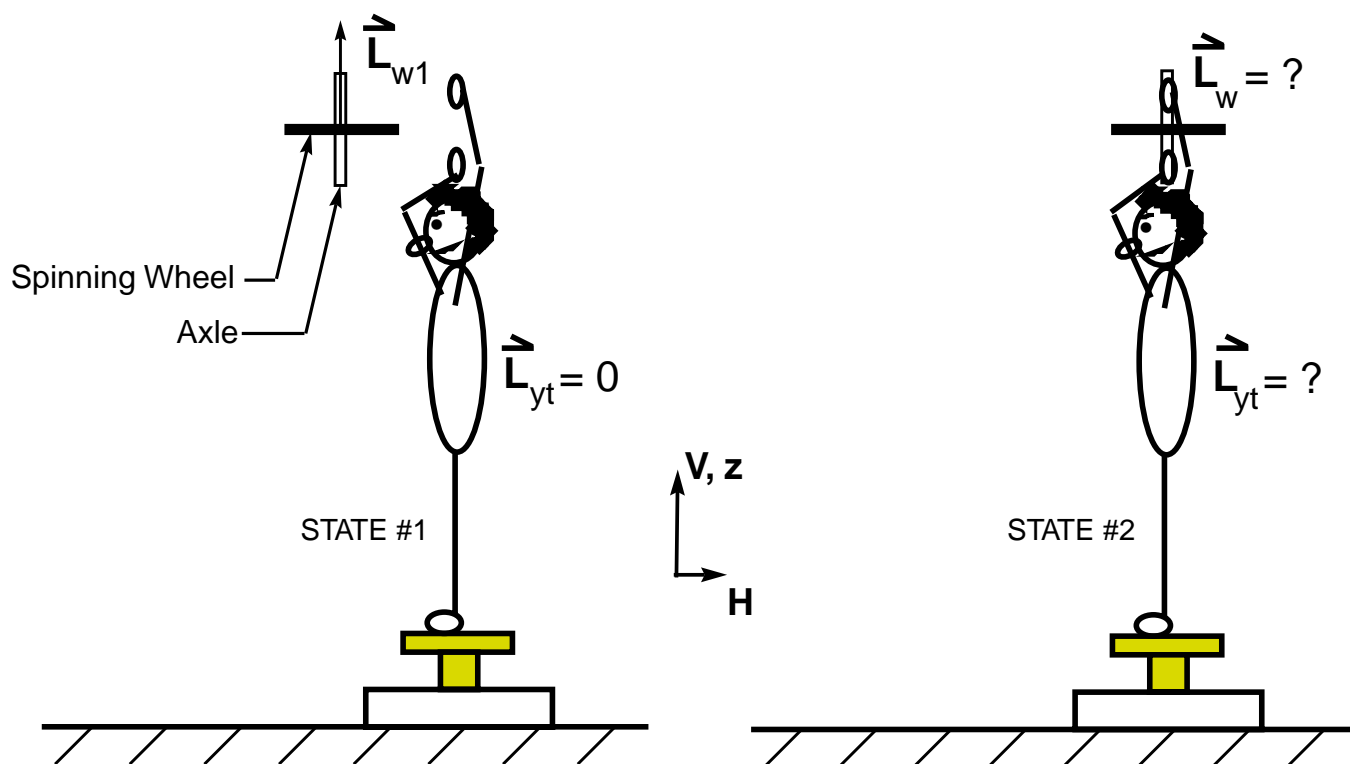


Fig. 12. You stand motionless on a turntable and accept a spinning wheel from your partner. Show your qualitative predictions for the angular momentum  $\vec{L}_w$  of the wheel and  $\vec{L}_{yt}$  of the system you plus the turntable in state #2.

2. *REAL EXPERIMENT:*

Perform the experiment indicated above three or four times so as to be sure the results are reproducible. In Fig. 13 show the qualitative results for the final state angular momentum  $\vec{L}_w$  of the wheel, and the final state  $\vec{L}_{ytw}$  of the system you, the turntable, and the wheel. In the final state you were (encircle one) rotating, not rotating, neither of these.

EXPERIMENTAL RESULTS

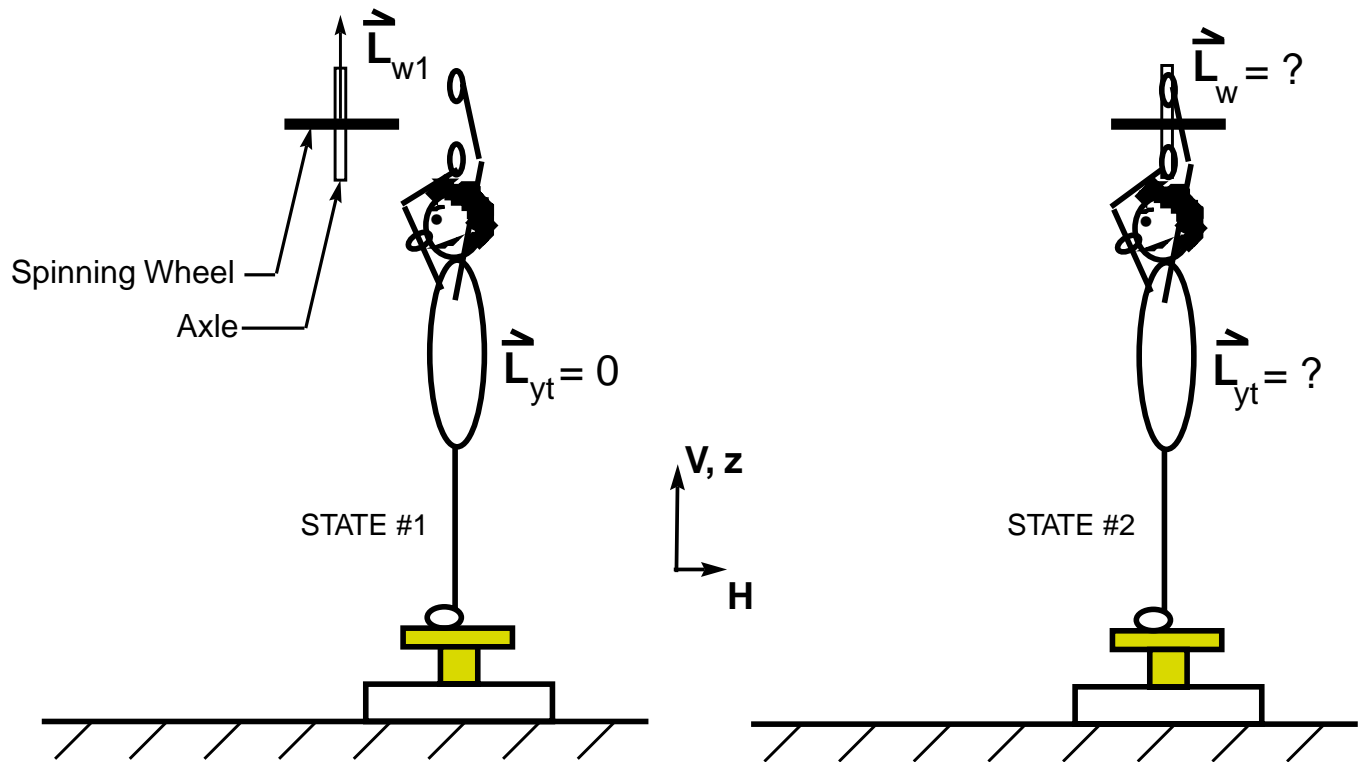


Fig. 13. You stand motionless on a turntable and accept a spinning wheel from your partner. Show your qualitative results for the angular momentum  $\vec{L}_w$  of the wheel and  $\vec{L}_{yt}$  of the system you plus the turntable in state #2.

3. Are your predictions confirmed? {Y, N, U, NOT}

4. Are your experimental results in qualitative accord with N2R in the form of Eq. (3),  $\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_z) / (\Delta t)$ ? {Y, N, U, NOT}

C. MANIPULATE A SPINNING WHEEL WHILE STANDING ON A TURNTABLE  
PREDICTIONS

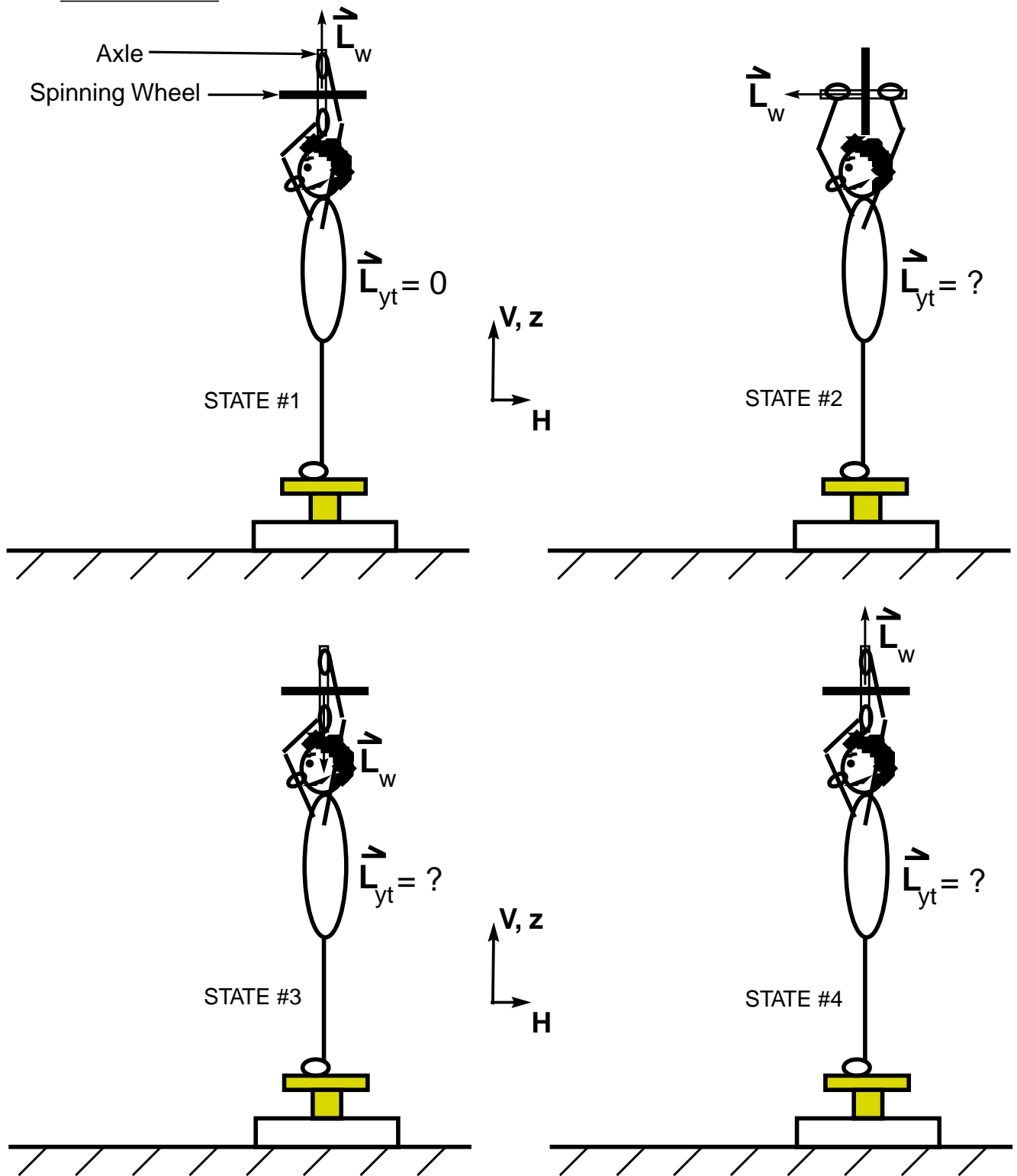


Fig. 14. You manipulate a spinning wheel through states 1, 2, 3, 4. Show your qualitative predictions for the angular momentum  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4.



