Fluid Flow in porous media

DARCY'S LAW

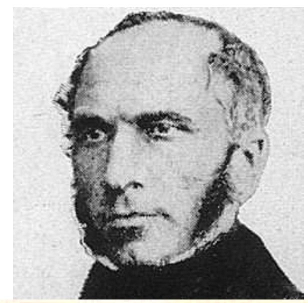
1. EXPERIMENTAL WORK
2. PERMEABILITY
3. HEAD
4. STRATIFICATION.
5. ON THE VALIDITY OF DARCY'S LAW
6. EQUATIONS.

PRACTICES

Darcy’s Law experiments

Dijon, why is it now a famous city?

- Famous industry?
- Football Team?
- Other solution?
-....



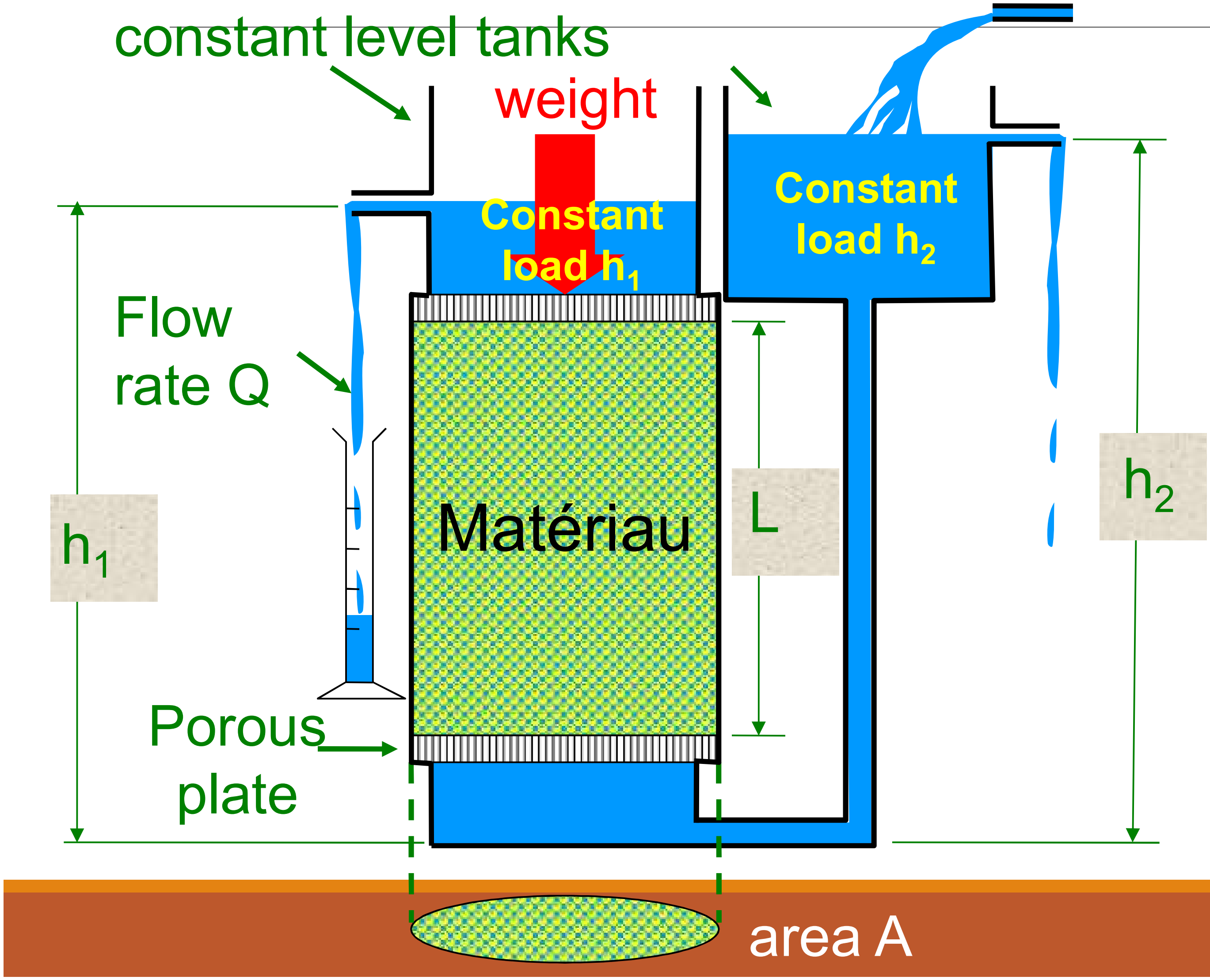
Darcy’s Law experiments

We know that groundwater flows from one point to another when there is a difference in hydraulic head h between these points. We now want to examine how fast it is doing it.

We will consider the simplest case: a regular and slow flow of water through a network of pores or cracks; network that is, if we average on a small volume, the same in each point and in each direction.

In this case one can use the law formulated by the French engineer Henry Darcy in 1856.

