TD 2

Exercice 1

$Q_L = 1$ l/s $\qquad Q_G = 0.2$ l/s $\qquad j_L$ and j_G

Tube diameter $D = 2$ cm

$$j_L = \frac{Q_L}{\frac{\pi D^2}{4}} = 3.18 \text{ m/s} \qquad j_G = \frac{Q_G}{\frac{\pi D^2}{4}} = 0.64 \text{ m/s}$$

flow pattern → dispersed bubbly flow

$j = j_L + j_G = U_M = 3.82$ m/s $\qquad$ Drift velocity of bubbles $U_\infty \sim 0.2$ m/s

1) $U_\infty << j$ → homogenous model relevant

$$\left(-\frac{dP}{dz}\right)_{friction} = \frac{4\tau_p}{D} = \frac{4}{D}\frac{1}{2} f_{p_M} \rho_M U_M^2 \qquad \rho_M,\ R_G,\ \mu_M,\ Re_M$$

$$U_G = U_L = \frac{j_G}{R_G} = \frac{j_L}{1-R_G} \Rightarrow \boxed{R_G = \frac{j_G}{j_G + j_L} = 0.168} \quad \begin{cases} \rho_M = \rho_L(1-R_G) + \rho_G R_G \\ \mu_M = \mu_L(1-R_G) + \mu_G R_G \end{cases}$$

$\rho_M = 832$ kg/m^3

$\mu_M = 8.35 \cdot 10^{-4}$ Pa.s

$$Re_M = \frac{\rho_M U_M D}{\mu_M} = 76189$$

$$f_{pM} = 0.079\, Re_M^{-0.25} = 0.00475$$

$$\frac{4\tau_p}{D} = \frac{2}{D} f_{pM} \rho_M U_M^2 = 5773 \text{ Pa/m}$$

$$\rho_M g = 8.32 \times 9.81 = 8116 \text{ Pa/m}$$

→ same order of magnitude.

2) Lockhart and Martinelli Model

$j_L, j_G \Rightarrow$ $Re_L = \frac{j_L D}{\nu_L}$, $Re_v = \frac{j_v D}{\nu_G}$ $\Rightarrow$ Laminar / Turbulent $\Rightarrow$ f_{PL}, f_{PV}, C; $\left(\frac{dP}{dz}\right)_L, \left(\frac{dP}{dz}\right)_v$

$$X = \sqrt{\frac{\left(\frac{dP}{dz}\right)_L}{\left(\frac{dP}{dz}\right)_v}} \Rightarrow \begin{matrix} \phi_L^2 = f(X, c) \\ \phi_G^2 = f(X, c) \end{matrix} \Rightarrow \left(-\frac{dP}{dz}\right)_{fric} = \phi_L^2 \left(\frac{dP}{dz}\right)_L = \phi_G^2 \left(\frac{dP}{dz}\right)_G$$

$Re_L = 63600$ → Turbulent $\qquad f_{PL} = 0.079\, Re_L^{-0.25} = 0.00497$

$Re_G = 853$ → Laminar $\qquad f_{PG} = \frac{16}{Re_v} = 0.0188$

Constant C in the Martinelli multiplier $C = 10$

$$\left(\frac{dP}{dz}\right)_L = \frac{2}{D} f_{PL} \rho_L j_L^2 = 5026 \text{ Pa/m}$$

$$\left(\frac{dP}{dz}\right)_G = 0.939 \text{ Pa/m} \qquad X = \frac{j_L}{j_G}\left[\frac{\rho_L f_{PL}}{\rho_G f_{PG}}\right]^{1/2} = 74$$

$$\Phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} = 1{,}135$$

$U_G < U_M$

$U_G < U_L$

$\Uparrow$

$$\frac{4\tau_P}{D} = \left(\frac{dP}{dz}\right)_{fric} = \phi_L^2 \times \left(\frac{dP}{dz}\right)_L = 5705 \text{ Pa/m}$$

$$L.M \Rightarrow R_G = \left(1 + X^{0.8}\right)^{-0.378} = 0.269 \Rightarrow U_G = \frac{j_G}{R_G} = 2.38 \text{ m/s}$$

Homogenous model $R_G = 0.168$ $\qquad U_M = 3.88$ m/s

$j_L = 3.18$ m/s

The mean gas velocity < Mixture velocity

$\Rightarrow$ not relevant in bubbly flow in vertical upward flow

Exercice 2

Ecoulement de R-12 à 333 kPa adiabatique

$\dot{m} = 250$ kg/m²/s

$x = 0,25$

Ecoulement annulaire sans changement de phase

$$-R_G \frac{dP}{dz} + \tau_{iG} \frac{S_i}{A} - \rho_G R_G g = 0 \qquad \times R_L$$

$$-R_L \frac{dP}{dz} - \tau_{iG} \frac{S_i}{A} - \rho_L R_L g + \frac{\tau_P S_P}{A} = 0 \qquad \times -R_G$$