1. *THOUGHT EXPERIMENT.*

Suppose you were to (a) initially stand motionless on a stationary turntable holding a spinning bicycle wheel above your head with its  $\vec{L}_w$  vector directed vertically up as shown in Fig. 14 (state #1), (b) turn the spinning wheel so that its  $\vec{L}_w$  vector is horizontal (state #2), then (c) turn the spinning wheel so that its  $\vec{L}_w$  vector is vertically down (state #3), then (d) turn the spinning wheel so that its  $\vec{L}_w$  vector returns to the vertically up position (state #4). In Fig. 14 show your qualitative predictions for the angular momentum of  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4. What do you predict for the total angular momentum  $\vec{L}_{ytw}$  of the system you plus turntable plus wheel in state #2? \_\_\_\_\_; in state #3? \_\_\_\_\_; in state #4? \_\_\_\_\_.

2. *REAL EXPERIMENT:*

Perform the experiment indicated above. On the next page diagram the qualitative results for the angular momentum of  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4. What is the total angular momentum  $\vec{L}_{ytw}$  of the system you plus turntable plus wheel in state #2? \_\_\_\_\_; in state #3? \_\_\_\_\_; in state #4? \_\_\_\_\_.

3. Are your predictions confirmed? {Y, N, U, NOT}

4. Are your experimental results in qualitative accord with N2R in the form of Eq. (3),  $\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{L}_z)/(\Delta t)$  ? {Y, N, U, NOT}

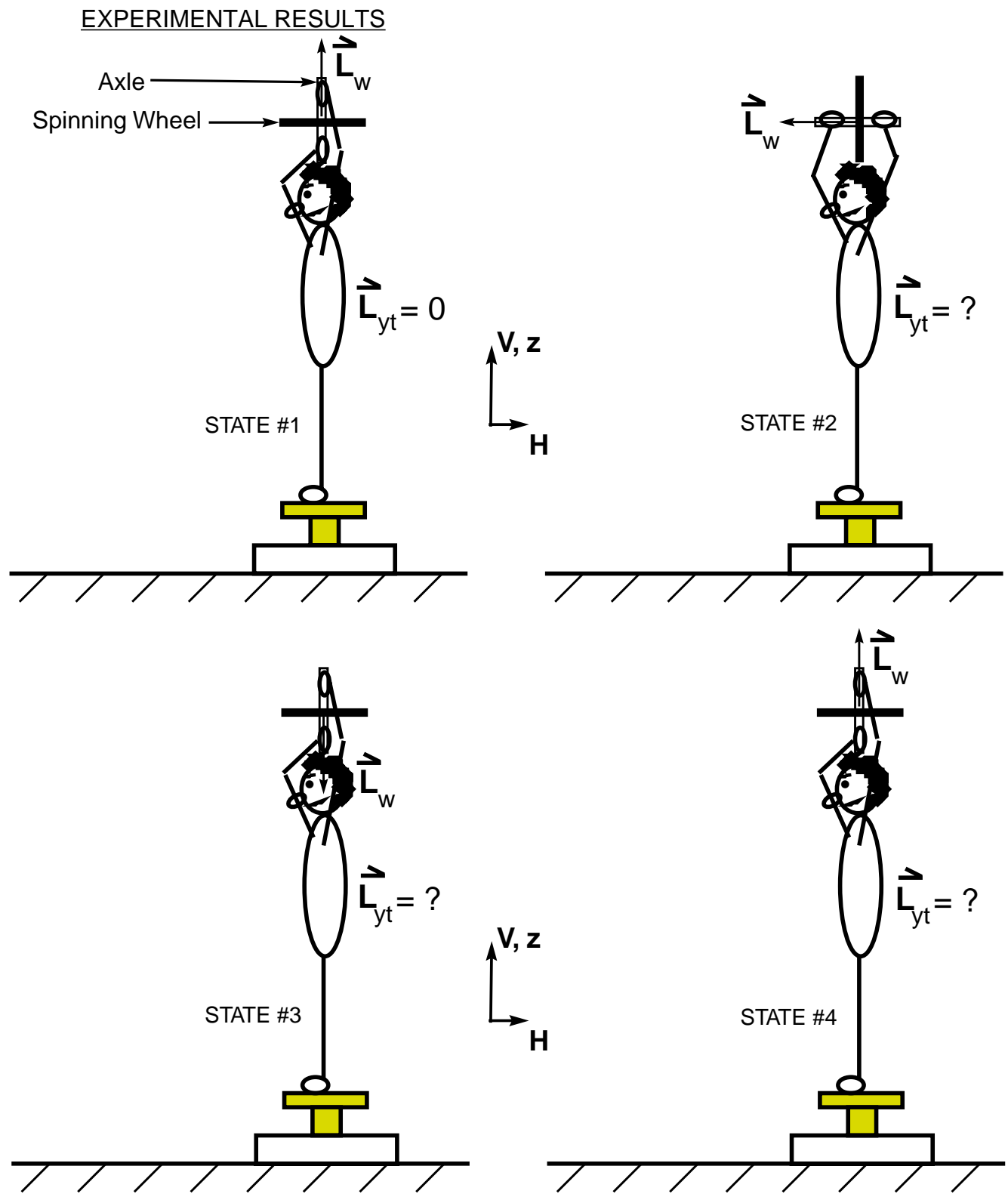


Fig. 15. You manipulate a spinning wheel through states 1, 2, 3, 4. Show the qualitative results for the angular momentum  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4.

D. SLOW A SPINNING WHEEL WHILE STANDING ON A TURNTABLE  
PREDICTIONS

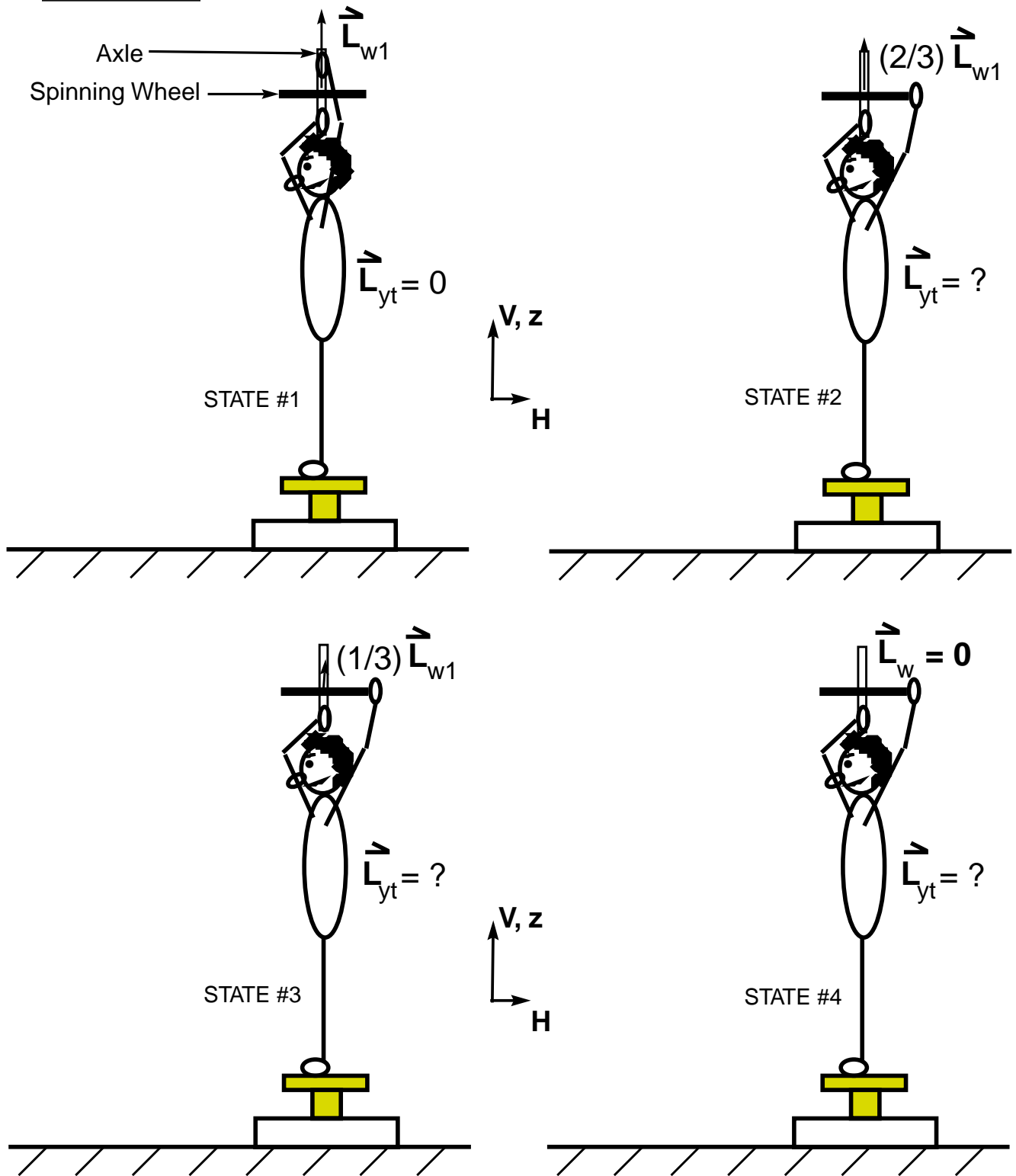


Fig. 16. You gradually slow a spinning wheel by holding your hand against it. Show your qualitative predictions for the angular momentum  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4.

1. *THOUGHT EXPERIMENT*

a. Suppose you were to (a) stand motionless on a stationary turntable holding a spinning bicycle wheel above your head (as shown in state #1 of Fig. 16) with its  $\vec{\mathbf{L}}_{w1}$  vector directed vertically up (the subscript "1" stands for "state #1," (b) gradually slow the spinning wheel by allowing the tire to rub against your free hand as shown in states #2, #3, and #4. In Fig. 16 show your qualitative predictions for the angular momentum of  $\vec{\mathbf{L}}_{yt}$  of the system you plus the turntable in states 2, 3, and 4. What do you predict for the total angular momentum  $\vec{\mathbf{L}}_{ytw}$  of the system you plus turntable plus wheel in state #2? \_\_\_\_\_; in state #3? \_\_\_\_\_; in state #4? \_\_\_\_\_ .

2. *REAL EXPERIMENT:*

Perform the experiment indicated above. On the next page, Fig. 17, diagram the qualitative results for the angular momentum of  $\vec{\mathbf{L}}_{yt}$  of the system you plus the turntable in states 2, 3, and 4. What is the total angular momentum  $\vec{\mathbf{L}}_{ytw}$  of the system you plus turntable plus wheel in state #2? \_\_\_\_\_; in state #3? \_\_\_\_\_; in state #4? \_\_\_\_\_ .

