8

Extended Bodies at Rest

Back pain is a major health problem. In 2010 in the United States, the medical costs related to back pain were around 80 billion dollars a year, about the same as the yearly cost of treating cancer. Back pain often results from incorrect lifting, which compresses the disks in the lower back, causing nerves to be pinched. Understanding the physics principles underlying lifting can help us develop techniques that minimize this compression and prevent injuries.

- **Why is it best to lift heavy objects with your knees bent and the object near your body?**
- **Why are doorknobs located on the side of the door opposite the hinges?**
- **Why is the force that your biceps muscle exerts on your forearm when lifting a barbell about seven times stronger than the force that Earth exerts on it?**

SO FAR IN THIS BOOK we have primarily been modeling moving objects as point-like with no internal structure. This method is appropriate when the shapes of objects do not affect the consequences of their interactions with each other—for example, an elevator moving up a shaft or an apple falling into a pile of leaves. However, objects in general and the human body in particular are extraordinarily complex, with many internal parts that rotate and move relative to one another. To study the body and other complex structures, we need to develop a new way of modeling objects and analyzing their interactions.

BE SURE YOU KNOW HOW TO:

- Define the point-like model for an object (Section 2.2).
- Draw a force diagram for a system (Section 3.1).
- Use the component form of Newton's second law (Section 4.2).

8.1 Extended and rigid bodies

In earlier chapters we focused on situations in which real objects that have nonzero dimensions could be reasonably modeled as point-like. Such modeling is possible when an object moves as a whole from one location to another, without turning. Such motion is called **translational motion**.

In this chapter we will be analyzing objects whose size and shape matter. We will start with the simplest case—objects at rest. (In Chapter 9 we will investigate their motion.) We'd like, for example, to understand how the contemporary dancers in **Figure 8.1** manage to maintain their unusual balance. Where should each force be exerted and how large should their magnitudes be in order for the dancers to remain stable? To answer these questions, we need a new model for extended objects and a new method for analyzing the forces that objects exert on each other. Our first task is to develop this new model for objects.

FIGURE 8.1 The point-like model of an object is not useful when we try to analyze the balance of these dancers.

Rigid bodies

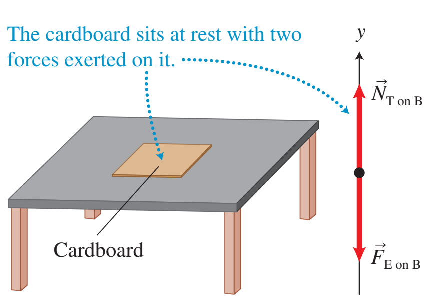
Notice that at the instant shown in the photo the various parts of the dancers' bodies are not moving with respect to each other. They are acting as a single rigid object. This observation motivates a new model for an extended object, in which the size of the object is not zero (it is not a point-like object), but parts of the object do not move with respect to each other. In physics, this model is called a **rigid body**.

Rigid body A rigid body is a model of a real extended object. When we model an extended object as a rigid body, we assume that the object has a nonzero size but the distances between all parts of the object remain the same (the size and shape of the object do not change).

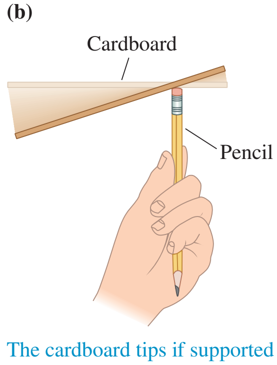
Many bones in your body can be reasonably modeled as rigid bodies, as can many everyday objects—buildings, bridges, streetlights, and utility poles. However, many objects cannot be modeled as rigid bodies. Objects that cannot be modeled as rigid bodies include ones that contain liquid or moving interior parts, such as water balloons or boxes of loose candy. In this chapter we will investigate what conditions are necessary for a rigid body to remain at rest.

FIGURE 8.2 The cardboard is stable in (a) but not in (b). The place where the supporting force is exerted on the board matters.

(a)



(b)

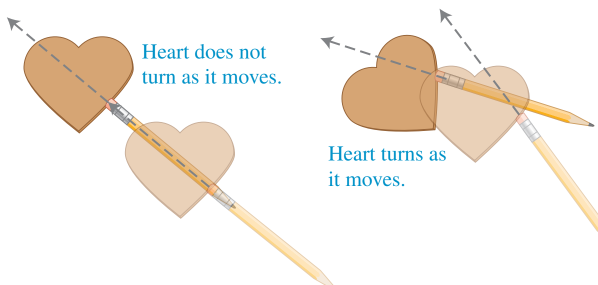
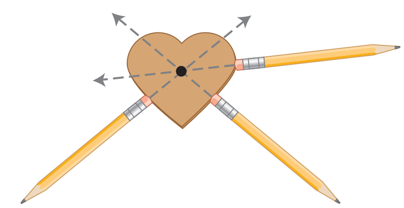
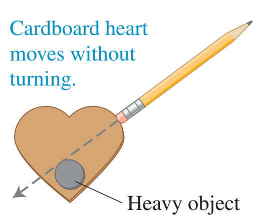
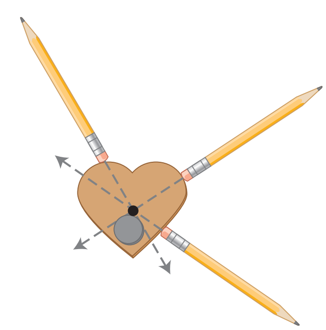


Center of mass

Let's start with some simple experiments. Place a piece of thin, flat cardboard on a very smooth table. If we consider the cardboard to be a point-like object, on a force diagram the upward normal force that the table exerts on the cardboard will balance the downward gravitational force that Earth exerts on the cardboard (**Figure 8.2a**)—the cardboard will not accelerate. Now, place the cardboard on a very small surface—like the eraser of a pencil. The cardboard tilts and falls off (Figure 8.2b). Does this result mean that the eraser cannot exert the same upward force on the cardboard that the table did, or is there some other explanation? The model of the point-like object cannot explain the tilting, since point-like objects do not tilt. Perhaps we need to model the cardboard as a rigid body. Before we do this, let us learn a little more about rigid bodies in Observational Experiment **Table 8.1**.

OBSERVATIONAL EXPERIMENT TABLE 8.1 Pushing a board so that it moves without turning

VIDEO OET 8.1

Observational experiment	Analysis
Experiment 1. We push with a pencil eraser on the edge of a heart-shaped piece of flat cardboard at different locations such that for each push the cardboard moves on a smooth surface without turning. We also push against the cardboard at points where it turns as it moves. 	We draw lines across the top of the cardboard from the places and in the directions that the cardboard did not turn as it moved. The lines along which the forces are exerted all cross at one point. 
Experiment 2. We now place a heavy object on the cardboard and repeat the experiment, each time pushing at different locations along the edge, trying to find the direction of pushing so that for each push the cardboard moves without turning. 	The lines along which the forces are exerted all cross at one point. This point is located somewhere between the point in experiment 1 and the position of the added object in experiment 2. 