The law predicts the volume of water per second, the flow rate Q, which passes through a column of the material (length L, area A) if a difference in hydraulic load h2 - h1 is maintained between the two ends



$$Q \propto (h_2 - h_1)$$

$$Q \propto A$$

$$Q \propto \frac{1}{L}$$

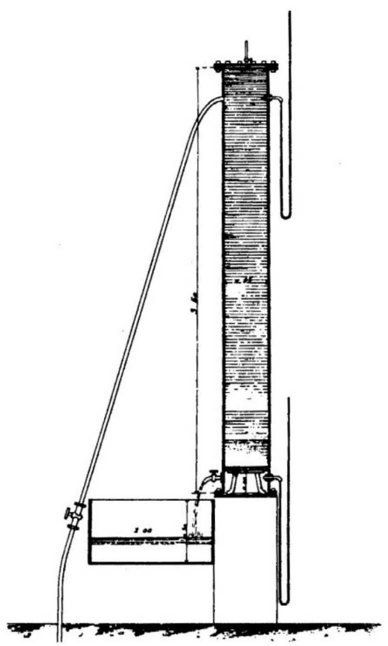
$$Q = K \frac{(h_2 - h_1)}{L} A$$

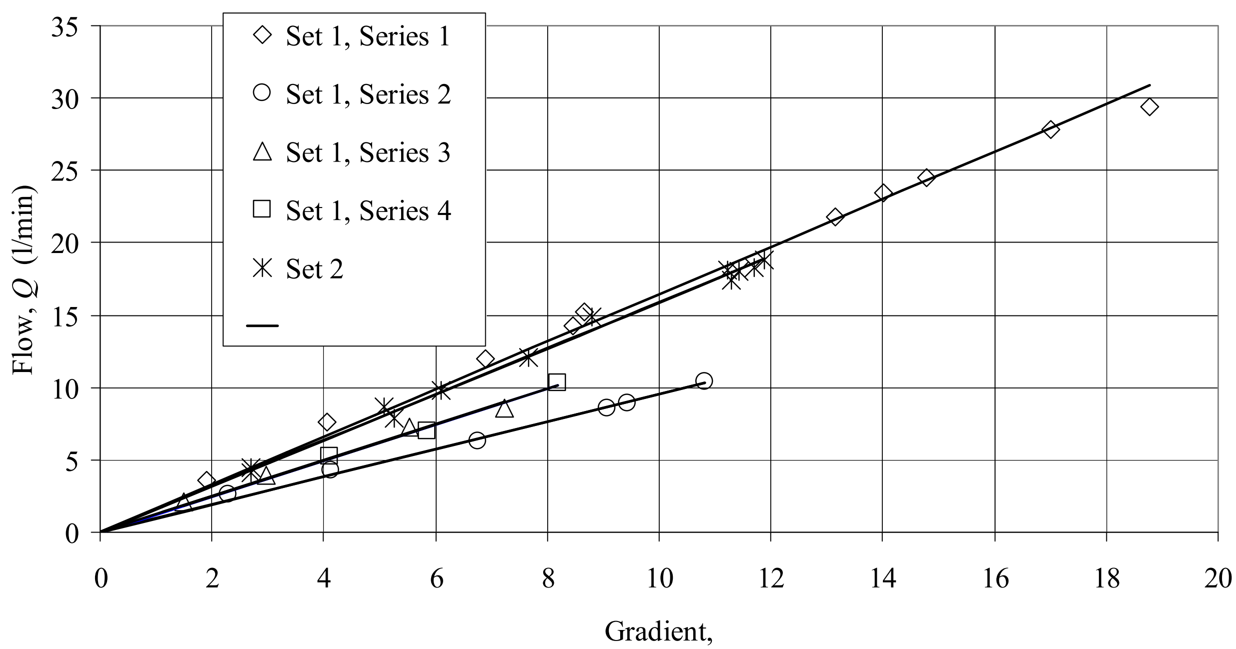
Darcy's Law - Experiments

FONTAINES PUBLIQUES
DE LA VILLE DE DIJON

EXPOSITION ET APPLICATION

DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER





Darcy’s Law - Experiments

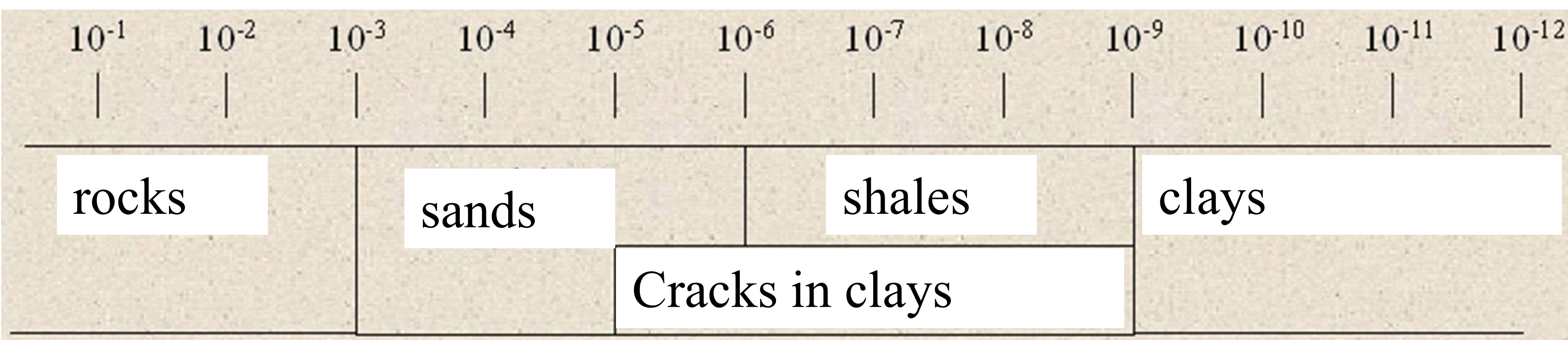
Pratice 1 : discovery of Darcy’s law

Darcy’s law-Permeability

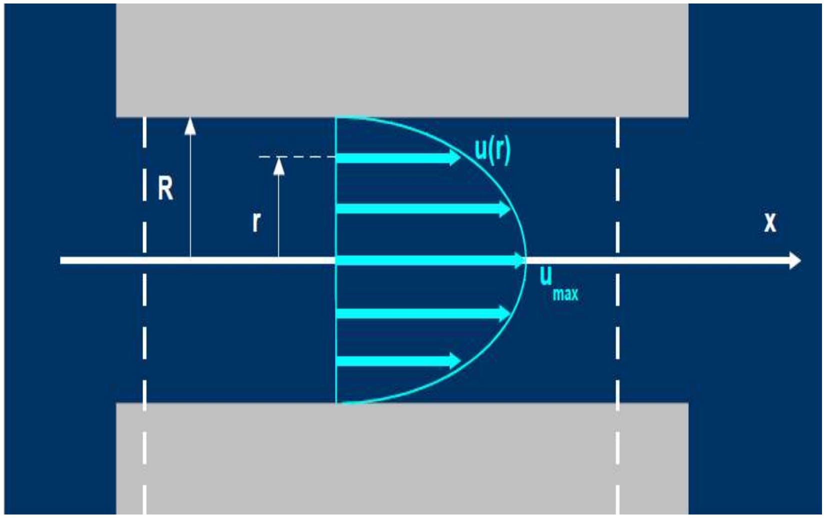
It measures the permeability of the granular medium, that is to say the ease with which water circulates. One will also find the hydraulic conductivity denoted K.

If we measure the flow rate Q in m3 / s, the load h in m, the length L in m and the area A in m2, then K is measured in m / s. But K is not a speed because h is not a classical height.

Here are some typical conductivity values (m / s):