3. Are your predictions confirmed? {Y, N, U, NOT}

4. Are your experimental results in qualitative accord with N2R in the form of Eq. (3),  $\vec{\tau}_z = \lim_{\Delta t \rightarrow 0} (\Delta \vec{\mathbf{L}}_z) / (\Delta t)$ ? {Y, N, U, NOT}

5. In the above experiment, does the *kinetic* energy of the system composed of the wheel, you, and the turntable remain constant as you slow the spinning wheel by allowing the tire to rub against your free hand? {Y, N, U, NOT}

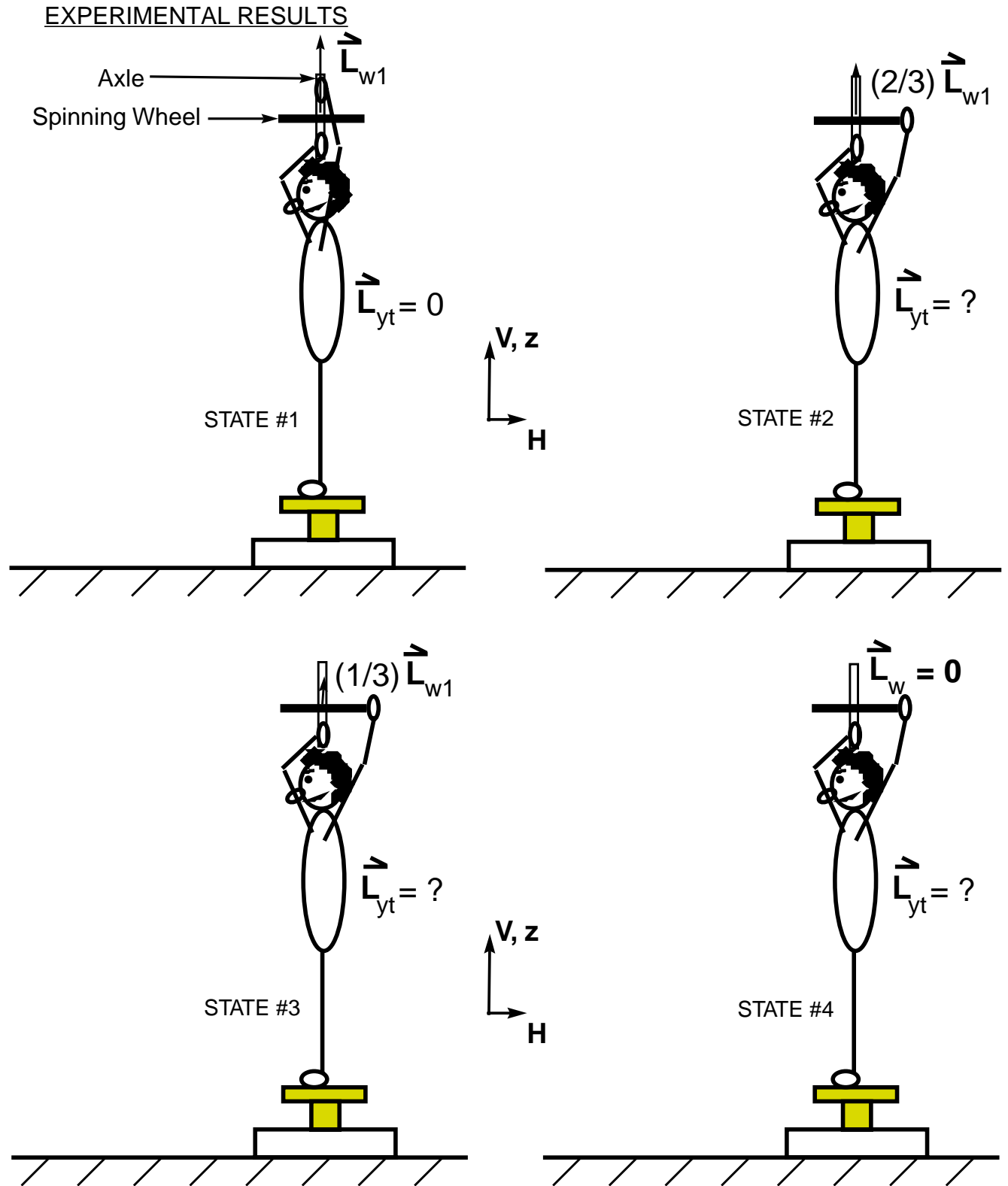


Fig. 17. You gradually slow a spinning wheel by holding your hand against it. Show the qualitative results for the angular momentum  $\vec{L}_{yt}$  of the system you plus the turntable in states 2, 3, and 4.

6. Does the *mechanical* energy of the system composed of the wheel, you, and the turntable remain constant as you slow the spinning wheel ? {Y, N, U, NOT}

7. Does the *total* energy of the system composed of the wheel, you, and the turntable remain constant as you slow the spinning wheel ? [HINT: Does your breaking hand become warm as you slow the wheel?] {Y, N, U, NOT}

8. Is the Law of Conservation of Energy obeyed in this experiment? {Y, N, U, NOT}

## E. THE CAT TWIST

1. Consider Fig. 18 below. At (a) a cat is held with angular momentum  $\vec{L} = I\vec{\omega} = 0$  and then dropped. By means of contortions (b) - (e) the cat does a  $180^\circ$  twist so that s(he) lands feet down at (f). It would appear that the angular momentum  $\vec{L} = I\vec{\omega}$  about the cat's center of mass (CM) must remain zero because there's no externally applied torque  $\vec{\tau}$  about the CM due to the gravitational force  $\vec{F}$  on cat by Earth. Even the experts have difficulty in explaining how cats manage their twists [see, e.g., Cliff Frohlich, "The Physics of Somersaulting and Twisting," *Scientific American* **242** (3), 154 (1980)] and the cats aren't talking. For now, can you simply figure out how the cat gets from (a) to (b)? {Y, N, U, NOT}

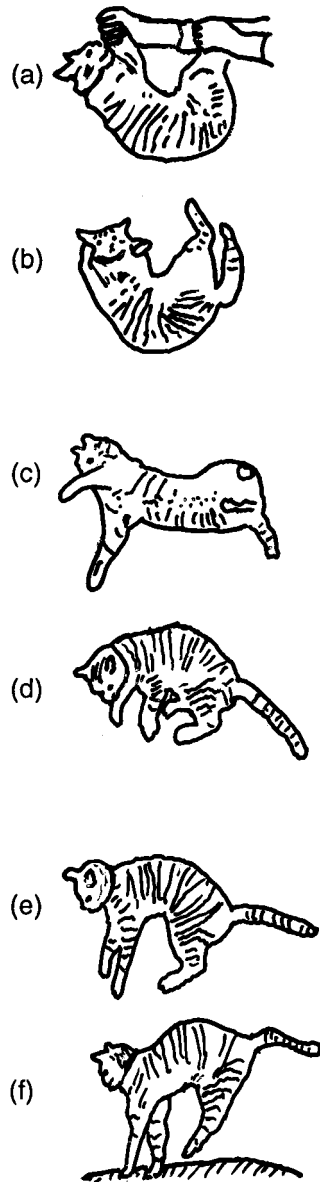


Fig. 18. A cat twist. (traced from photographs taken by Stephen Dalton).

2. Can you demonstrate a cat twist? {Y, N, U, NOT} (*Before you try this, please complete Sec. F1, 2 on p. 33.*) Have two partners hold you near the ceiling as in (a) on the preceding page and release you. Or, if you prefer, simply stand on on a turntable, Fig. 19, and do a twist about a vertical axis. Have a partner steady you so that your initial  $\vec{L} = 0$ . Draw a diagram in the space below which explains how you did your twist.

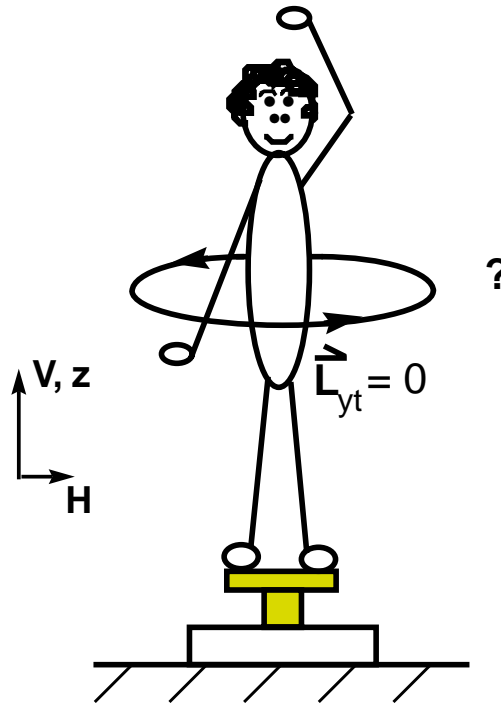
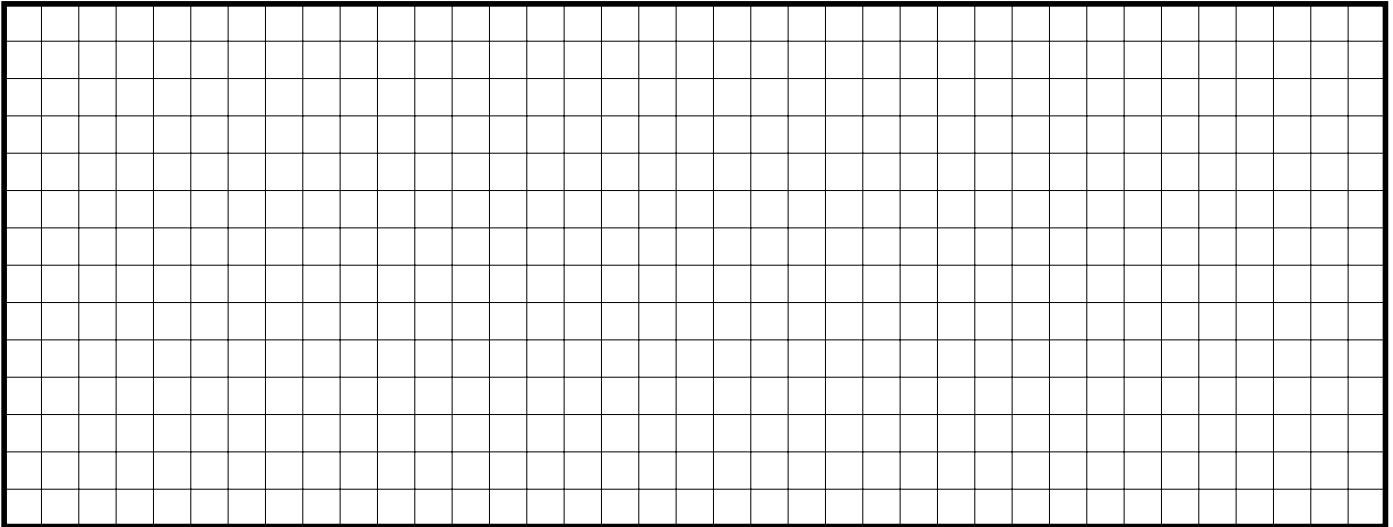


Fig. 19. A stationary student about to perform a cat twist on a turntable.