Pattern

1. In both experiments, all of the lines along which we had to push to move the cardboard without it turning pass through a common point on the cardboard.
2. Pushing at the same locations in other directions causes the cardboard to turn as it moves.

The analysis we did in Table 8.1 indicates that a rigid body possesses a special point. If a force exerted on that object points directly toward or away from that point, the object will not turn. We call this point the object's **center of mass**. It is found experimentally that the center of mass is not necessarily located at the geometrical center of the object, but depends on the distribution of mass in the object.

Center of mass (qualitative definition) The center of mass of an object is a point where a force exerted on the object pointing directly toward or away from that point will not cause the object to turn. The location of this point depends on the mass distribution of the object.

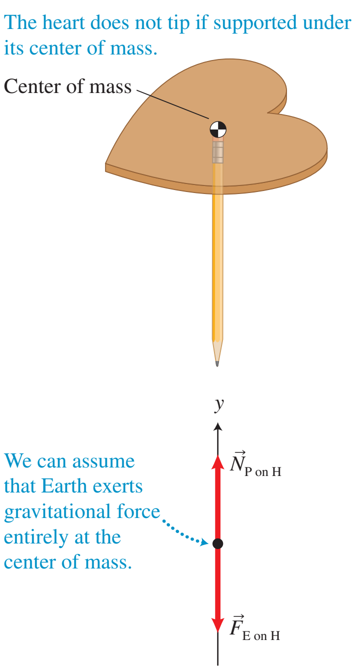
TIP Although the location of the center of mass depends on the mass distribution of the object, the mass of the object is not necessarily evenly distributed around the center of mass. We will learn more about the properties of the center of mass; we just want to caution you against taking the name of this point literally.

Where is the gravitational force exerted on a rigid body?

At the beginning of the chapter we found that we could not balance cardboard on the eraser of a pencil. Why did the cardboard fall off? Imagine drawing the forces exerted on the cardboard. The cardboard interacts with two objects: the eraser and Earth. The eraser exerts an upward normal force on the cardboard at the point where it touches it. Earth exerts a downward gravitational force on every part of the cardboard. Is it possible to simplify the situation and to find one location at which we can assume that the entire gravitational force is exerted on the cardboard?

Let us go back to the heart-shaped cardboard from Experiment 1 in Table 8.1. If we place the eraser exactly under the cardboard's center of mass, the cardboard does not tip over and fall (**Figure 8.3**). If the cardboard does not tip, it means that all forces exerted on it, including the force exerted by Earth and the force exerted by the eraser, pass through the center of mass. The normal force exerted by the eraser passes through the place where it contacts the cardboard—below the center of mass (indicated on the figure with a checked circle).

FIGURE 8.3 Balancing cardboard.



Earth exerts a small force on each small part of the board, but we can assume that the total force is exerted exactly at the center of mass. That is why sometimes the object's center of mass is called the object's **center of gravity**. Note that the center of gravity is located at the same spot as the center of mass only when the object is so small that we can neglect the change of g throughout the object. For example, if you think of a large spaceship close to Earth, the value of g at the points closer to Earth is larger than at the points that are farther.

When we model something as a point-like object, we model it as if all of the object's mass is located at its center of mass. Likewise, we can apply what we know about translational motion for point-like objects to rigid bodies, as long as we apply the rules to their centers of mass. **When multiple forces are exerted on a rigid body, the center of mass of the rigid body accelerates translationally according to Newton's second law $\vec{a} = \Sigma\vec{F}/m$.**

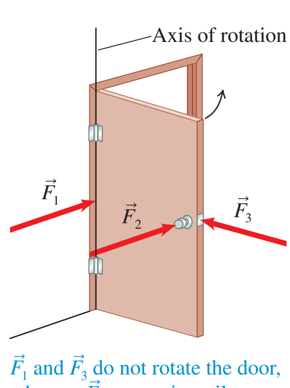
REVIEW QUESTION 8.1 You have an oval framed painting. How do you determine where you should insert a single nail into the frame so that the painting is correctly oriented in both the vertical and horizontal directions?

8.2 Torque: a new physical quantity

We learned in the previous section that the turning effect of an individual force depends on where and in which direction the force is exerted on an object. The translational acceleration of the object's center of mass is still determined by Newton's second law, independently of where the force is exerted. In this section we will learn about the turning ability of a force that an object exerts on a rigid body.

Axis of rotation

FIGURE 8.4 Different forces have different effects in turning a door about its axis of rotation.



When objects turn around an axis, physicists say that they undergo **rotational motion**. The axis may be a fixed physical axis, such as the hinge of a door, or it may not, as in the case of a spinning top. In this chapter we will focus on the conditions under which objects that could potentially rotate do not do so.

Consider a door. When you push on the doorknob perpendicular to the door's surface ($\vec{F}_2$ in **Figure 8.4**), it rotates easily about the door hinges. We call the imaginary line passing through the hinges the **axis of rotation**. You know from experience that pushing a door at or near the axis of rotation ($\vec{F}_1$ in Figure 8.4) is not as effective as pushing the doorknob. You also know that the harder you push near the knob, the more rapidly the door starts moving. Lastly, pushing on the outside edge of the door toward the axis of rotation ($\vec{F}_3$ in Figure 8.4) does not move the door at all.

These observations suggest that three factors affect the turning ability of a force: (1) the place where the force is exerted, (2) the magnitude of the force, and (3) the direction in which the force is exerted. Next, let's construct a quantitative expression for this turning ability.

The role of position on the turning ability of a force

In order to quantify the turning ability of a force, we take a 0.10-kg meter stick and suspend it at its center of mass from spring scale 2 (**Figure 8.5**); $\vec{F}_2$ is the force exerted on the meter stick by scale 2 at the point of suspension. Spring scale 1 pulls perpendicularly on the stick, exerting a downward force $\vec{F}_1$ at different locations on the left side, and scale 3 exerts a downward perpendicular force $\vec{F}_3$ at different locations on the right side. Earth exerts a 1.0-N force $\vec{F}_{\text{E on M}}$ on the stick's center of mass, which is the point of suspension.

When either scale 1 or scale 3 pulls alone on the stick, the stick rotates (when scale 1 pulls on the stick, it rotates counterclockwise; when 3 pulls, it rotates clockwise). If we pull equally on scales 1 and 3 ($\vec{F}_1 = \vec{F}_3$) but the scales are located at different distances from the axis of rotation, the stick rotates (**Figure 8.6a**). We can use trial and error to find the combinations where the scales can be placed and pulled such that the stick does not rotate. For example, if scale 1 is twice as far from the axis of rotation and pulls half as hard as scale 3, the stick remains in equilibrium (Figure 8.6b). Similarly, if scale 1 is three times farther from the axis of rotation and exerts one-third of the force compared to scale 3, the stick remains in equilibrium (Figure 8.6c). When the stick does not rotate and does not move translationally, it is in a state of **static equilibrium**.

FIGURE 8.5 An experiment to determine a condition necessary for multiple forces to keep a meter stick in a horizontal position.