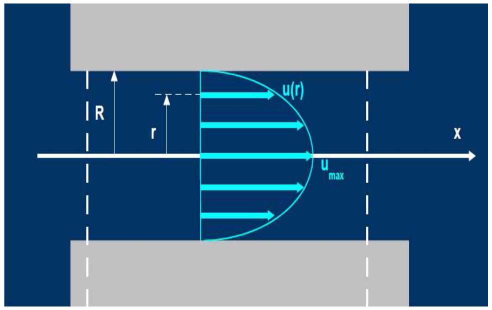
Darcy’s law-Permeability



$$\nabla.\mathbf{v} = 0$$

$$\left(\frac{\partial \mathbf{v}}{\partial t}\right) + \mathbf{v}.\nabla\mathbf{v} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{v} + \mathbf{f}$$

Darcy's law-Permeability

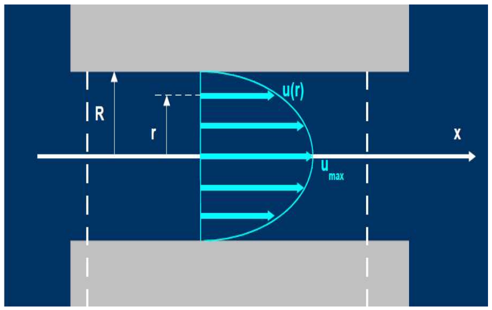


$$\frac{\partial u_r}{\partial t} + \mathbf{u}.\nabla u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\Delta u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)$$

$$\frac{\partial u_\theta}{\partial t} + \mathbf{u}.\nabla u_\theta + \frac{u_r u_\theta}{r}) = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \nu\left(\Delta u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)$$

$$\frac{\partial u_x}{\partial t} + \mathbf{u}.\nabla u_x = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\Delta u_x$$

Darcy's law-Permeability



Steady state

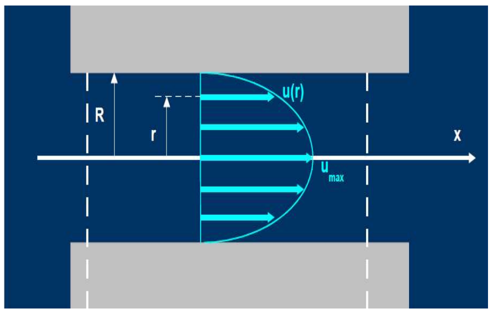
$\partial/\partial t=0$, $u(r,\theta,x)=u(r)$, $P(x)$

$$\frac{\partial P}{\partial r}=0$$

$$\frac{\partial P}{\partial \theta}=0$$

$$\frac{-\partial P}{\partial x}+\mu\frac{1}{r}\frac{d}{dr}\left(r\frac{du_x}{dr}\right)=0$$