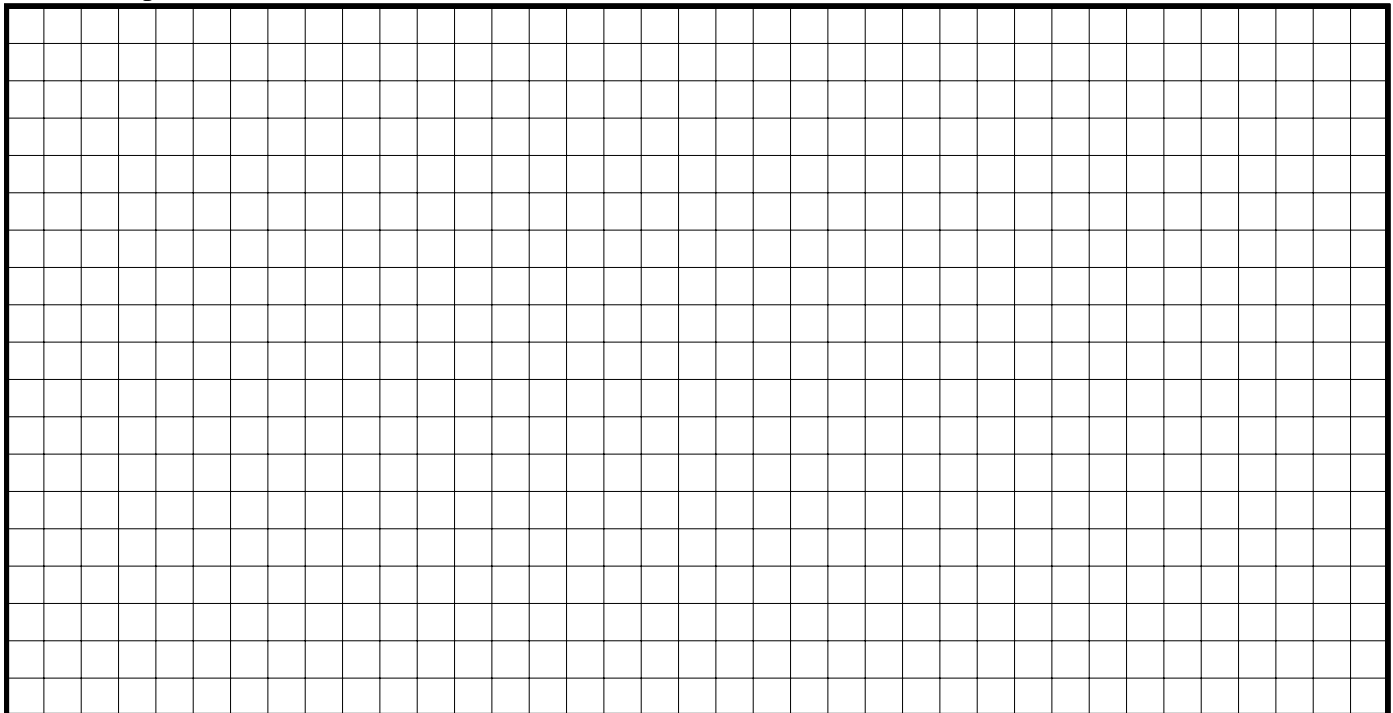


F. SKY LAB FILM CLIP

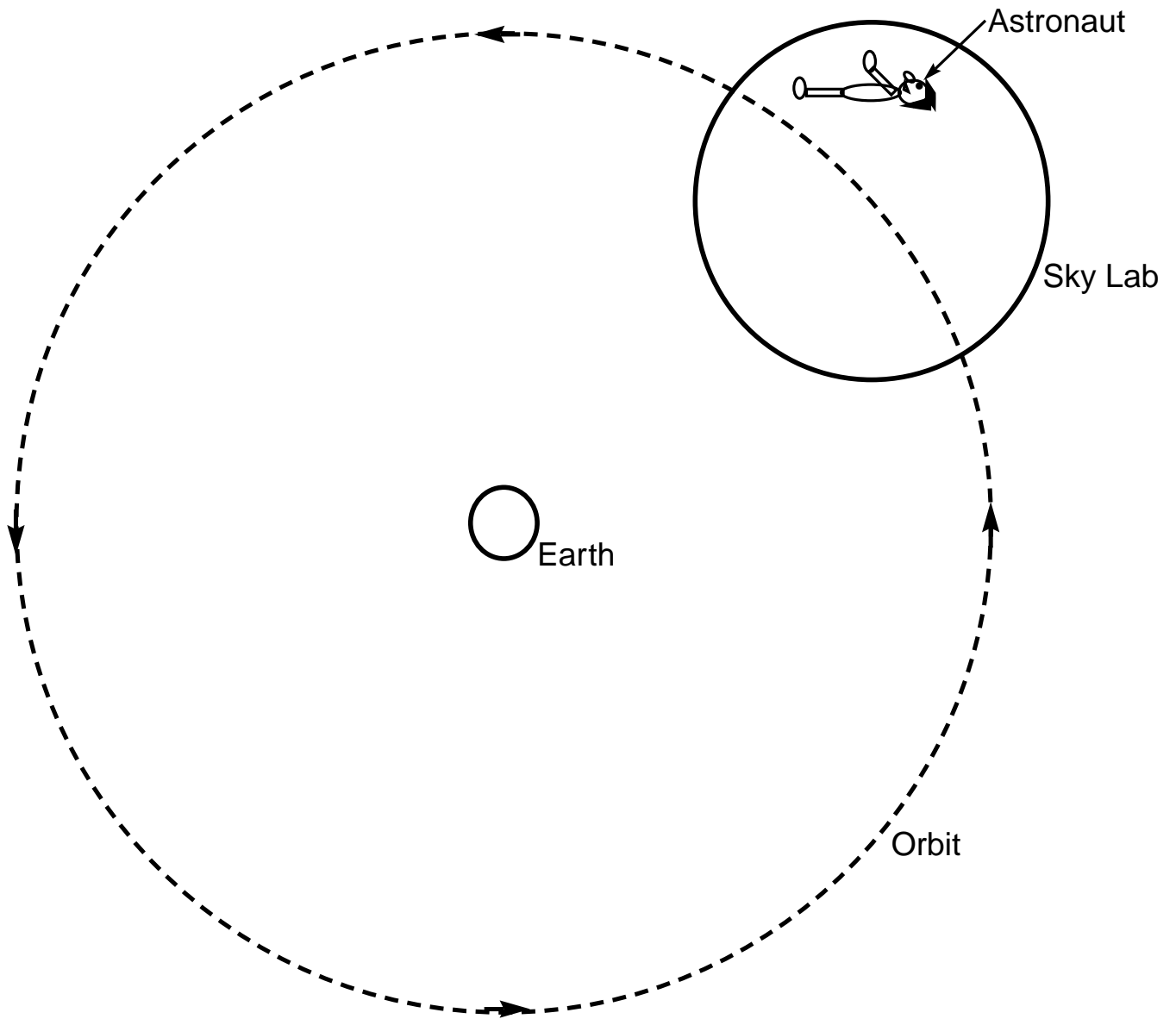
1. Study the sky lab film clip. Can you explain the "cat twist " (see Sec. E) performed by the astronaut in the sequence "Initial Conditions: zero velocity, zero rotation?" {Y, N, U, NOT} (As usual, a few labeled sketches are worth many terawords.)



2. Do you understand all the other gyrations of the astronauts? {Y, N, U, NOT} (If not then discuss them with your instructor.) You may wish to make a few sketches of some of the astronaut's movements in the space below.



3. Draw in all the force vectors acting on the astronaut in the *not-to-scale* sketch below.



4. Why do some people say that the astronauts in Sky Lab are "weightless"? (See the excellent essay by physicist-astronaut Sally Ride "Physics in Weightlessness" in D. Halliday and R. Resnick, *Fundamentals of Physics* (John Wiley, third ed., 1988), Chap. 15.)

ACKNOWLEDGEMENTS: This lab has benefited from the helpful comments of Professors Fred Lurie and James Sowinski, who served as SDI lab instructors during the Spring semester of 1993.

SDI Lab #5 Appendix : A STUDY OF NEWTON'S SECOND LAW OF ROTATION USING AN AIR SUSPENDED GYROSCOPE (ASG)\*

A. DISK AT END OF ANTENNA.

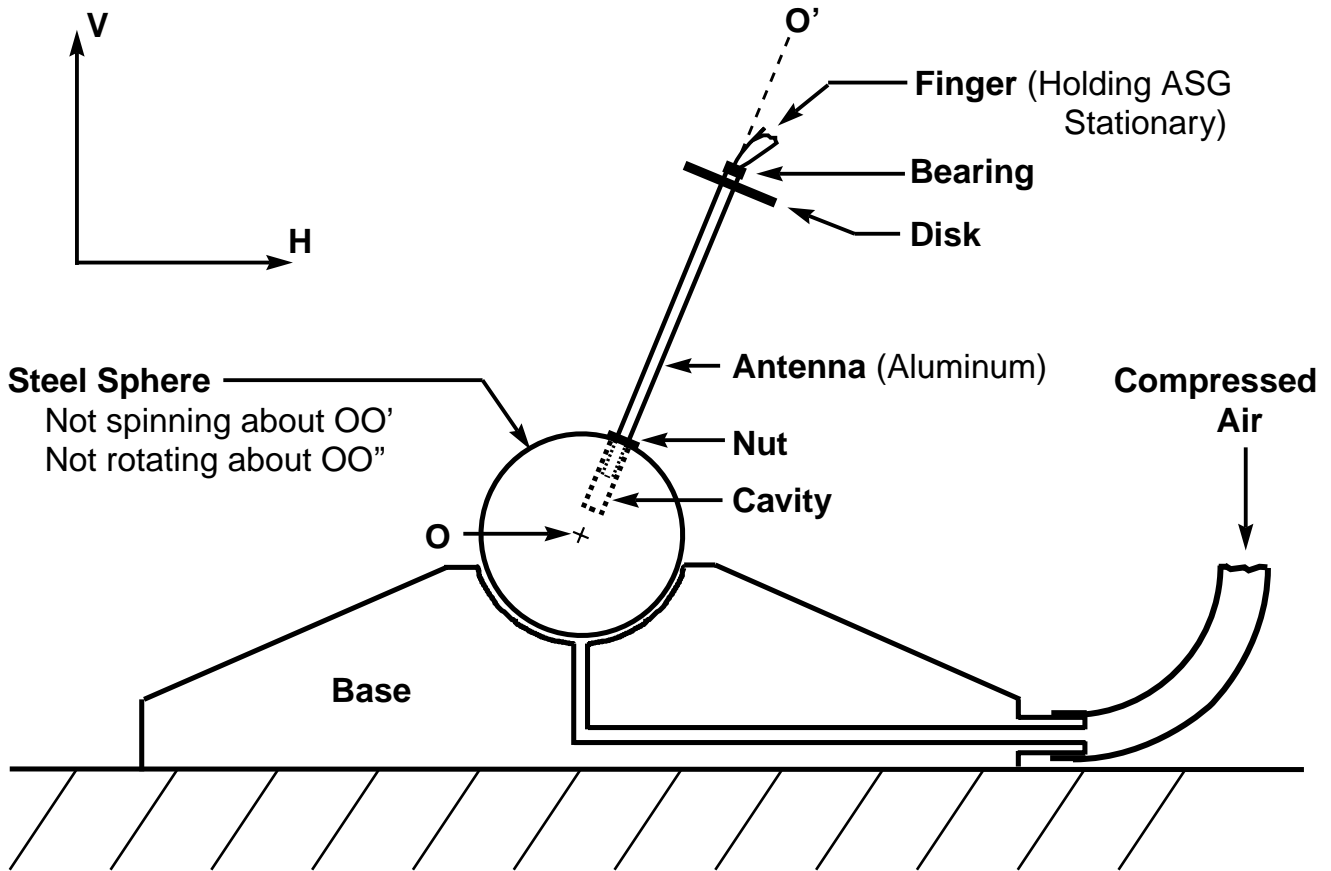


Fig. 1. The Ealing Air Suspended Gyroscope (ASG). The 4-in diameter steel ball is suspended by the flow of compressed air. Here the disk has been placed at the end of the antenna so that the ASG is *unbalanced* and would rotate *clockwise* about OO', an axis through O and perpendicular to the paper, unless held by a finger. Here and below "ASG" refers to the steel ball and its appendages: antenna, nut, bearing, and disk (if present).