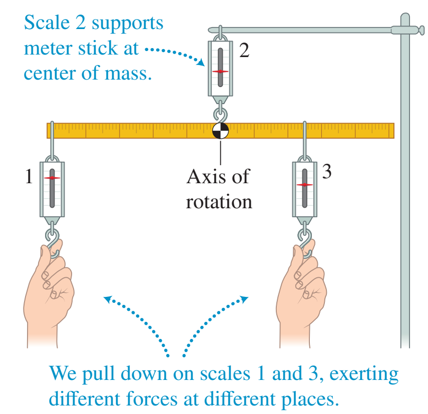
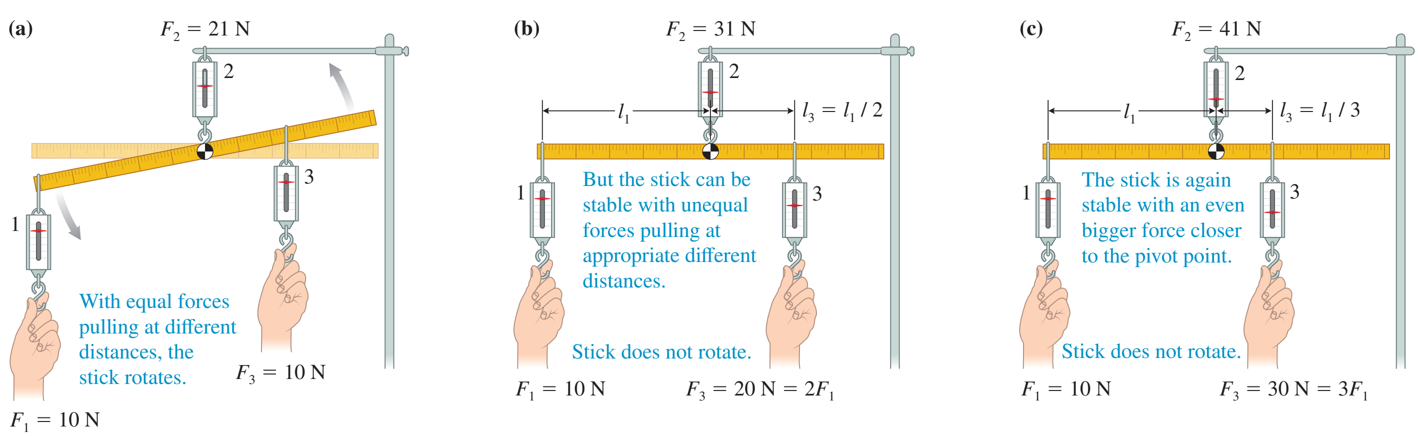


FIGURE 8.6 (a) The meter stick does not balance even though equal downward forces are exerted on each side. (b) and (c) A greater force on one side nearer the pivot point balances a smaller force on the other side farther from the pivot point.



> **Static equilibrium** An object is said to be in static equilibrium when it *remains* at rest (does not undergo either translational or rotational motion) with respect to a particular observer in an inertial reference frame.

If we explore more situations in which the stick is not rotating, we find that when we have two springs pulling perpendicular to the stick, the stick remains in static equilibrium when the product of the magnitude of the force exerted by the scale on the stick (F_1 or F_3 in our experiment) and the distance between where the force is exerted and the axis of rotation (l_1 or l_3 in our experiment) is the same for both forces:

$$F_1 l_1 = F_3 l_3$$

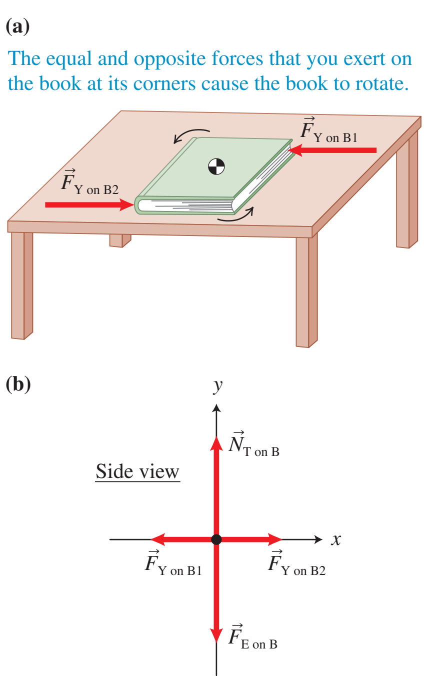
In other words, the turning ability of the force on the left cancels the turning ability of the force on the right, and the object is in static equilibrium.

The role of magnitude on the turning ability of a force

In addition, we notice that independently of whether the stick rotated or not, the reading of scale 2, F_2, supporting the stick always equals the sum of the readings of scales 1 and 3, $F_1 + F_3$, plus the magnitude of the force exerted by Earth on the stick $F_{\text{E on S}}$. In other words, in all cases the sum of the forces exerted on the stick was zero ($\Sigma F_y = 0$). This finding is consistent with what we know from Newton's laws—an object does not accelerate translationally if the sum of the forces exerted on it is zero. If it is originally at rest and does not accelerate translationally, then it remains at rest.

However, as we have seen from the experiment with the cardboard on the eraser, this sum-of-forces-equals-zero rule does not guarantee the rotational stability of rigid bodies (see Figure 8.2). Even when the sum of the forces exerted on the cardboard was zero, it could still start turning.

FIGURE 8.7 The turning effect of a force must depend on more than F and l.



Another simple experiment helps illustrate this idea. Take a book with a glossy cover, place it on a smooth table (to minimize friction), and push it, exerting the same magnitude, oppositely directed force on each of two corners (as shown in **Figure 8.7a**). The force diagram in Figure 8.7b shows that the net force exerted on the book is zero—there is no translational acceleration. However, the book starts turning. The forces exerted by Earth and the table on the book pass through the book's center of mass; thus they do not cause turning. But the forces that you exert on the corners of the book do cause it to turn. Notice that these forces are of the same magnitude and are exerted at the same distance from the center of mass. You can imagine that there is an invisible axis of rotation passing through the center of mass perpendicular to the desk's surface. You would think that the turning effect caused by each force around this imaginary axis is the same; thus the two turning effects should cancel, as they did for scales 1 and 3 in the experiment described earlier. However, this does not happen. The fact that the book turns tells us that not only are the magnitude and placement of the force on the object important to describe the turning ability of the force, but the direction in which this force causes turning (for example, clockwise or counterclockwise around an axis of rotation) is also important.

By convention, physicists call counterclockwise turning about an axis of rotation positive and clockwise turning negative. So far, this is what we know about the new quantity that characterizes the turning ability of a force:

(a) It is equal to the product of the magnitude of the force and the distance the force is exerted from the axis of rotation.
(b) It is positive when the force tends to turn the object counterclockwise and negative when the force tends to turn the object clockwise.
(c) When one force tends to rotate an object counterclockwise and the other force tends to rotate an object clockwise, their effects cancel if $(F_{\text{counterclockwise}} l_1) + (-F_{\text{clockwise}} l_2) = 0$. In this case the object does not rotate.