Darcy's law-Permeability



Steady state $\partial/\partial t=0$, $u(r,\theta,x)=u(r)$, $P(x)$

$$\frac{\partial P}{\partial x}=A$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_x}{dr}\right)=A/\mu$$

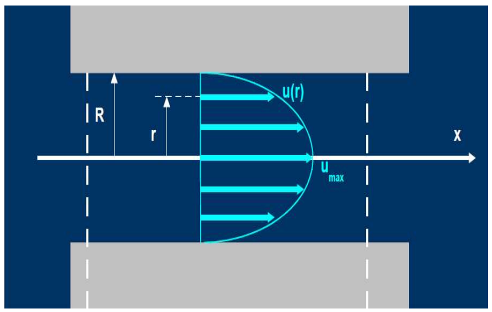
$$A=\frac{ps-pe}{L}$$

$$u_x(r)=\frac{ps-pe}{4\mu L}(R^2-r^2)$$

We have demonstrated Poiseuille flow ! Let's go level 2

Darcy's law-Permeability

Flow rate



$$Q=\int_0^R 2\pi r u_x(r)dr$$

$$Q=\frac{ps-pe}{4\mu L}2\pi\int_0^R(R^2-r^2)rdr=\frac{ps-pe}{8\mu L}\pi R^4$$

$$v_m=Q/(\pi R^2)=\frac{ps-pe}{8\mu L}R^2=\frac{ps-pe}{L}\frac{d^2}{32\mu}$$

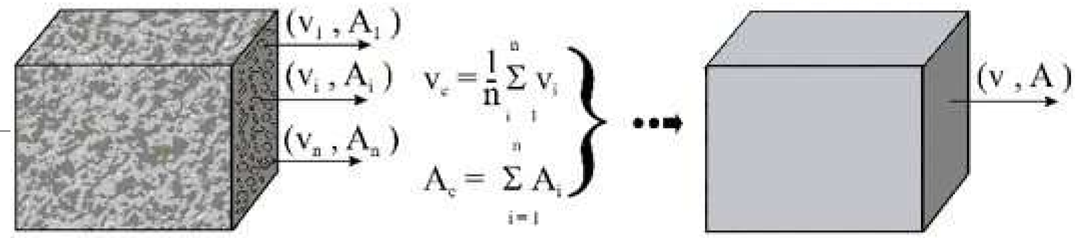
If we use P=$\rho g h$

$$\bar{v}_m=\frac{d^2\rho g}{32\mu}\frac{hs-he}{L}=K\frac{hs-he}{L}$$

$$K=\frac{k\rho g}{\mu}$$

$$K=\frac{d^2\rho g}{32\mu}\Rightarrow k=\frac{d^2}{32}$$

Upscaling?



Darcy velocity is an apparent velocity (denoted by v_f , v_D or simply v) as if the whole cross-section would be available for the water transfer.

The porous medium is replaced by a representative continuum derived from a macroscopic concept → Darcy's law provides thus a global description of the microscopic behavior.

Darcy's flux is an average value of the microscopic fluxes from a Representative Elementary Volume (REV.

The microscopic concept would involve the use of the microscopic velocities, associated with the actual paths of the water particles. Because in practice it is impossible to measure the real microscopic velocities, an average value of the real velocities is accepted.

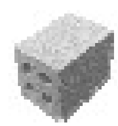
Darcy's law-Permeability

$$K = \frac{k\rho g}{\mu}$$

$$\varepsilon = \frac{Vfluid}{Vtotal} \qquad \text{Sspec} = \frac{Ssolid}{Vsolid}$$

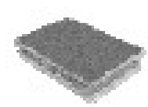
Several models allow us to determine permeability:

