Proposition de sujet de stage : Exploratory study of machine and deep learning methods on Lie groups.

Keywords: Machine Learning, Lie group, Statistical signal processing.

1 Context

In a context of engineering for aeronautics and space, using of machine learning and deep learning methods has become crucial and fundamental during the last decades. Indeed, in a different tasks in image and signal processing applied for aeronautics and space, these methods are now classically used to resolve some estimation, prediction and classification problems. For example, we can quote **supervised methods as Neural Networks and Support Vector Machines** which are performed for target radar classification [1] (for instance to classify type of aircrafts) or object detection tasks (for instance to detect anomaly in the trajectory of satellites) [2]. Also, **unsupervised methods** are very useful to segment radar image or recognize targets observed by a teledetection system (LIDAR or GNSS) [3].

Nevertheless, in some application problems, the data to predict or classify have to be constrained by its geometrical properties. The example the most speaking is the set of orthogonal matrices. Indeed, in applications such as navigation or computer vision, the sensor provides angular measurements characterized by rotation matrices which in essence are orthogonal. Another example is the set of covariance matrices classically used to model the dispersion of the data for instance in radar signal processing. Consequently, the prediction or the classification algorithms have to take into account the constraints imposed by their structure. In the state-of-the-art literature, machine learning methods using geometric properties especially the **Riemmanian structure** are scarce but exist [4]. For instance, an EM algorithm to separate covariance matrices data has been proposed in [5]. Also, deep neural networks based on the same structure has been formalized in [6].

In the context of this work, we propose to deal with **Lie groups**, matrix space of which two practical examples are the set of the rotation matrices and the set of covariance matrices. Mathematically speaking, it is a space equipped with Riemmanian structure but also with a structure of group [7]. Its advantage is to leverage mathematical tools (especially statistics and optimization) making it suitable for statistical learning problems. Thus, machine learning methods on Lie groups exist in a supervised framework for tasks of estimation and filtering [8] [9] [10]. Nevertheless, in the context of classification and prediction, so in a supervised way, works are almost non-existent and everything remains to be done.

2 Objectives of the Internship

Recently, clustering algorithm on Lie groups, **K-means** and **EM algorithm**, have been theorized on the Lie group $SE(2)$ and tested on simulated data. Also, they have provided promising performance on almost-real data. The target application is the clustering of curved shapes. This kind of shape can be encountered in different spatial applications as the tracking of cluster of space debris or the characterization of the uncertainty on the position of a rover.

The main objectives of this internship is to analyse these algorithm but also to implement new machine learning algorithms for data lying on Lie groups in a **supervised framework**. More precisely:

- A primary aim will be to understand the unsupervised approaches already theorized and implemented. Modifications of the latter could be made for specific scenarios of applications, especially for other Lie groups of interest.

- Then, an second aim will be to theorize and implement supervised machine and deep learning algorithms. An interest will be granted to generalize **Regression methods** and **Multilayer Perceptron Neural Networks**. Ideally, the proposed methods will be tested numerically on realistic scenario.

3 Info

- Disciplines: applied mathematics, machine learning, statistical estimation.
- Tools: MATLAB or Python.
- Required curriculum: MSc. or Dipl. Ing. in applied mathematics, computer science or statistics.
- Duration: 5 to 6 months.
- Starting date: February to Early April.
- Applications (resume, transcript, motivation letter) and informal inquiries are to be e-mailed to Samy Labsir (IPSA/TéSA), samy.labsir@ipsa.fr.
- Location: TéSA laboratory, Toulouse, France.
- Salary: $\sim$ 550 euros /Month

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