Let's apply what we have devised so far to check whether this new quantity is useful for explaining other situations. Here is another simple experiment that you can do at home. Place a full milk carton or something of similar mass into a grocery bag. The bag with the milk carton by itself is not too heavy and you can easily lift it with one hand. Now hang the grocery bag from the end of a broomstick (**Figure 8.8a**). Try to support it by holding only the handle end of the broomstick with your hands close together. It is very difficult. Why?

The broomstick, the object of interest, can turn around an axis through the hand that is closest to you. The bag exerts a force on the broomstick far from this axis of rotation. This means that l_{Bag}, a quantity that we must find in order to determine the turning ability of the force exerted by the bag on the broomstick, is very large (Figure 8.8b). Your other hand, which is very close to the axis of rotation (distance l_{Hand}), must balance the effect of the bag. But since l_{Hand} is so small, the force your hand exerts must be very large. The outcome of this experiment agrees with what we learned so far about the turning ability of a force.

Holding the broomstick perpendicular to your body is quite difficult. However, if you hold the broomstick at an angle above the horizontal (Figure 8.8c), you find that the bag becomes easier to support. Why? Perhaps this has to do with the direction the force is exerted relative to the broomstick (see Figure 8.8d).

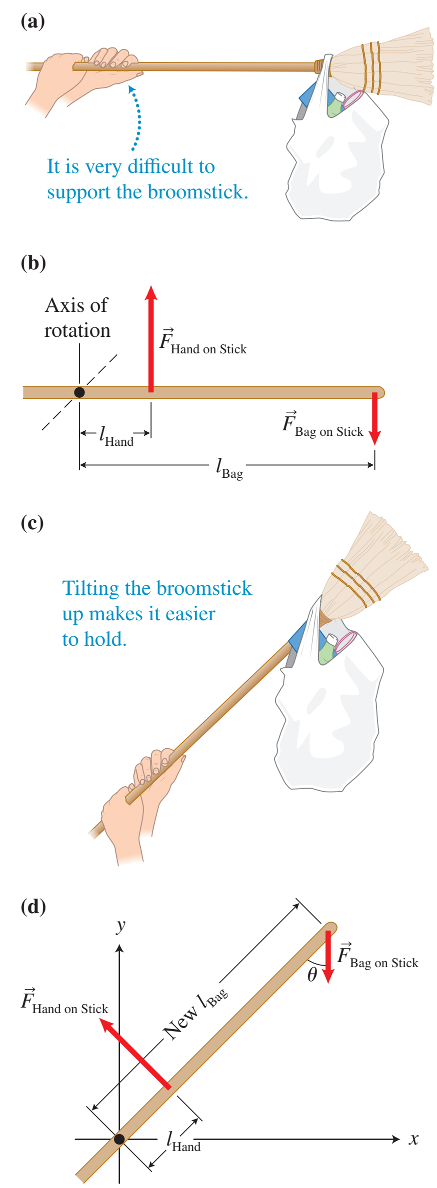


FIGURE 8.8 Holding a bag at the end of a stick is more difficult when the stick is horizontal than when the stick is tilted up.

The role of angle on the turning ability of a force

Our current mathematical model of the physical quantity that characterizes the turning ability of the force ($\pm Fl$) takes into account the direction in which an exerted force can potentially rotate an object (clockwise or counterclockwise) but does not take into account the actual direction of the force. However, our experiment with the broomstick indicates that the angle at which we exert a force relative to the object affects the turning ability of the force. We know from experience that pushing on a door on its outside edge directly toward the hinges does not cause it to rotate. The direction of the push must matter. How can we improve our model for the physical quantity to take the direction of the force into account?

To investigate this question we can change our experiment with the meter stick slightly by making scale 1 pull on the stick at an angle other than 90° (**Figure 8.9**). Scale 2, on the far right end of the meter stick, 0.50 m from the suspension point, will exert a constant force of 10.0 N downward at a 90° angle. Scale 1 on the far left end of the stick will pull at different angles θ so that the meter stick remains horizontal. The results are shown in **Table 8.2**. In all cases the stick is horizontal—therefore, the turning ability of the force on the right is balanced by the turning ability of the force on the left.

FIGURE 8.9 An experiment to determine the angle dependence of the turning ability caused by a force.

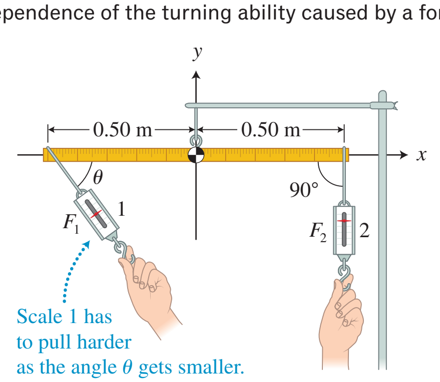


TABLE 8.2 Magnitude, location, and direction of force and its turning ability

Magnitude of $\vec{F}_1$	Distance to the axis of rotation	Angle θ between $\vec{F}_1$ and the stick	Turning ability produced by $\vec{F}_2$
10.0 N	0.50 m	90°	$-(10.0\text{ N})(0.50\text{ m}) = -5.0\text{ N}\cdot\text{m}$
12.6 N	0.50 m	53°	$-(10.0\text{ N})(0.50\text{ m}) = -5.0\text{ N}\cdot\text{m}$
14.2 N	0.50 m	45°	$-(10.0\text{ N})(0.50\text{ m}) = -5.0\text{ N}\cdot\text{m}$
20.0 N	0.50 m	30°	$-(10.0\text{ N})(0.50\text{ m}) = -5.0\text{ N}\cdot\text{m}$

Using the data in the table, we see the effects of the magnitude and the angle of force $\vec{F}_1$ on its turning ability: the smaller the angle between the direction of the force and the stick, the larger the magnitude of the force that is necessary to produce the same turning ability. Thus we find that there are four factors that affect the turning ability of a force: (1) the direction (counterclockwise or clockwise) that the force can potentially rotate the object, (2) the magnitude of the force F, (3) the distance l of the point of application of the force from the axis of rotation, and (4) the angle θ that the force makes relative to a line from the axis of rotation to the point of application of the force. If we combine these four factors, the physical quantity characterizing the turning ability of a force takes a form such as

Counterclockwise (CCW) or clockwise (CW)

Angle

$$\pm Flf(\theta)$$

Magnitude Distance

where $f(\theta)$ is some function of the angle θ.