a) Capillary: $k = \varepsilon \frac{d^2}{32}$



ε is the void fraction

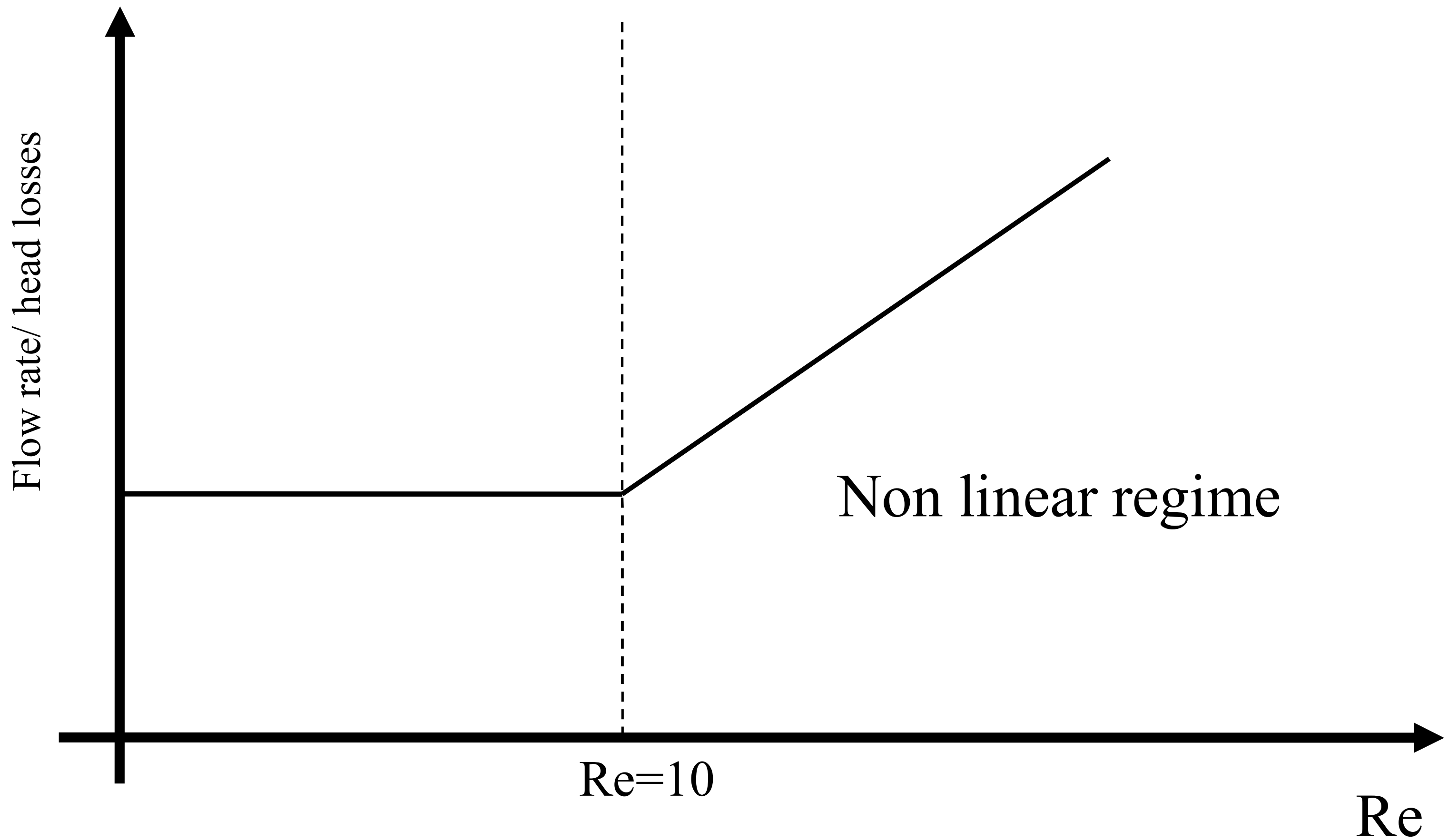
b) Hele Shaw: $k = \varepsilon \frac{b^2}{12}$



c) Kozeny (1927) : $k = C_0 \frac{\varepsilon^3}{Sspec}$ Co=0.5m circle or 0.562m if square

d) Kozeny- Carman (1937) $k = \frac{d_m^2}{180} \frac{\varepsilon^3}{(1-\varepsilon)^2}$ $\qquad dm = \frac{6(1-\varepsilon)}{Sspec}$

Darcy's Law- Validity



Why?

The ratio between the inertial forces and the viscous forces driving the flow is computed by the Reynolds number, which is used as a criterion to distinguish between the laminar flow, the turbulent flow and the transition zone.

For porous media, the Reynolds number is defined as:

$$Re = \frac{\rho q d}{\mu} = \frac{q d}{\nu}$$

where:

- Re is dimensionless
- ρ the fluid density (M/L^3 ; kg/m^3)
- q specific discharge (L/T ; m/s)
- d usually, the mean grain diameter or the mean pore dimension (L; m)
- μ dynamic viscosity ($M/T.L$; $kg/s.m$)
- ν kinetic viscosity (L^2/T ; m^2/s)

According to Bear (1972), Darcy' law which supposes a laminar flow is valid for Reynolds number less than 1, but the upper limit can be extended up to 10