1. Call in your instructor to properly adjust the compressed air flow rate to the ASG: high enough to levitate the sphere but low enough to insure "laminar flow" of the air. Under these conditions the air exerts a pressure on the sphere which is everywhere perpendicular to the surface of the sphere, the *net* force on the sphere by the air acts vertically upward at "O" the geometric center of the sphere, and the effective suspension point of the ASG is at O. O is at rest in the Earth's reference frame, and Newton's Second Law of Rotational Motion  $\vec{\tau}_O = I_O \vec{\alpha}_O$  can be applied such that the torque  $\vec{\tau}$ , the moment of inertia I, and the angular acceleration  $\vec{\alpha}$  are all taken with respect to O.

\*Covers about the same physics as Sec. IIA on the bicycle wheel.

2. Remove your finger from the ASG in the position shown in Fig. 1 and carefully observe the motion as the ASG rotates clockwise about OO', an axis through O and perpendicular to the paper in Fig. 1. Fig. 2 shows the ASG at an instant when it has rotated about halfway to the "fully down" position. In Fig. 2 show ALL the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist). [HINT: The force  $\vec{F}$  on ASG by Earth acts at the center of mass (cm) of the ASG. From symmetry, the cm must be along the axis OO'.]

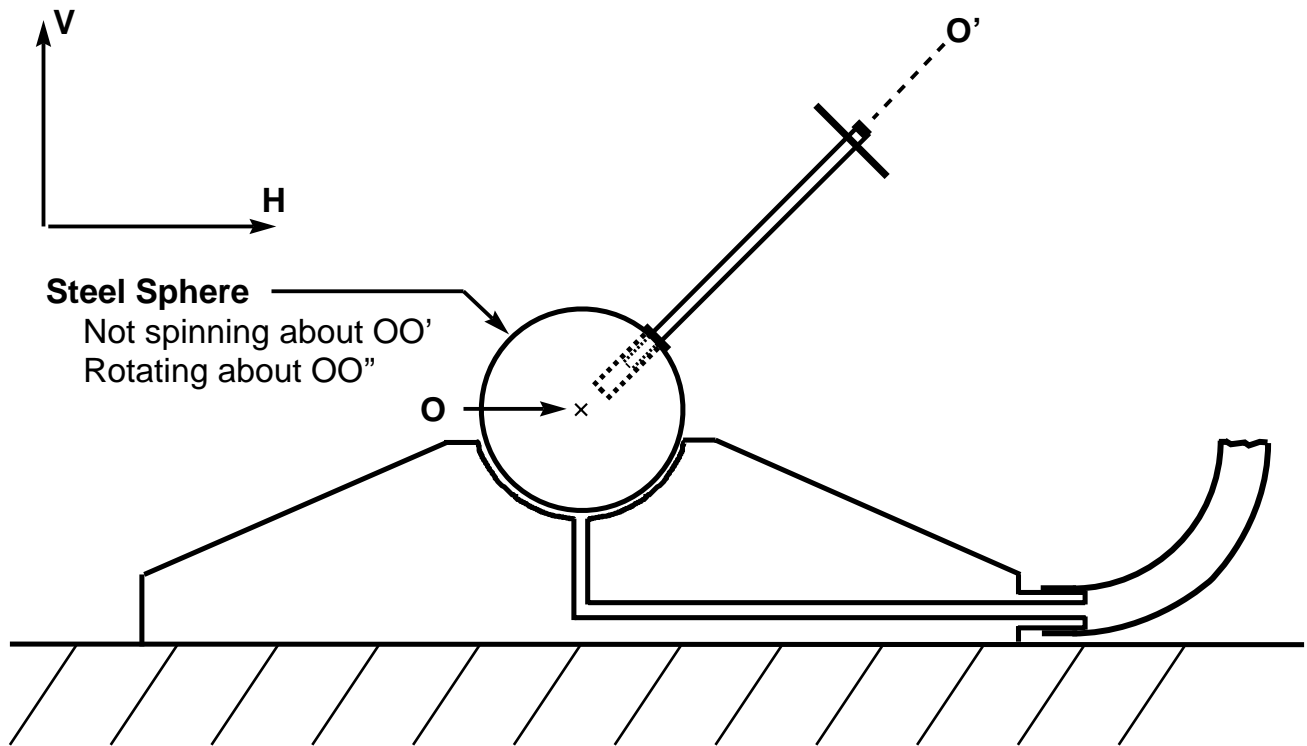


Fig. 2. The ASG has been released from the *stationary* position shown in Fig. 1 and is shown at an instant of time when it has rotated about half way to the "fully down" position.

a. Do you understand why the ASG **does** fall i.e, rotate downwards? {Y, N, U, NOT}

[HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta\vec{\omega}_o)/(\Delta t)$ . Compare with the question of SDI Lab #3, "Circular Motion and Frictional Forces," regarding water in a bucket held stationary over your head: "Do you understand why the water **does** fall out of the bucket?"]

3. *THOUGHT EXPERIMENT:*

a. Suppose you were to hold the ASG *at rest* in the position of Fig. 2 and then were to set it spinning with its spin angular velocity  $\vec{\omega}$  vector  $\vec{\omega}_{\text{spin}}$  *upward* and along the antenna, i.e., upward and along the axis OO'. Now suppose you remove your hand. *Just after you remove your hand* would the torque  $\vec{\tau}$  applied to the ASG be the same as in Fig. 2 where the non-spinning ASG is rotating downward? {Y, N, U, NOT}

b. Can you predict the direction of the precession vector  $\vec{\Omega}$  (if it exists) of the spinning ASG at a time just after you release it from the position shown in Fig. 2? {Y, N, U, NOT} [HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t)$ .] Show your predicted  $\vec{\Omega}$  in Fig. 3 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta \vec{\omega}$ , and  $\vec{\alpha}$  vectors.

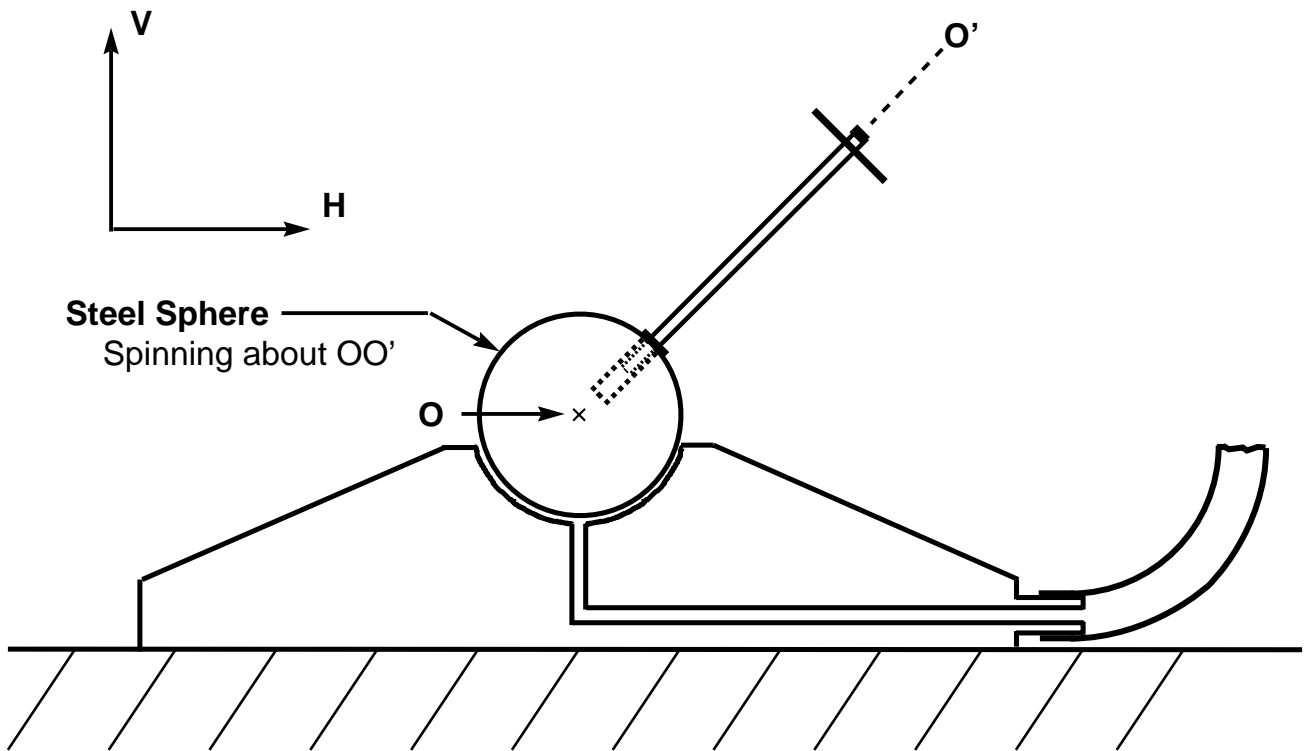


Fig. 3. The spinning ASG with  $\vec{\omega}_{\text{spin}}$  *upward* and along the antenna at a time just after having been released from the position shown in Fig. 2. Indicate your *predictions* for the vectors indicated in the thought experiment of "3" above.

4. *REAL EXPERIMENT*: Do the experiment indicated in "3" above. To set the ASG spinning with a high  $\vec{\omega}$ ,<sup>†</sup> hold the bearing between the fingers of one hand and rotate the sphere rapidly with the other. Use the right hand rule to be sure you impart an  $\vec{\omega}$  *upward* and along the antenna. Show the observed  $\vec{\Omega}$  in Fig. 4 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ , and  $\vec{\alpha}$  vectors.

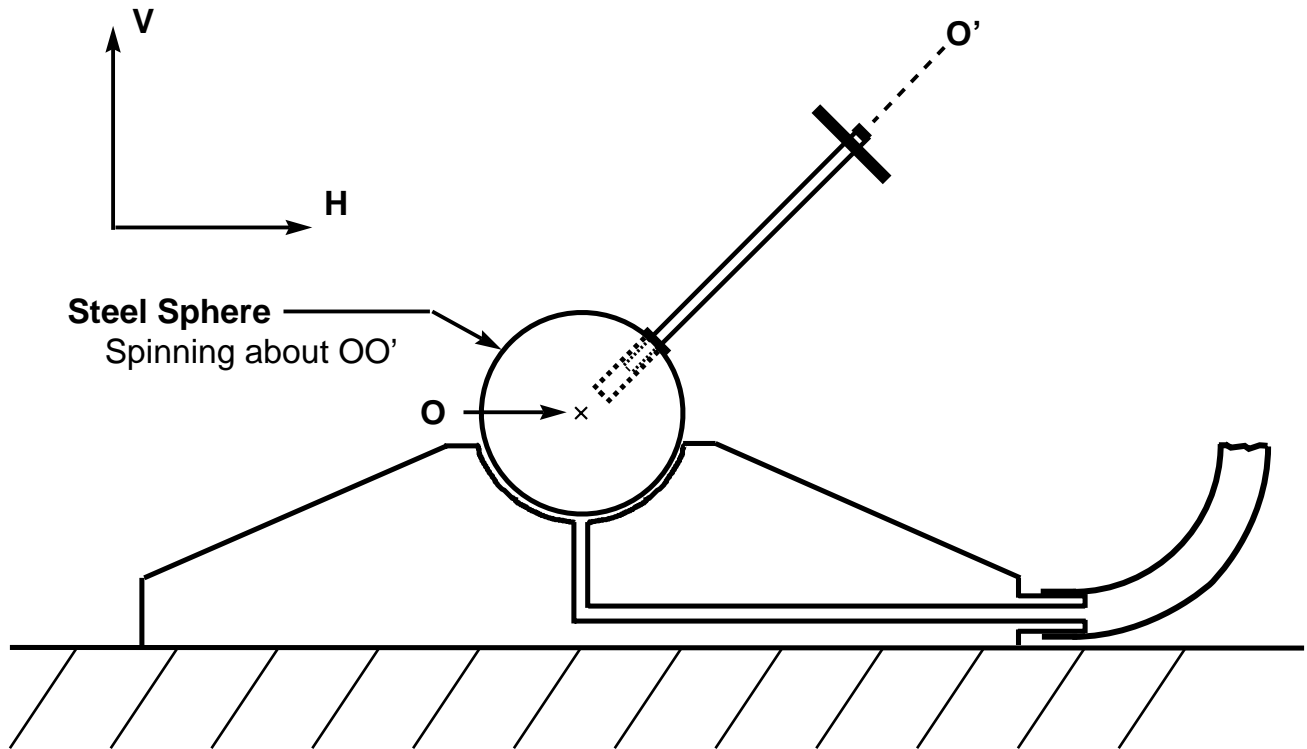


Fig. 4. Same as Fig. 3: The spinning ASG with  $\vec{\omega}_{\text{spin}} = \vec{\omega}$  (see footnote) *upward* and along the antenna at a time just after having been released. Indicate the vectors observed in the real experiment of "4" above.