Consider the last row of Table 8.2. The force exerted by scale 1 is 0.50 m from the axis of rotation ($l = 0.50$ m), and the scale exerts a 20-N force F_1 on the stick at a 30° angle relative to the stick. This force produces the counterclockwise $+5.0\ \text{N}\cdot\text{m}$ effect needed to balance the $-5.0\ \text{N}\cdot\text{m}$ clockwise effect of scale 2. What value would $f(\theta)$ have to be to get this rotational effect?

$$+5.0\ \text{N}\cdot\text{m} = (20.0\ \text{N})(0.5\ \text{m})\ f(30°)$$

or $f(30°)$ must be 0.50. Recall that $\sin 30° = 0.50$. Maybe the function $f(\theta)$ is the sine function, that is, $\tau = Fl \sin\theta$. Is this consistent with the other rows in Table 8.2?

$$\begin{aligned} +(10.0\ \text{N})(0.5\ \text{m})(\sin 90°) &= +5.0\ \text{N}\cdot\text{m}\ (1.00) = +5.0\ \text{N}\cdot\text{m} \\ +(12.6\ \text{N})(0.5\ \text{m})(\sin 53°) &= +6.3\ \text{N}\cdot\text{m}\ (0.80) = +5.0\ \text{N}\cdot\text{m} \\ +(14.2\ \text{N})(0.5\ \text{m})(\sin 45°) &= +7.1\ \text{N}\cdot\text{m}\ (0.71) = +5.0\ \text{N}\cdot\text{m} \end{aligned}$$

This expression ($\tau = Fl\sin\theta$) is the mathematical definition of the new physical quantity that characterizes the ability of a force to turn (rotate) a rigid body. This physical quantity is called a **torque**. The symbol for torque is τ, the Greek letter tau.

Torque τ produced by a force The torque produced by a force exerted on a rigid body about a chosen axis of rotation is

$$\tau = \pm Fl\sin\theta \qquad (8.1)$$

where F is the magnitude of the force, l is the magnitude of the distance between the point where the force is exerted on the object and the axis of rotation, and θ is the angle that the force makes relative to a line connecting the axis of rotation to the point where the force is exerted (see **Figure 8.10**).

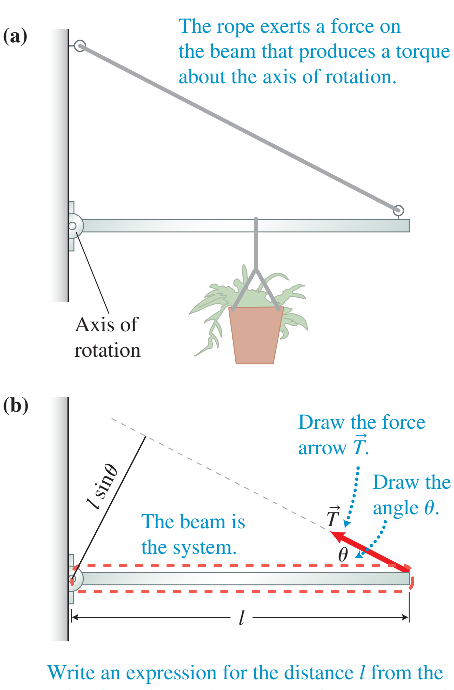


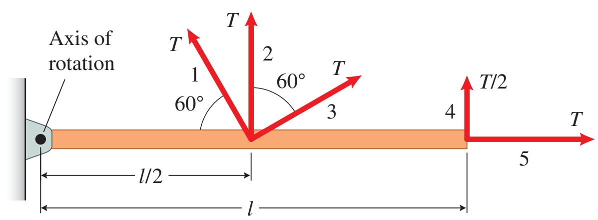
FIGURE 8.10 A method to determine the torque (turning ability) produced by a force.

Figure 8.10 illustrates the method for calculating the turning ability (torque) due to a particular force. In this case, we are calculating the torque due to the force that the slanted rope exerts on the end of a beam that supports a load hanging from the beam. The torque is positive if the force has a counterclockwise turning ability about the axis of rotation, and negative if the force has a clockwise turning ability. The SI unit for torque is newton·meter, N·m (the British system unit is lb·ft). Note that the expression $l\sin\theta$ gives us the shortest distance from the axis of rotation to the line along which the force producing the torque is exerted.

TIP Notice that the units of torque $(\text{N}\cdot\text{m})$ are the same as the units of energy $(\text{N}\cdot\text{m} = \text{J})$. Torque and energy are very different quantities. We will always refer to the unit of torque as newton · meter $(\text{N}\cdot\text{m})$ and the unit of energy as joule (J).

QUANTITATIVE EXERCISE 8.1 Rank the magnitudes of the torques

Suppose that five strings pull one at a time on a horizontal beam that can pivot about a pin through its left end, which is the axis of rotation. The magnitudes of the tension forces exerted by the strings on the beam are either T or $T/2$. Rank the magnitudes of the torques that the strings exert on the beam, listing the largest magnitude torque first and the smallest magnitude torque last. Indicate if any torques have equal magnitudes. Try to answer the question before looking at the answer below.



Represent mathematically A mathematical expression for the torque produced by each force is shown below. To understand why each torque is positive, imagine in what direction each string would turn the beam about the axis of rotation if that were the only force exerted on it. You will see that each string tends to turn the beam counterclockwise (except string 5).

Torque due to string 1: $\tau_1 = +T(l/2)\sin 60° = +0.43\ Tl$

Torque due to string 2: $\tau_2 = +T(l/2)\sin 90° = +0.50\ Tl$

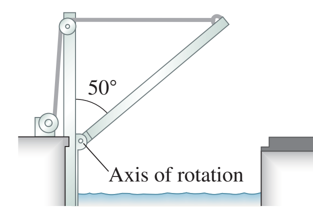
Torque due to string 3: $\tau_3 = +T(l/2)\sin 150° = +0.25\ Tl$

Torque due to string 4: $\tau_4 = +(T/2)l\sin 90° = +0.50\ Tl$

Torque due to string 5: $\tau_5 = +Tl\sin 0° = 0$

Solve and evaluate Notice that the angle used for the torque for the force exerted by rope 3 was 150° and not 60°—the force makes a 150° angle relative to a line from the pivot point to the place where the string exerts the force on the beam. String 5 exerts a force parallel to the line from the pivot point to the place where it is exerted on the beam; as a result, torque 5 is zero. The rank order of the torques is $\tau_2 = \tau_4 > \tau_1 > \tau_3 > \tau_5$.

Try it yourself Determine the torque caused by the cable pulling horizontally on the inclined drawbridge shown below. The force that the cable exerts on the bridge is 5000 N, the bridge length is 8.0 m, and the bridge makes an angle of 50° relative to the vertical support for the cable system.

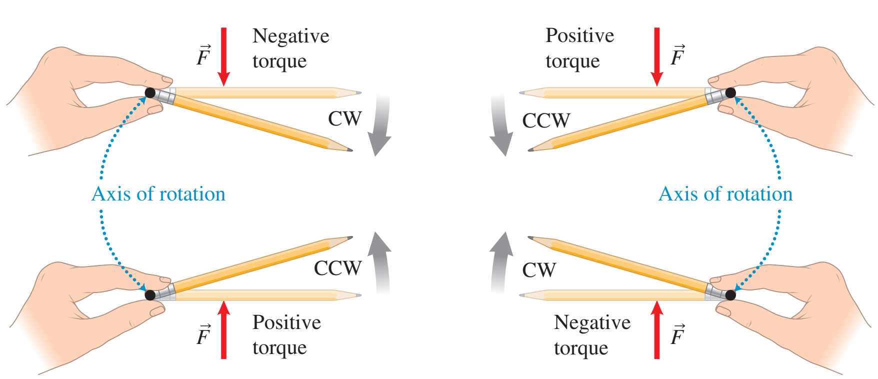


Answer $\tau = +(5000\ \text{N})(8.0\ \text{m})\sin 40° = +26{,}000\ \text{N}\cdot\text{m}$. Note that we did not use 50° in our calculation. Why?