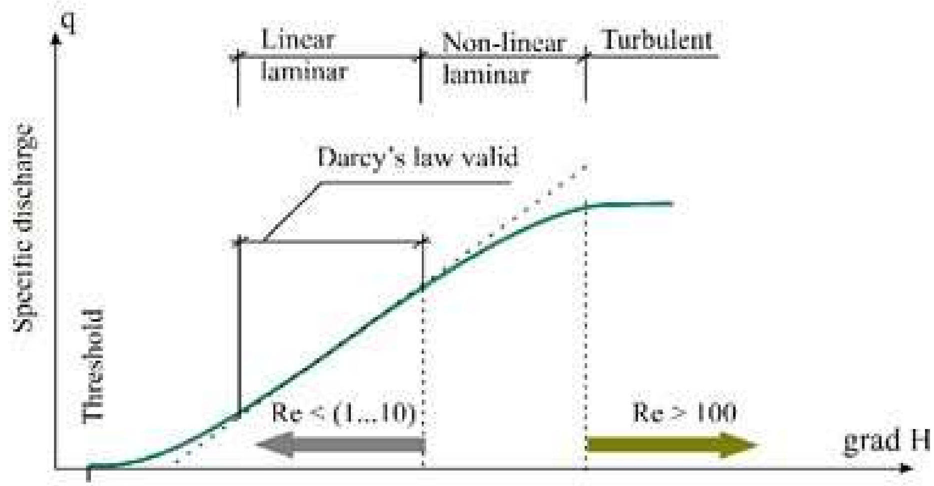
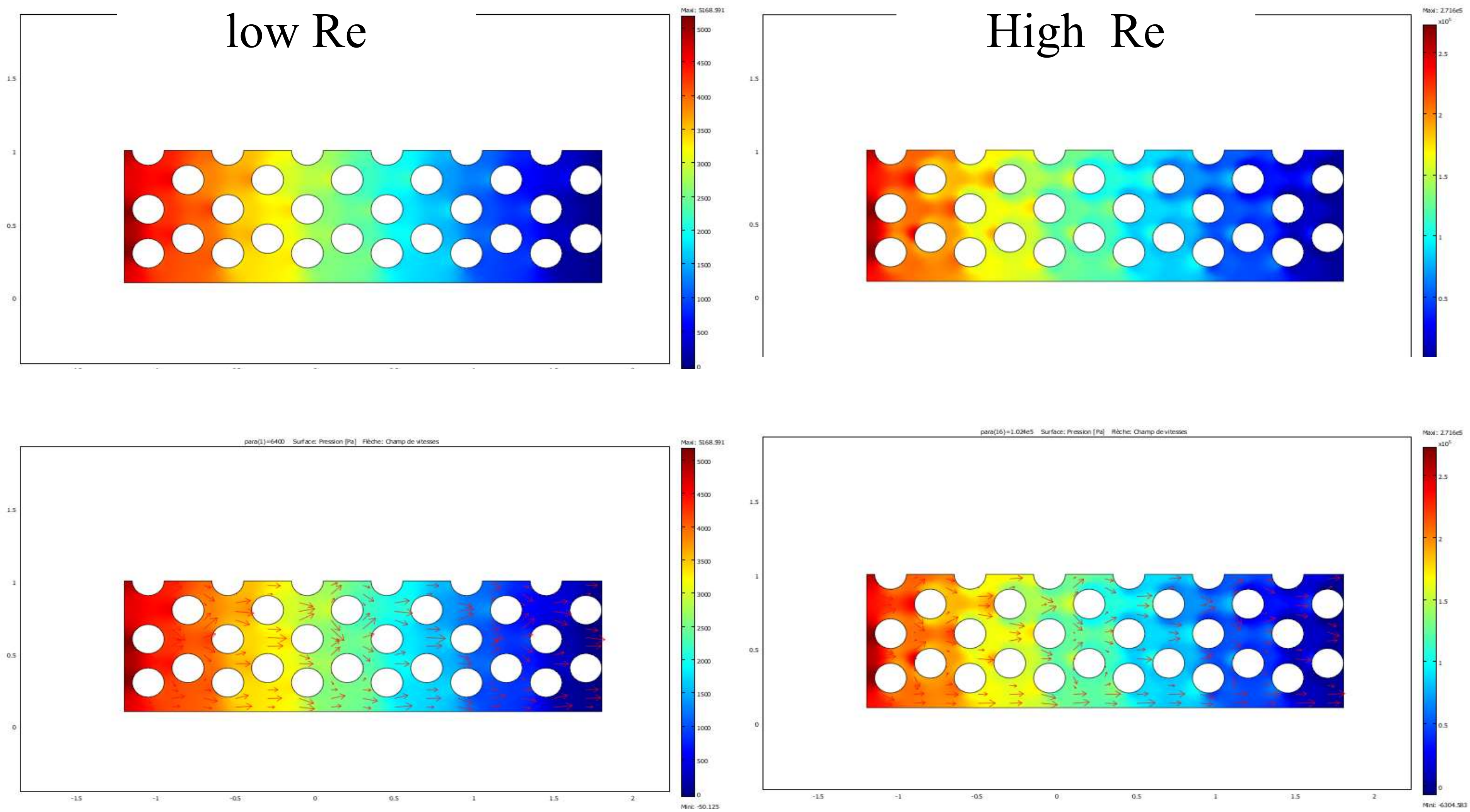