Are the vectors as shown in Fig. 4 in agreement with your predictions in Fig. 3? {Y, N, U, NOT}

<sup>†</sup> If  $\vec{\Omega}$  exists then the net angular velocity  $\vec{\omega}$  of the ASG is the vector sum of  $\vec{\omega}_{\text{spin}}$  and  $\vec{\Omega}$ , i.e.,  $\vec{\omega} = \vec{\omega}_{\text{spin}} + \vec{\Omega}$ . However, for the conditions here and below  $\vec{\omega}_{\text{spin}}$  is much greater than  $\vec{\Omega}$ , i.e.,  $\vec{\omega}_{\text{spin}} \gg \vec{\Omega}$ . Hence, for simplicity, here and below, **we shall regard  $\vec{\omega}$  as being equivalent to  $\vec{\omega}_{\text{spin}}$** . Also, for simplicity, we shall ignore the slight wobbling (nutation) of the antenna as it precesses. The nutation (studied in more advanced classes) is sensitive to the way in which the ASG is released.

5. Do you understand why the spinning ASG of "4" (Fig. 4) **does NOT** fall, i.e., rotate downward? {Y, N, U, NOT} [HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta \vec{\omega}_o)/(\Delta t)$ . Compare with the question of SDI Lab #3 regarding water in a bucket rotated rapidly over your head: "Do you understand why the water **does NOT** fall out of the bucket?"]

6. Repeat part "4" (Fig. 4) above, with the  $\vec{\omega}$  magnitude about the same but with  $\vec{\omega}$  now pointed *downward* and parallel to the antenna. Release the ASG and note the  $\vec{\Omega}$  direction. Show the observed  $\vec{\Omega}$  in Fig. 5 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta \vec{\omega}$ , and  $\vec{\alpha}$  vectors.

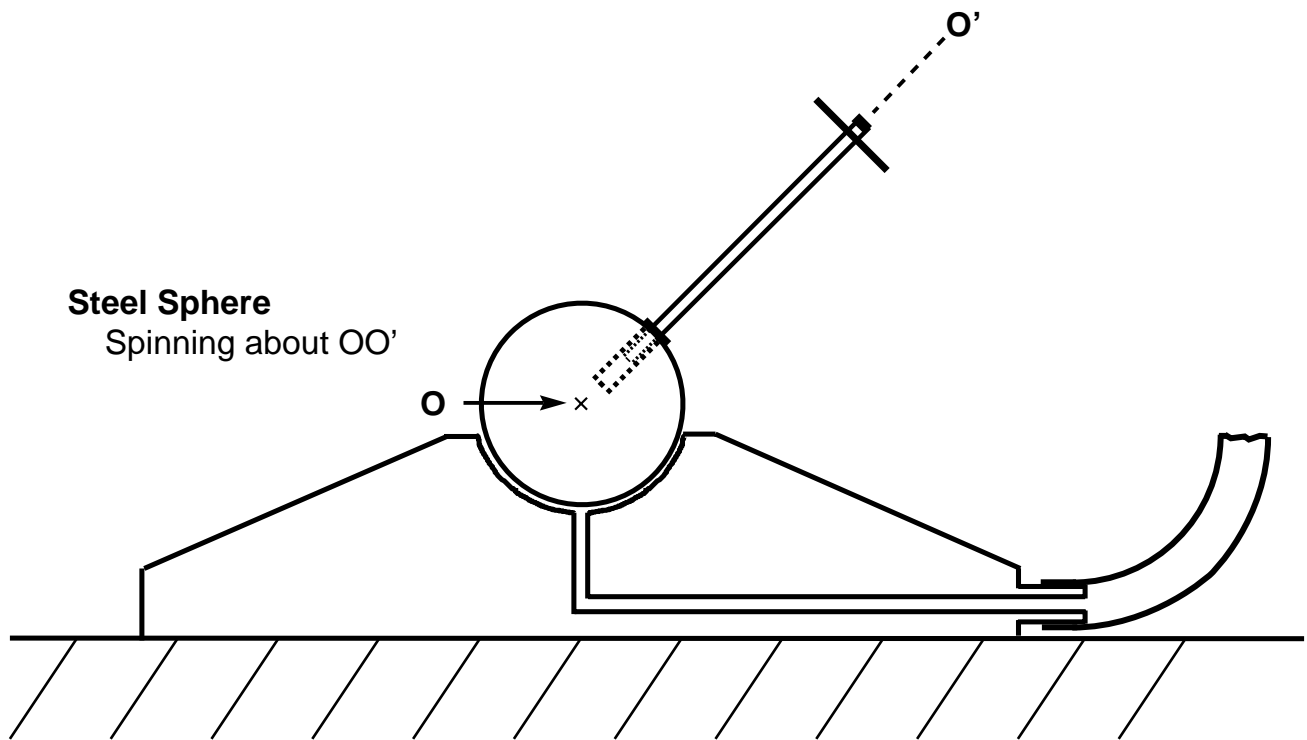


Fig. 5. The spinning ASG with  $\vec{\omega}$  *downward* and parallel to the antenna at a time just after having been released. Indicate the vectors observed in the real experiment of "6" above.

7. Considering Figs. 4 and 5, can you give a qualitative relationship between the *directions* of  $\vec{\omega}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

8. Is your relationship in qualitative accord with N2R,  $\vec{\tau}_o = I_o\vec{\alpha}_o$ ? {Y, N, U, NOT}

9. Repeat part "4" (Fig. 4) above except now spin the ASG either much slower or much faster than you did in "4." Show the observed  $\vec{\Omega}$  in Fig. 6 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ , and  $\vec{\alpha}$  vectors. Take care to display the *qualitative* magnitudes of all vectors relative to those of "5" with its much different  $\vec{\omega}$ .

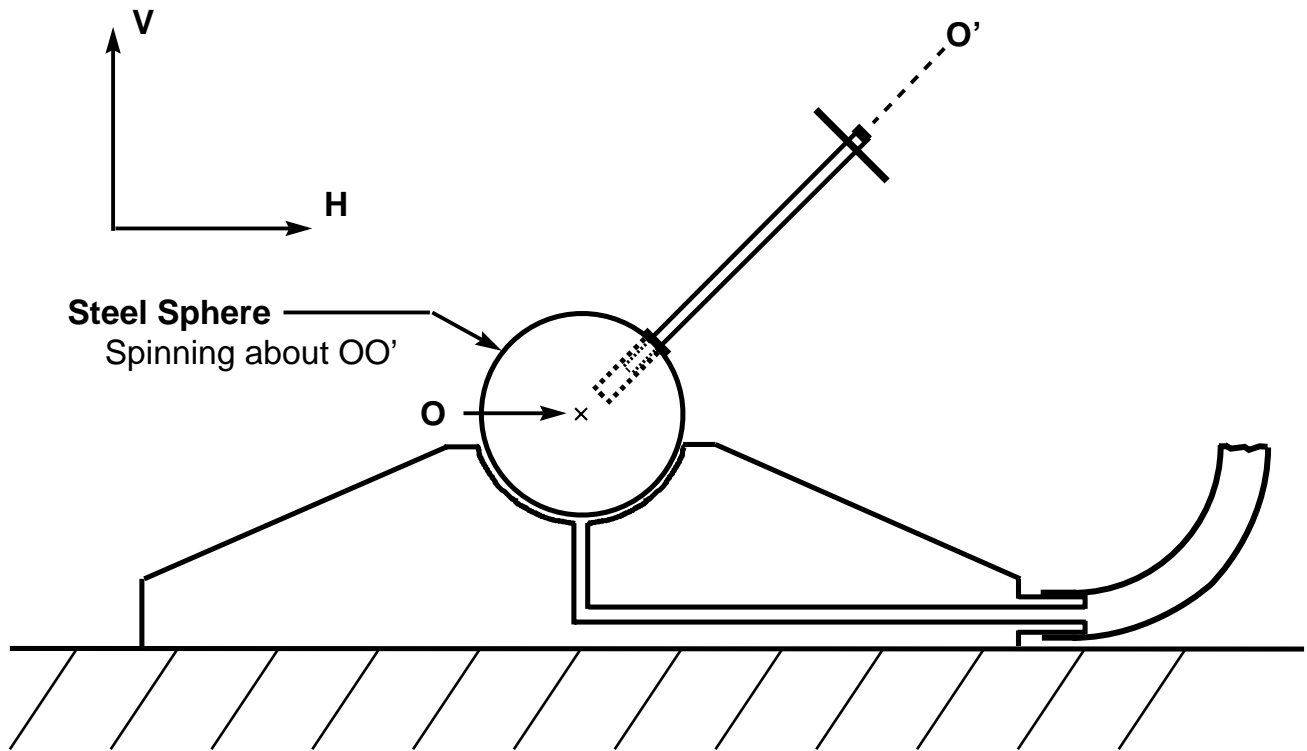


Fig. 6. The spinning ASG with  $\vec{\omega}$  *upward* and along the antenna as in Fig. 4 but with the magnitude of  $\vec{\omega}$  much different than in Fig. 4. The ASG is shown at a time just after having been released. Indicate the vectors observed in experiment of "9" above.



10. Considering Figs. 4 and 6, can you give a qualitative relationship between the *magnitudes* of  $\vec{\omega}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

11. Is this relationship consistent with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o$ ? {Y, N, U, NOT}

12. Repeat part "5" (Fig. 4) with about the same magnitude and direction of  $\vec{\omega}$ , but with one of the heavier loading disks at your table on the end of the antenna so that the torque magnitude is markedly increased. Fig. 7 shows essentially the same situation as Fig. 4, except for the heavier loading weight and thus increased  $\vec{\tau}$ . Show the observed  $\vec{\Omega}$  in Fig. 7 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ , and  $\vec{\alpha}$  vectors. Take care to display the *qualitative* magnitudes of all vectors relative to those of "4" with its much different  $\vec{\tau}$ .