To decide the sign of the torque that a particular force exerts on a rigid body about a particular axis, pretend that a pencil is the rigid body. Hold it with two fingers at a place that represents the axis of rotation (**Figure 8.11**) and exert a force on the pencil representing the force whose torque sign you wish to determine. Does that force cause the pencil to turn counterclockwise (a + torque) or clockwise (a − torque) about the axis of rotation?

In this chapter we treat torque as a scalar quantity that can be positive or negative. The sign of the torque indicates the specific direction of rotation (compare this to work, the sign of which means adding or subtracting). The directional meaning of the sign hints that torque has a vector nature. You will learn more about the vector nature of torque in Chapter 9.

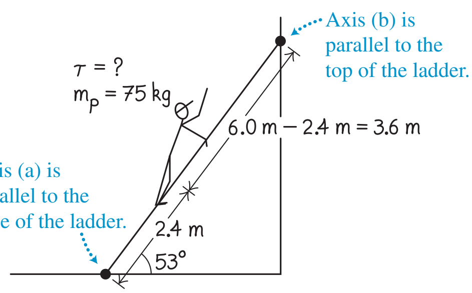
FIGURE 8.11 A method to determine the sign of a torque.



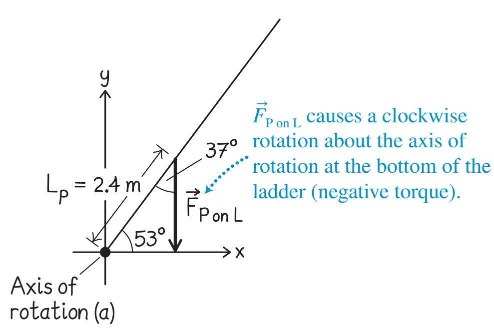
EXAMPLE 8.2 A painter on a ladder

A 75-kg painter stands on a 6.0-m-long 20-kg ladder tilted at 53° relative to the ground. He stands with his feet 2.4 m up the ladder. Determine the torque produced by the force exerted by the painter on the ladder for two choices of axis of rotation: (a) an axis parallel to the base of the ladder where it touches the ground and (b) an axis parallel to the top ends of the ladder where it touches the wall of the house.

Sketch and translate A sketch of the situation with the known information is shown below. The ladder is our system.



Simplify and diagram Four objects interact with the ladder: Earth, the painter's feet, the wall, and the ground. Our interest in this problem is only in the torque that the painter's feet exert on the ladder. Thus, we diagram for the ladder and the downward force $\vec{F}_{\text{P on L}}$ that the painter's feet exert on the ladder. The diagrams are for the two different axes of rotation.



Represent mathematically We use Eq. (8.1) for the torque caused by the force that the painter's feet exert on the ladder.

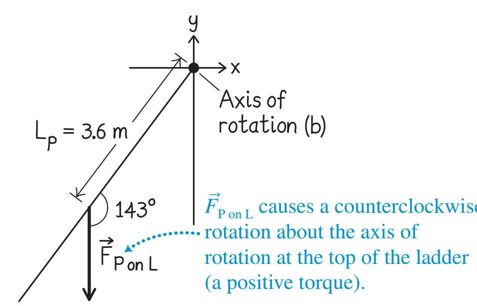
Solve and evaluate (a) For the first calculation, we choose the axis of rotation at the place where the feet of the ladder touch the ground. The torque produced by the force exerted by the painter's feet on the ladder is

$$\tau = -F_{\text{P on L}} l \sin\theta = -(m_{\text{P}}g) l \sin\theta$$
$$= -(75\ \text{kg})(9.8\ \text{N/kg})(2.4\ \text{m})\sin 37° = -1100\ \text{N}\cdot\text{m}$$

Note that the force exerted by the painter's feet on the ladder has the same magnitude as the force mg that Earth exerts on the painter, but it is not the same force, as it is exerted on a different object and is a contact force and not a gravitational force. That force tends to rotate the ladder clockwise about the axis of rotation (a negative torque). The force makes a 37° angle relative to a line from the axis of rotation to the place where the force is exerted.

(b) We now choose the axis of rotation parallel to the wall at the top of the ladder where it touches the wall. The torque produced by the force exerted by the painter's feet on the ladder is

$$\tau = +F_{\text{P on L}} l \sin\theta = +(m_{\text{P}}g) l \sin\theta$$
$$= +(75\ \text{kg})(9.8\ \text{N/kg})(3.6\ \text{m})\sin 143°$$
$$= +1600\ \text{N}\cdot\text{m}$$



When we choose the axis of rotation at the top of the ladder, the downward force exerted by the painter's feet on the ladder tends to rotate the ladder counterclockwise about the axis at the top (a positive torque). This force makes a 143° angle with respect to a line from the axis of rotation to the place where the force is exerted.

Note that the torque depends on where we place the axis of rotation. You cannot do torque calculations without carefully defining the axis of rotation.

Try it yourself Write an expression for the torque produced by the upward normal force exerted by the floor on the ladder about the same two axes: (a) at the base of ladder and (b) at the top of the ladder.

Answer

(a) The torque will be 0 at the bottom, because the normal force passes through the axis. (b) About the axis at the top of the ladder, the torque is $\tau = -(6.0\ \text{m})N_{\text{Floor on Ladder}} \sin 37°$. Note that the floor tends to push the ladder clockwise about an axis through the top of the ladder. The normal force makes a 37° angle relative to a line from the top axis to the place where the force is exerted.

REVIEW QUESTION 8.2 Give an example of a situation in which (a) a torque produced by a force is zero with respect to one choice of axis of rotation but not zero with respect to another; (b) a force is not exerted at the axis of rotation, but the torque produced by it is zero anyway; and (c) several forces produce nonzero torques on an object, but the object does not rotate.

8.3 Conditions of equilibrium

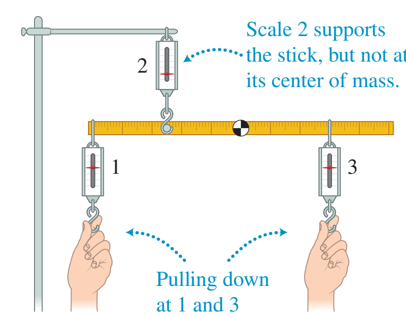
We can combine our previous knowledge of forces and our new knowledge of torque to determine under what conditions rigid bodies remain in static equilibrium, that is, at rest. Recall that we defined static equilibrium as a state in which an object *remains* at rest with respect to a particular observer in an inertial reference frame.