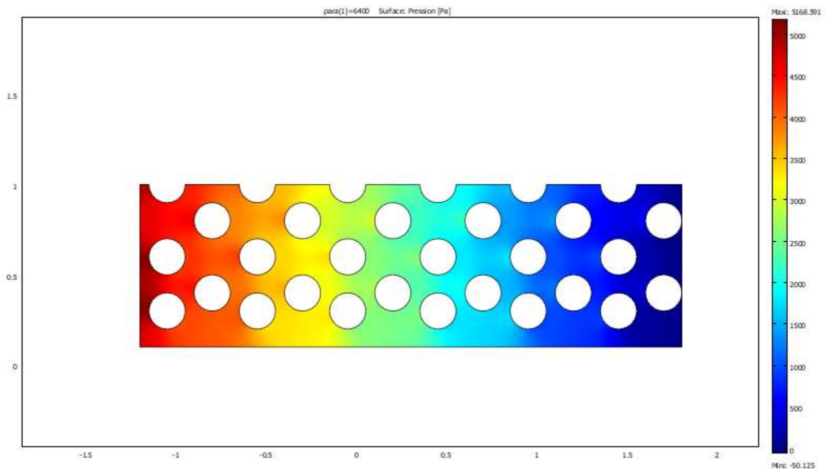
Figure. Range of validity of Darcy's law

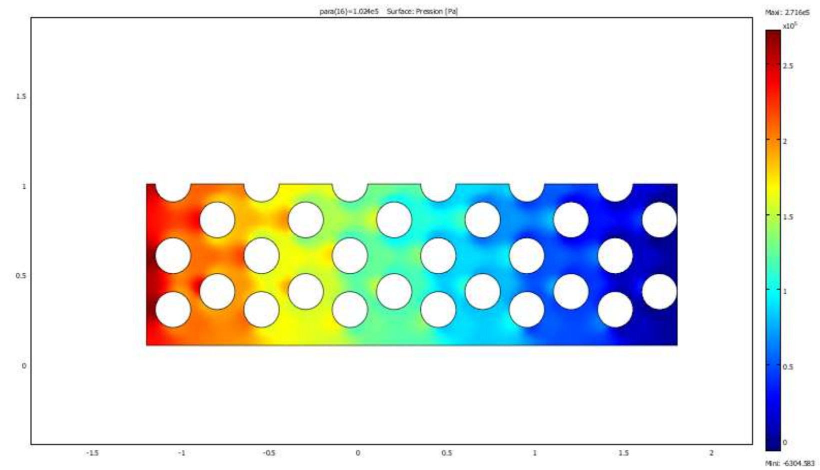
The inception of the turbulent flow can be located at Reynolds numbers greater than 60…100. Between the laminar and the turbulent flow there is a transition zone, where the flow is laminar but non-linear.