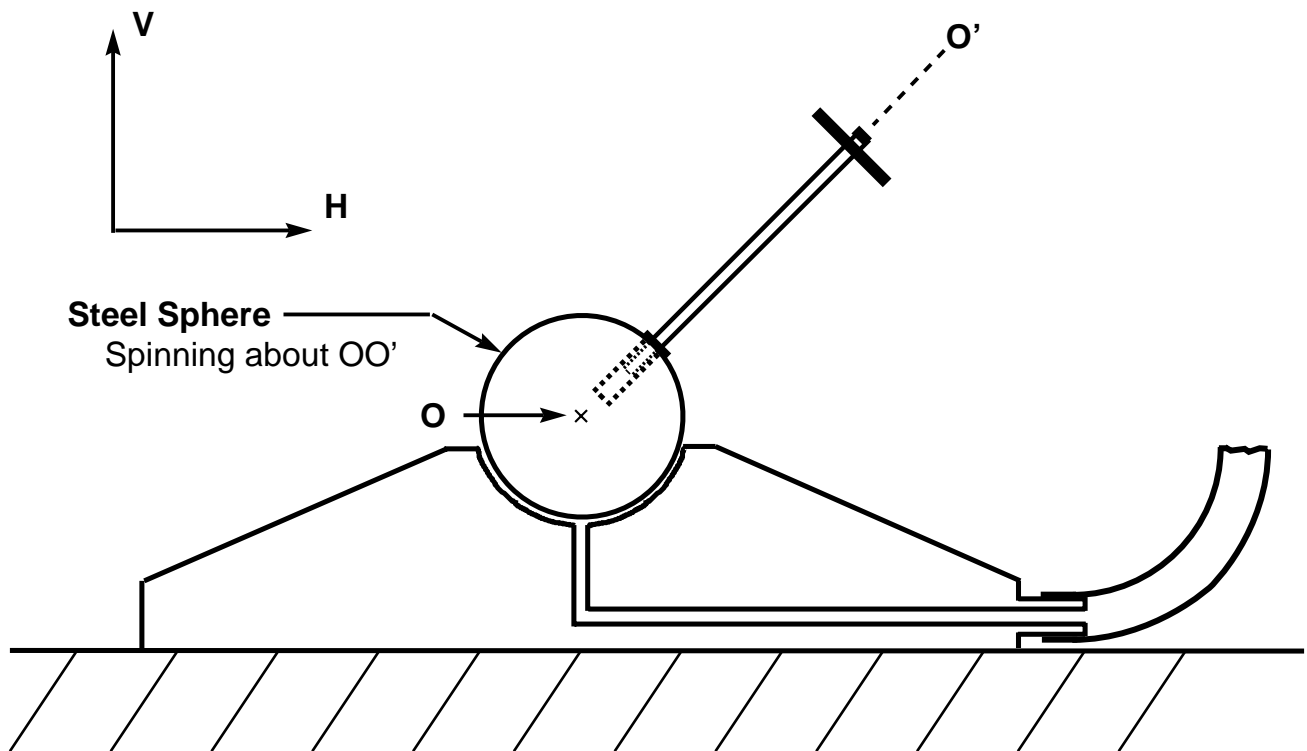


Fig. 7. The spinning ASG with the direction and magnitude of  $\vec{\omega}$  about the same as in Fig. 4 but with the magnitude of  $\vec{\tau}$  much larger than in Fig. 4 because of the heavier loading disk at the end of the antenna. The ASG is shown at a time just after having been released. Indicate the vectors observed in experiment "12" above.

14. Considering Figs. 4 and 7, can you give a qualitative relationship between the *magnitudes* of  $\vec{\tau}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

15. Is this relationship consistent with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o$ ? {Y, N, U, NOT}

B. DISK REMOVED FROM ANTENNA

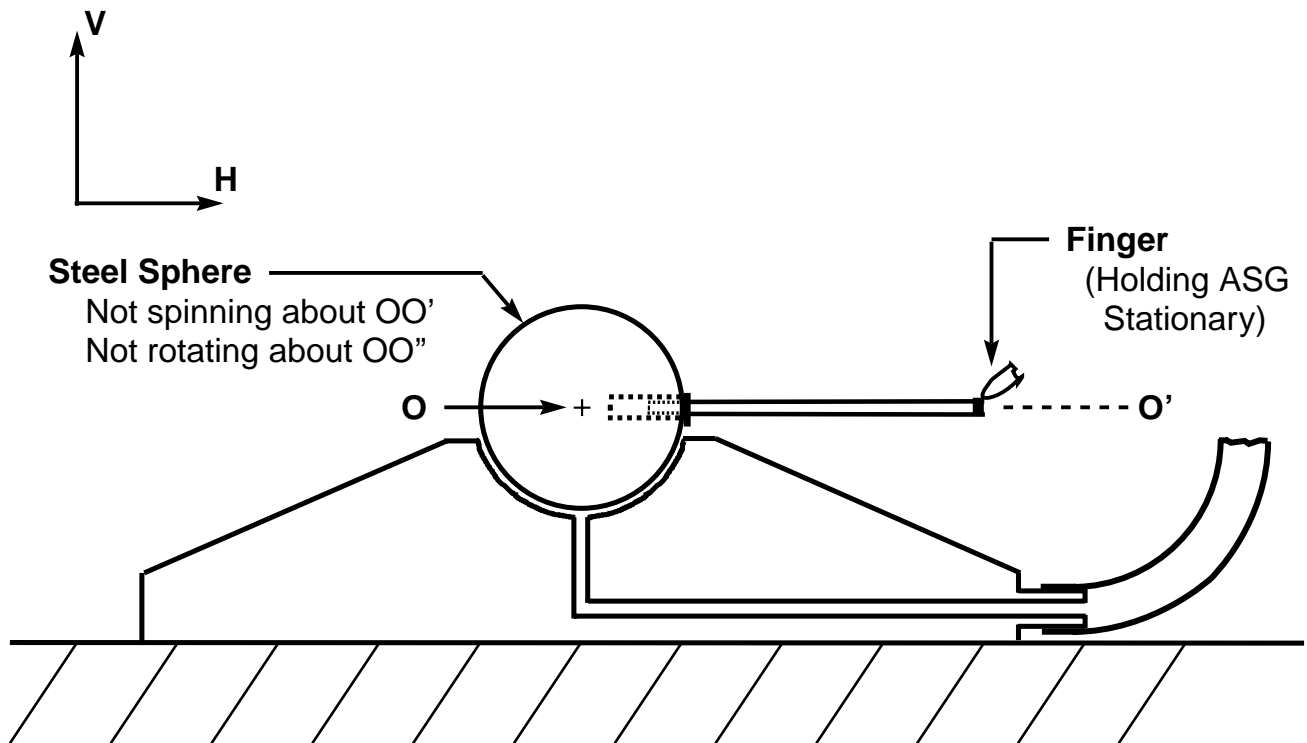


Fig. 8. All weighting disks have been removed from the end of the antenna. Because of the cavity in the steel sphere, the ASG is now unbalanced and would rotate *counterclockwise* about OO'', an axis through O and perpendicular to the paper, unless held by the finger.

1. Remove your finger from the ASG in the position shown in Fig. 8 and carefully observe the motion as the ASG rotates *counterclockwise* about  $OO''$ , an axis through  $O$  and perpendicular to the paper in Fig. 8. The ASG will oscillate about the vertical but for now, we are only interested in the initial upward rotation. Fig. 9 shows the ASG at an instant when it has rotated about halfway to the "fully up" position.

a. In Fig. 9 show ALL the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist).

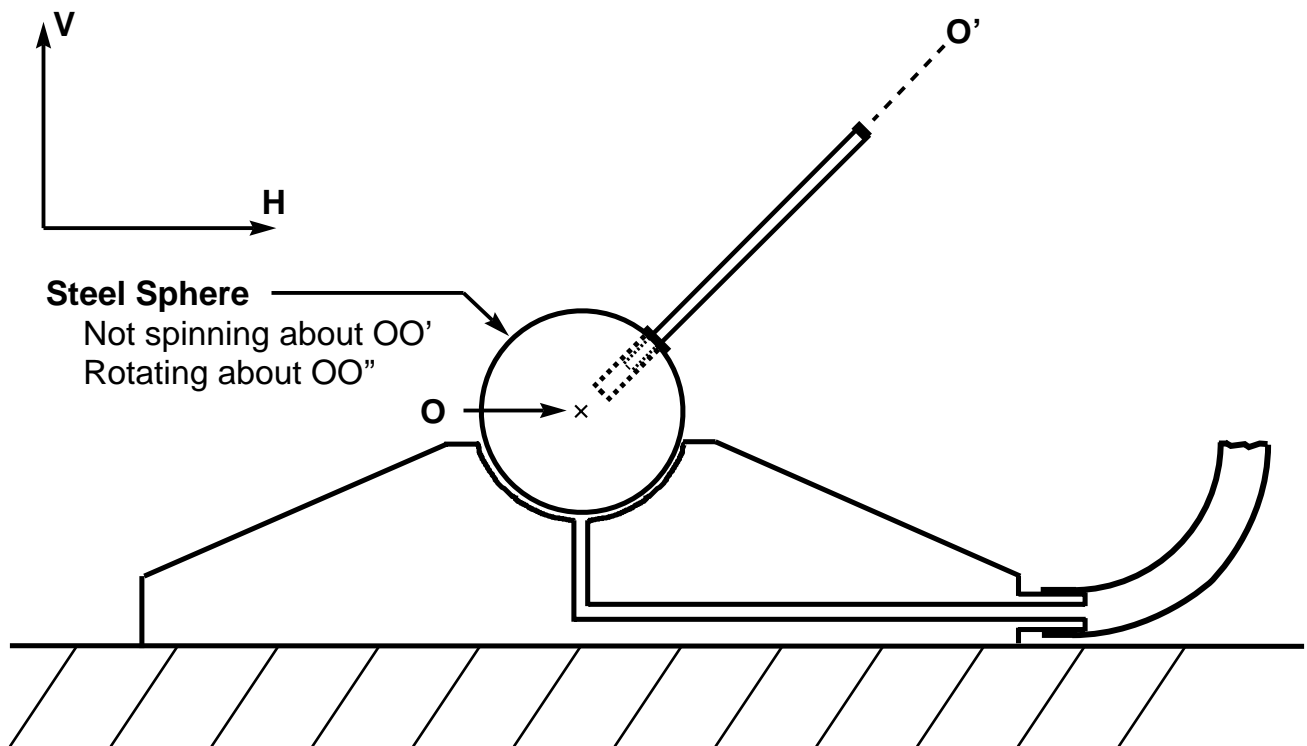


Fig. 9. The ASG has been released from the *stationary* position shown in Fig. 8 and is shown at an instant of time when it has rotated about half way to the "fully up" position.

b. Do you understand why the ASG **does** rise, i.e., rotate upwards? {Y, N, U, NOT} [HINT: Use N2R in the form  $\vec{\tau}_o = I_o \vec{\alpha}_o = I_o \lim_{\Delta t \rightarrow 0} (\Delta\vec{\omega}_o)/(\Delta t)$ . Compare with the question under Fig. 2 regarding the downwards rotation of the ASG when the antenna is weighted.]

2. Hold the ASG *at rest* in the position of Fig. 9 and then set it spinning with an angular velocity  $\vec{\omega}$  of about the same magnitude and direction as in "1e" (Fig. 4), i.e., with  $\vec{\omega}$  *upward* and along the antenna. Show the observed  $\vec{\Omega}$  in Fig. 10 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ , and  $\vec{\alpha}$  vectors.