It is possible for an object to be at rest briefly. For example, a ball thrown upward stops for an instant at the top of its flight, but it does not remain at rest. Thus, the word *remains* is important in the expression "remains at rest"—the object has to stay where it is. The words "with respect to an observer in an inertial reference frame" are also an important part of the definition of static equilibrium. Recall from the chapter on Newtonian mechanics (Chapter 3) that if an observer is not in an inertial reference frame, an object can accelerate with respect to the observer even if the sum of the forces exerted on it is zero. In this chapter we will only consider observers who are at rest with respect to Earth, since that is the most common point of view for observing real-life situations involving static equilibrium.

We again suspend the same 0.1-kg meter stick from spring scale 2, as shown in **Figure 8.12**. However, the suspension point is no longer at the center of mass of the meter stick. You and your friend again pull on the stick at different positions with spring scales 1 and 3. When pulled as described in Observational Experiment **Table 8.3**, the stick does not rotate. Pulling at other positions while exerting the same forces, or pulling at the same positions while exerting different forces, causes the stick to rotate. We need to find a pattern in the combinations of forces and torques exerted on the meter stick that keep the stick in static equilibrium.

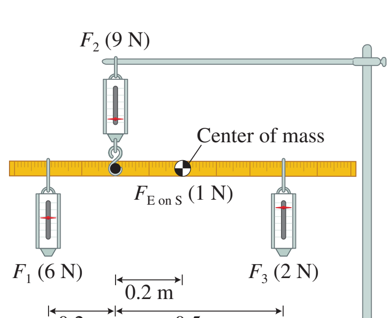
For most situations that we analyze in this chapter, we assume that the objects rotate in the x-y plane and that the axis of rotation goes through the origin of the coordinate system and is perpendicular to the x-y plane.

FIGURE 8.12 Multiple objects exert forces on a meter stick.

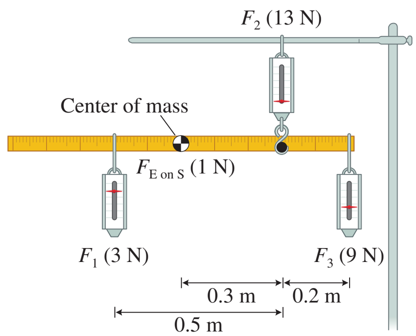


OBSERVATIONAL EXPERIMENT TABLE 8.3 Meter stick in static equilibrium

VIDEO OET 8.3

Observational experiment	Analysis
Experiment 1. Three spring scales and Earth exert forces on a meter stick at locations shown below. Examine the forces and torques exerted on the stick. Choose the axis of rotation at the place where the string from scale 2 supports the stick. This choice determines the distances in the torque equations for each force. 	$\Sigma F_y = (-6.0\text{ N}) + 9.0\text{ N} + (-1.0\text{ N}) + (-2.0\text{ N}) = 0$ Counterclockwise torques: $\tau_1 = (F_1)(l_1) = (6.0\text{ N})(0.20\text{ m}) = 1.2\text{ N}\cdot\text{m}$ Clockwise torques: $\tau_2 = (F_2)(l_2) = (9.0\text{ N})(0) = 0$ $\tau_E = -(F_{\text{E on S}})(l_{\text{CM}}) = -(1.0\text{ N})(0.2\text{ m}) = -0.2\text{ N}\cdot\text{m}$ $\tau_3 = -(F_3)(l_3) = -(2.0\text{ N})(0.50\text{ m}) = -1.0\text{ N}\cdot\text{m}$ $\Sigma\tau = \tau_1 + \tau_2 + \tau_E + \tau_3$ $= +1.2\text{ N}\cdot\text{m} + 0 - 0.2\text{ N}\cdot\text{m} - 1.0\text{ N}\cdot\text{m} = 0$

(CONTINUED)

Observational experiment	Analysis
Experiment 2. Three spring scales and Earth exert forces on a meter stick at locations shown below. Examine the forces and torques exerted on the stick. Choose the axis of rotation at the place where the string from scale 2 supports the stick. 	$\Sigma F_y = -3.0\text{ N} + (-1.0\text{ N}) + 13\text{ N} + (-9.0\text{ N}) = 0$ Counterclockwise torques: $\tau_1 = (F_1)(l_1) = (3.0\text{ N})(0.50\text{ m}) = 1.5\text{ N}\cdot\text{m}$ $\tau_E = (F_{E\text{ on }S})(l_{CM}) = (1.0\text{ N})(0.3\text{ m}) = 0.3\text{ N}\cdot\text{m}$ Clockwise torques: $\tau_2 = (F_2)(l_2) = (13\text{ N})(0) = 0$ $\tau_3 = -(F_3)(l_3) = -(9.0\text{ N})(0.20\text{ m}) = -1.8\text{ N}\cdot\text{m}$ $\Sigma\tau = \tau_1 + \tau_E + \tau_2 + \tau_3$ $= +1.5\text{ N}\cdot\text{m} + 0.3\text{ N}\cdot\text{m} + 0 - 1.8\text{ N}\cdot\text{m} = 0$

Patterns

- In both cases the net force exerted on the meter stick in the vertical direction is zero: $\Sigma F_y = 0$.
- In both cases the sum of the torques exerted on the meter stick equals zero: $\Sigma\tau = 0$.

TIP Notice that you cannot determine the torque produced by a force without specifying the point at which the force is exerted on the object relative to the axis of rotation.

The first pattern in both experiments is familiar to us. It is simply Newton's second law applied to the vertical axis of the meter stick for the case of zero translational acceleration. Because the sum of the vertical forces exerted on the meter stick is zero, there is no vertical acceleration. We had no horizontal forces, so the meter stick could not accelerate horizontally. The second pattern shows that in both cases the net torque is zero and the meter stick does not start turning.

We can now summarize these conclusions as follows:

TIP Remember that all the gravitational forces exerted by Earth on the different parts of the rigid body can be combined into a single gravitational force being exerted on the center of mass of the rigid body.

Condition 1. Translational (force) condition of static equilibrium An object modeled as a rigid body is in translational static equilibrium with respect to a particular observer if it is at rest with respect to that observer and the components of the sum of the forces exerted on it in the perpendicular x- and y-directions are zero:

$$\Sigma F_{\text{on O }x} = F_{1\text{ on O }x} + F_{2\text{ on O }x} + \cdots + F_{n\text{ on O }x} = 0 \quad (8.2x)$$

$$\Sigma F_{\text{on O }y} = F_{1\text{ on O }y} + F_{2\text{ on O }y} + \cdots + F_{n\text{ on O }y} = 0 \quad (8.2y)$$

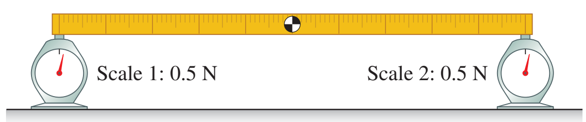
The subscript n indicates the number of forces exerted by external objects on the rigid body.