$$\tau_{iG} \frac{S_i}{A} - \rho_G R_G R_L g + \rho_L R_L R_G g - R_G \tau_P \frac{S_P}{A} = 0$$

$$\tau_{iG} \frac{4}{D}\sqrt{R_G} + (\rho_L - \rho_G) R_G R_L g - \frac{4}{D} R_G \tau_P = 0$$

$$-\frac{2}{D} f_i \rho_G (U_G - U_L)^2 \sqrt{R_G} + (\rho_L - \rho_G) R_G R_L g + \frac{2}{D} R_G f_{P_L} \rho_L U_L^2 = 0$$

$$U_G = \frac{j_G}{R_G} = \frac{\dot{m} x}{\rho_G R_G} \qquad f_i = 0,005\left(1 + 300 \frac{\delta}{D}\right) = 0,005\left(1 + 150(1 - \sqrt{R_G})\right)$$

$$U_L = \frac{j_L}{R_L} = \frac{\dot{m}(1-x)}{\rho_L R_L} \qquad Re_L = \frac{U_L D}{\mu_L}\rho_L = \frac{\dot{m}(1-x)}{\mu_L R_L}$$

$$f_{P_L} = 0,079 \left[\frac{\dot{m}(1-x)}{\mu_L(1-R_G)}\right]^{-0,25}$$

$$-\rho_G \times 0,005(1 + 150(1 - \sqrt{R_G}))\, \dot{m}^2 \left[\frac{x}{\rho_G R_G} - \frac{(1-x)}{\rho_L R_L}\right]^2 \sqrt{R_G} + (\rho_L - \rho_G)\frac{D}{2} R_L R_G g$$

$$+ R_G \times 0,079 \left[\frac{\dot{m}(1-x) D}{\mu_L R_L}\right]^{-0,25} \frac{\dot{m}^2 (1-x)^2}{R_L^2 \rho_L} = 0$$

$$\underbrace{-6000 \sqrt{R_G}\,(151 - 150\sqrt{R_G}) \left[\frac{0,00130}{R_G} - \frac{0,00054}{(1-R_G)}\right]^2}_{-8,79} + \underbrace{67,139\, R_G(1-R_G)}_{8,23}$$

$$+ \underbrace{0,02176 \cdot \frac{R_G}{(1-R_G)^{1,75}}}_{0,56} = 0$$

$f_i = 0,06569$ $\qquad f_{PL} = 0,005282$

$R_G = 0,85696$ $\qquad U_G = 3,798$ m/s $\qquad U_L = 0,9444$ m/s

$$\frac{dP}{dz} = \frac{\tau_{iG}}{R_G}\frac{S_i}{A} - \rho_G g = \frac{4}{D\sqrt{R_G}} \tau_{iG} - \rho_G g = \frac{-2}{D\sqrt{R_G}} f_i \rho_G (U_G - U_L)^2 - \rho_G g$$

$$= -2058.8 - 188 = -2247 \text{ Pa/m}$$

La modélisation de f_i joue peu sur R_G

$$-\frac{dP}{dz} - (\rho_L R_L + \rho_G R_G)g + \frac{\tau_P S_P}{A} = 0$$

$$\frac{dP}{dz} = -(\rho_L R_L + \rho_G R_G)g - \frac{2}{D} f_{P_L} \rho_L U_L^2 = -2273 - 1307{,}82 = -3580{,}8 \text{ Pa/m}$$

$$2109 \qquad 3416{,}9$$

Estimation avec L.M.

$Re_L = 7151$ $\qquad$ $j_L = 0{,}135$ m/s

$Re_G = 54\,528$ $\qquad$ $j_G = 3{,}255$ m/s

$$\left(\frac{dP}{dz}\right)_L = -\frac{2}{D} f_{P_L} \frac{\dot{m}^2(1-x)^2}{\rho_L} = -43{,}519 \text{ Pa/m}$$

$$\left(\frac{dP}{dz}\right)_V = -\frac{2}{D} f_{PG} \frac{\dot{m}^2 x^2}{\rho_V} = -206{,}06 \text{ Pa/m}$$

$$X = \sqrt{\frac{\left(\frac{dP}{dz}\right)_L}{\left(\frac{dP}{dz}\right)_V}} = 0{,}4596$$

$$\phi_L^2 = 1 + \frac{20}{X} + \frac{1}{X^2} = 49{,}25$$

$$R_G = \left[1 + 0{,}28\, X^{0{,}71}\right]^{-1} = 0{,}861$$

$$\left(\frac{dP}{dz}\right)_f = -2143 \text{ Pa/m}$$

$$\left(\frac{dP}{dz}\right)_G = -(\rho_L R_L + \rho_G R_G)g = -2054{,}8 \text{ Pa/m}$$

$$\left(\frac{dP}{dz}\right)_{total} = -4198 \text{ Pa/m}$$

Ecart d'environ 20%