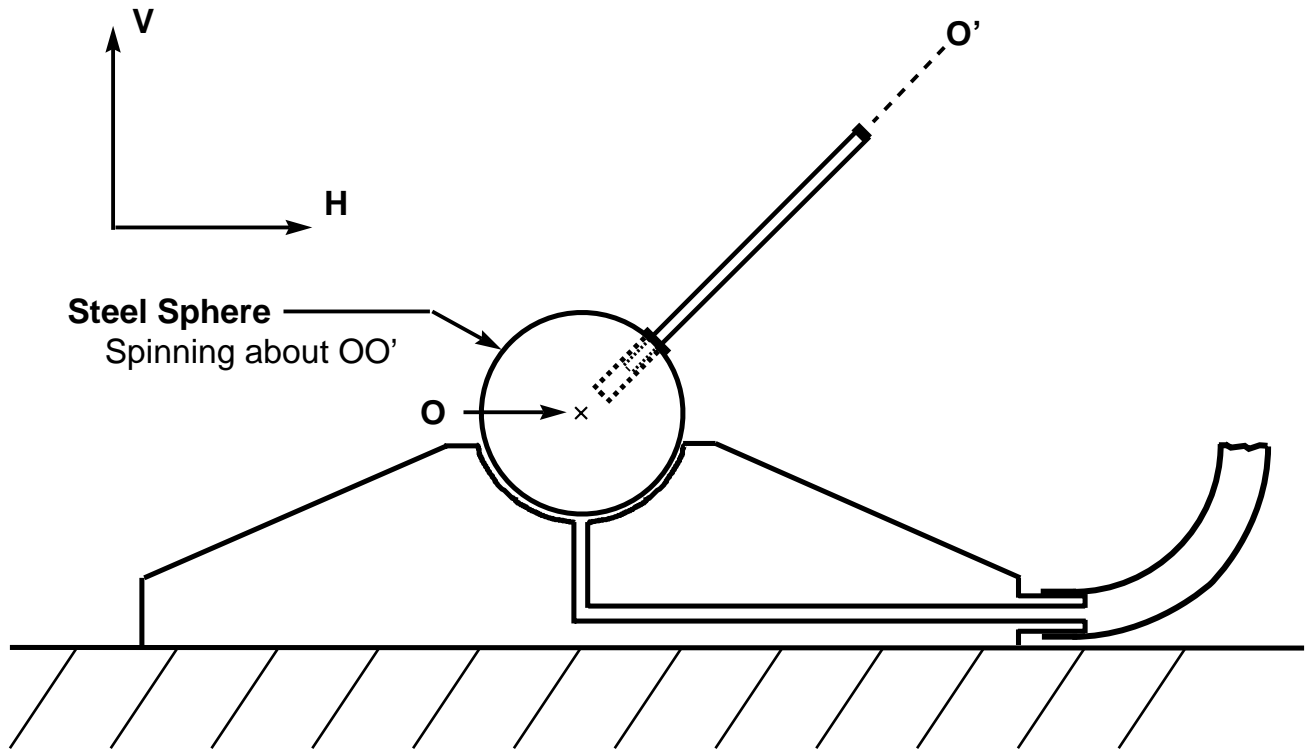


Fig. 10. The spinning ASG with the direction and magnitude of  $\vec{\omega}$  about the same as in Fig. 4 but with the direction of  $\vec{\tau}$  opposite to that in Fig. 4 because the loading weights have been removed from the end of the antenna. The ASG is shown at a time just after having been released.

3. Considering Figs. 4 and 10, can you give a qualitative relationship between the *directions* of  $\vec{\tau}$  and  $\vec{\Omega}$ ? {Y, N, U, NOT}

4. Is this relationship consistent with N2R,  $\vec{\tau}_O = I_O \vec{\alpha}_O$ ? {Y, N, U, NOT}

### C. DISK AT THE BALANCE POINT

1. Place the lighter disk of Fig. 4 back on the antenna and carefully adjust its position until the ASG remains nearly motionless for any position of the ASG. In Fig. 11 show all the  $\vec{F}$ ,  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ ,  $\vec{\Omega}$  (precessional angular velocity), and  $\vec{\alpha}$  vectors (if they exist) for this balanced non-spinning condition.

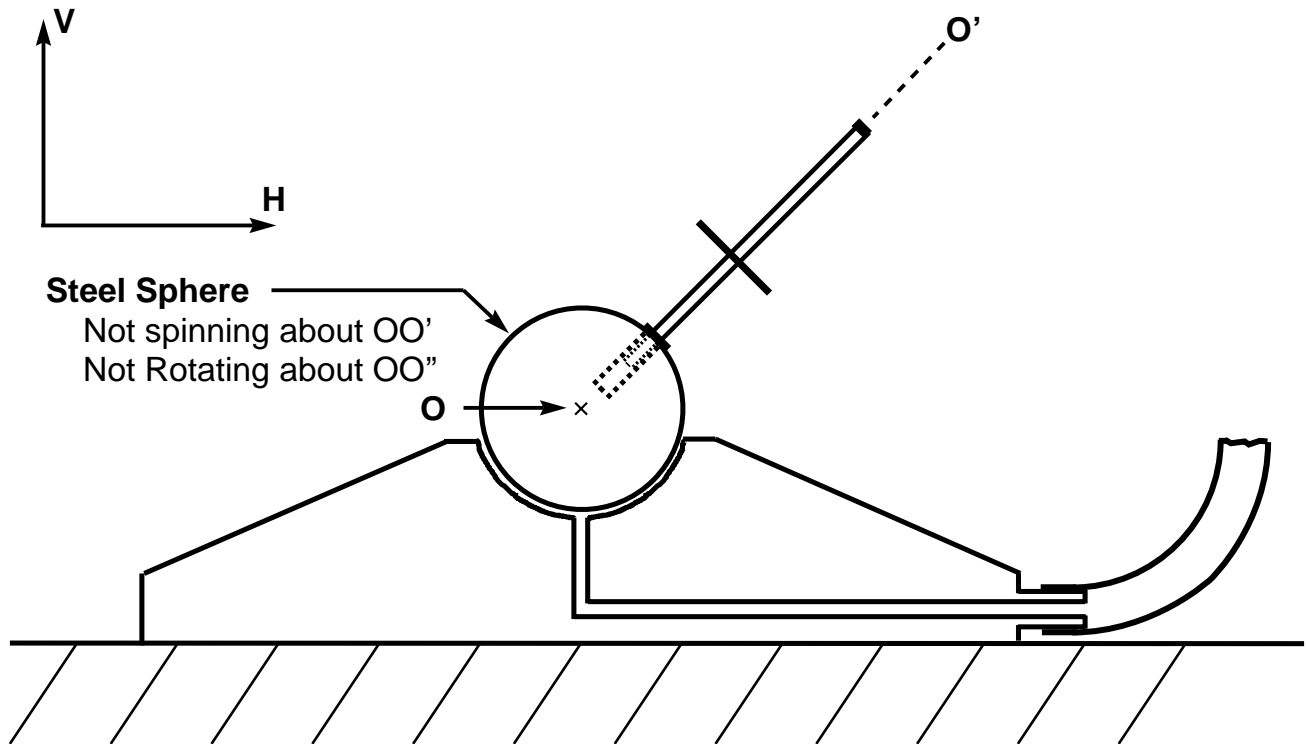


Fig. 11. Here the disk has been placed so that the non-spinning and non-rotating ASG remains nearly motionless in any position, i.e., the ASG is nearly perfectly balanced about the suspension point  $O$ .

2. Are the vectors shown by you in Fig. 11 in qualitative accord with N2R,  $\vec{\tau}_O = I_O \vec{\alpha}_O$ ?  
 {Y, N, U, NOT}

3. With the ASG *at rest* in the position of Fig. 11, set it spinning with an angular velocity  $\vec{\omega}$  of about the same magnitude and direction as in "1e" (Fig. 4), i.e., with  $\vec{\omega}$  *upward* and along the antenna. Show the observed  $\vec{\Omega}$  in Fig. 12 below, as well as the  $\vec{\tau}$ ,  $\vec{\omega}$ ,  $\Delta\vec{\omega}$ , and  $\vec{\alpha}$  vectors, if they exist.

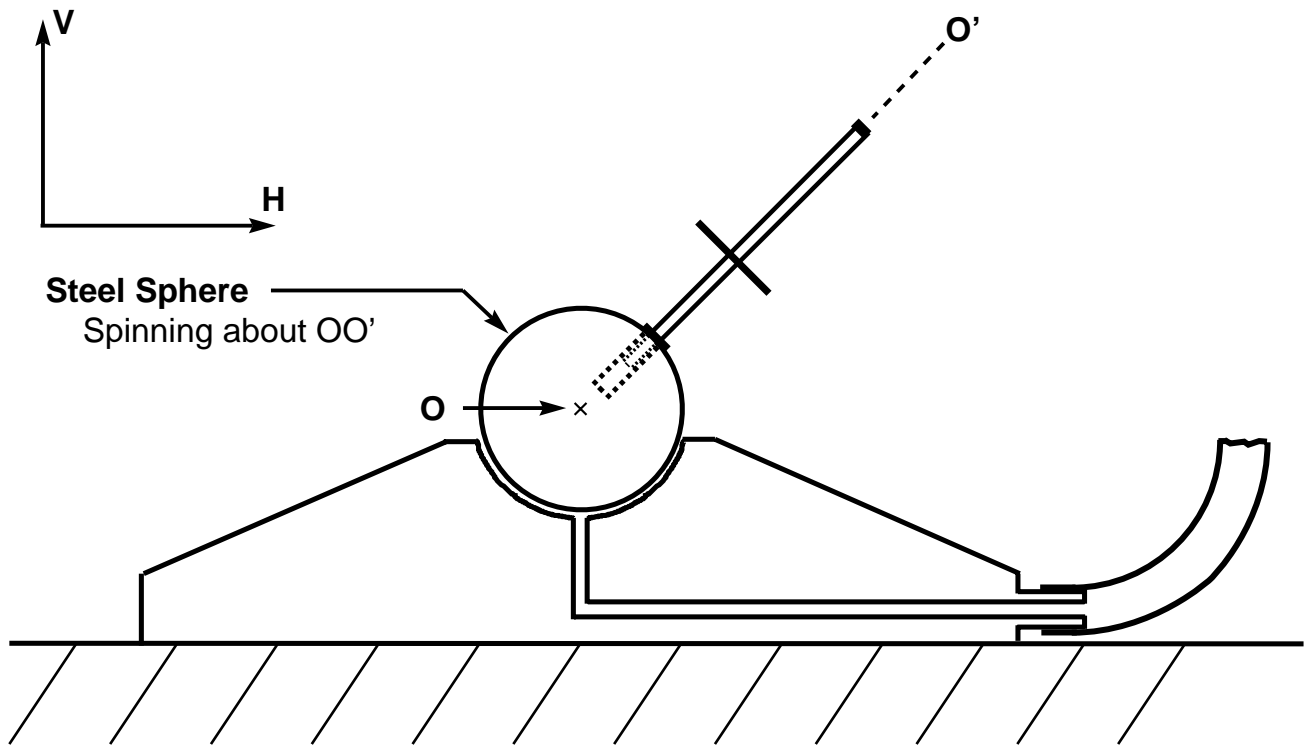


Fig. 12. The balanced ASG has been set spinning with an  $\vec{\omega}$  of about the same magnitude and direction (i.e.,  $\vec{\omega}$  is upward and along the antenna) as in Fig. 4.

4. Are the vectors shown by you in Fig. 12 in qualitative accord with N2R,  $\vec{\tau}_o = I_o \vec{\alpha}_o$ ?  
 {Y, N, U, NOT}

5. Could the ASG in the condition shown in Fig. 12 be used as a navigational gyroscope?  
{Y, N, U, NOT}

6. Suppose you were on the equator and set a *perfectly balanced* gyroscope spinning with its  $\vec{\omega}$  horizontal and first parallel and then perpendicular to the equator. Would you observe an  $\vec{\Omega}$  for either of these conditions? {Y, N, U, NOT} If so, what would be its approximate magnitude?