Darcy's law- Validity

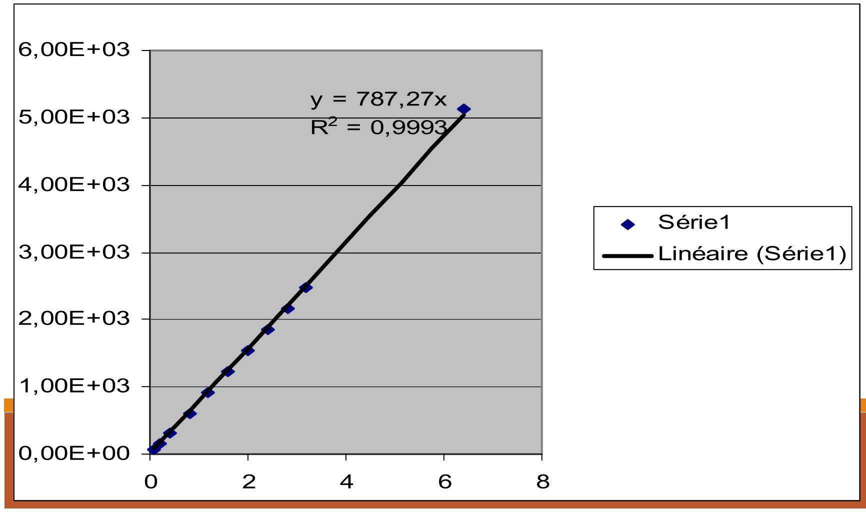


Darcy's Law- Validity

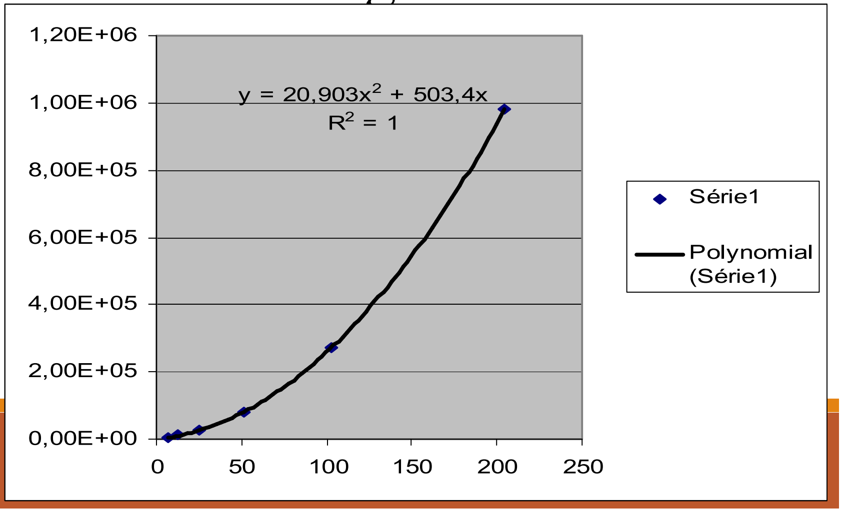




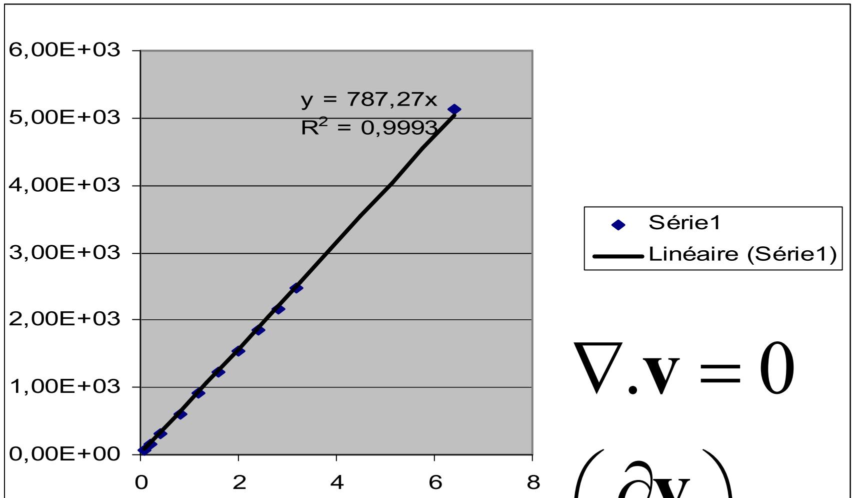
Low Re

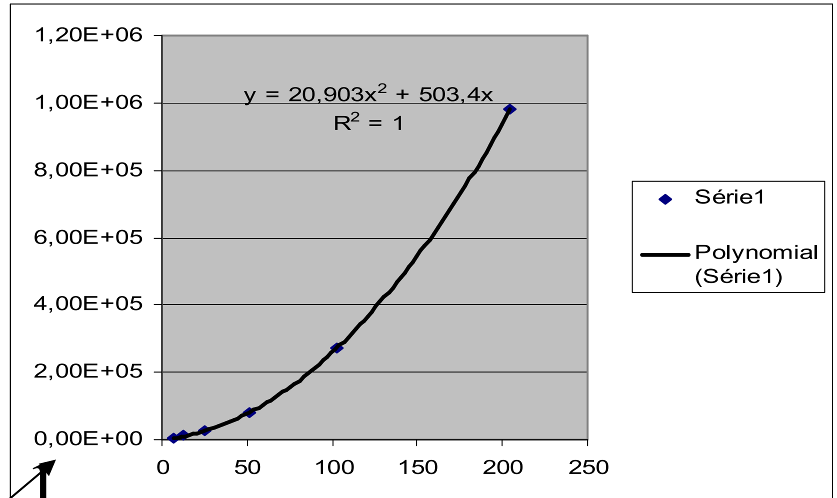


high Re



Darcy's Law- Validity





$$\nabla.\mathbf{v} = 0$$

$$\left(\frac{\partial \mathbf{v}}{\partial t}\right) + \mathbf{v}.\nabla\mathbf{v} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{v} + \mathbf{f}$$

$$-\nabla P = \frac{\mu}{k}V + \beta\rho V^2$$

If Re<<1, Darcy's law appears

Take this with care, β depends on the flow rate

Quadratic correction of the Darcy' law : **Forchheimer relation**

Darcy’s law-Continuity

Darcy's law-Continuity

If we don't have deformations of the porous medium and no compressibility effects, we can write:

$$\nabla.\mathbf{v} = 0 \qquad Q = K\frac{(h_2 - h_1)}{L}S \qquad \mathrm{v} = \frac{Q}{S}$$

$$\mathbf{v} = -K\,\overrightarrow{Grad}\,H \qquad k = \frac{\mu K}{\rho g} \qquad dp^{*=\rho.g.dH} = dp + \rho.g.dz$$

$$\mathbf{v} = -\frac{K}{\rho g}\nabla H = -\frac{k}{\mu}(\nabla p^*)$$

$$div\ \rho\left(-\frac{k}{\mu}\overrightarrow{Grad}\ p^*\right) = 0$$

$$\Delta p^* = 0 \Rightarrow \Delta H = 0$$

Darcy's law-Continuity

If we don't have deformations of the porous medium and no compressibility effects, we can write:

$$\nabla \, . \mathbf{v} \; = \; 0 \qquad \mathrm{v} = \frac{Q}{S}$$

Heterogeneous anisotropic media :

$$\mathbf{v} = -\frac{\overline{\overline{K}}}{\rho g} \overrightarrow{Grad}\, p^{*}$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{with} \quad u_i = -K_i \frac{\partial H}{\partial x_i}$$

$$K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} + K_z \frac{\partial^2 H}{\partial z^2} = 0$$