Condition 2. Rotational (torque) condition of static equilibrium A rigid body is in turning or rotational static equilibrium if it is at rest with respect to the observer and the sum of the torques $\Sigma\tau$ (positive counterclockwise torques and negative clockwise torques) about any axis of rotation produced by the forces exerted on the object is zero:

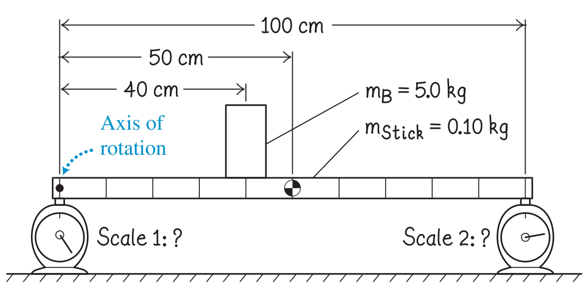
$$\Sigma\tau = \tau_1 + \tau_2 + \cdots + \tau_n = 0 \quad (8.3)$$

EXAMPLE 8.3 Testing the conditions of static equilibrium

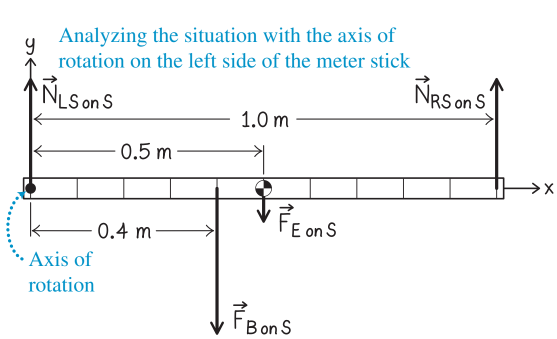
Place the ends of a standard meter stick on two scales, as shown below. The scales each read 0.50 N. From this, we infer that the mass of the meter stick is about 0.10 kg (the gravitational force that Earth exerts on the meter stick would be $(0.10\ \text{kg})(9.8\ \text{N/kg}) = 1.0\ \text{N}$). Predict what each scale will read if you place a 5.0-kg brick 40 cm to the right of the left scale.



Sketch and translate A labeled sketch of the situation is shown below. We choose the stick as the system of interest and use a standard *x-y* coordinate system. We choose the axis of rotation at the place where the left scale touches the stick. By doing this, we are making the torque produced by the normal force exerted by the left scale on the stick zero—that force is exerted exactly at the axis of rotation. With this choice, we remove one of the unknown quantities from the torque condition of equilibrium and will be able to use that condition to find the force exerted by the right scale on the meter stick.



Simplify and diagram We model the meter stick as a rigid body with a uniform mass distribution (its center of mass is at the midpoint of the stick). We model the brick as a point-like object, and assume that the scales push up on the stick at the exact ends of the stick. We then draw a force diagram showing the forces exerted on the stick by Earth, the brick, and each of the scales. As noted, the left end of the stick has been chosen as the axis of rotation.



Represent mathematically According to the conditions of static equilibrium, the sum of the forces exerted on the meter stick should equal zero, as should the sum of the torques around the axis of rotation. The gravitational force exerted by Earth and the normal force exerted by the brick on the stick both have clockwise turning ability around the axis of rotation and produce negative torques. The force exerted by the right scale on the stick has counterclockwise turning ability and produces a positive torque. The force exerted by the left scale on the stick produces zero torque since it is exerted at the axis of rotation. The two conditions of equilibrium are then the following:

Translational (force) condition $(\Sigma F_y = 0)$:

$$(-F_{\text{E on S}}) + (-F_{\text{B on S}}) + N_{\text{RS on S}} + N_{\text{LS on S}} = 0$$

Rotational (torque) condition $(\Sigma\tau = 0)$:

$$-F_{\text{E on S}}(0.50\ \text{m}) - F_{\text{B on S}}(0.40\ \text{m}) + N_{\text{RS on S}}(1.00\ \text{m}) = 0$$

Since none of the forces have *x*-components, we didn't apply the *x*-component form of the force condition of equilibrium.

Earth exerts a downward gravitational force on the brick of magnitude:

$$F_{\text{E on B}} = m_B g = (5.0\ \text{kg})(9.8\ \text{N/kg}) \approx 50\ \text{N}$$

Thus, the stick must exert a balancing 50-N upward force on the brick. According to Newton's third law $(\vec{F}_{\text{B on S}} = -\vec{F}_{\text{S on B}})$, the brick must exert a downward 50-N force $\vec{F}_{\text{B on S}}$ on the stick.

Solve and evaluate We have two equations with two unknowns ($N_{\text{RS on S}}$ and $N_{\text{LS on S}}$). We first use the torque equilibrium condition to determine the magnitude of the force exerted by the right scale on the stick (we use 10 N/kg instead of 9.8 N/kg for simplicity):

$$-[(0.10\ \text{kg})(10\ \text{N/kg})](0.50\ \text{m}) - [(5.0\ \text{kg})(10\ \text{N/kg})](0.40\ \text{m}) + N_{\text{RS on S}}(1.00\ \text{m}) = 0$$

or

$$N_{\text{RS on S}} = 20.5\ \text{N}$$

We can use this result along with the force equilibrium condition equation to determine the magnitude of the force exerted by the left scale on the stick.

$$-(0.10\ \text{kg})(10\ \text{N/kg}) - (5.0\ \text{kg})(10\ \text{N/kg}) + 20.5\ \text{N} + N_{\text{LS on S}} = 0$$

or

$$N_{\text{LS on S}} = 30.5\ \text{N}$$

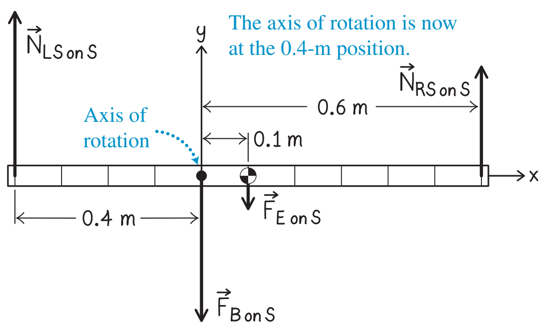
These predictions make sense because the sum of these two upward forces equals the sum of the two downward forces that Earth and the brick exert on the meter stick. Also, the force on the left end is greater because the brick is positioned closer to it, which sounds very reasonable. Performing this experiment, we find that the outcome matches the predictions.

Using a different axis of rotation Remember that we had the freedom to choose whatever axis of rotation we wanted. Let's try it again with the axis of rotation at 40 cm from the left side, the location of the brick. See the force diagram on the next page. The force condition

(CONTINUED)

of equilibrium will not change since it does not depend on the choice of the axis of rotation:

$$(-F_{\text{E on S}}) + (-F_{\text{B on S}}) + N_{\text{RS on S}} + N_{\text{LS on S}} = 0$$



The torque condition will change:

$$[-F_{\text{E on S}}\,(0.10\text{ m})] + [-N_{\text{LS on S}}\,(0.40\text{ m})]$$
$$+ N_{\text{RS on S}}(0.60\text{ m}) = 0$$

Now we have two unknowns in each of the two equations and have to solve them simultaneously to determine the unknowns. This will be somewhat harder than when we chose the axis of rotation at the left end of the stick. Let's solve the force condition equation for $N_{\text{RS on S}}$ and substitute the result into the torque condition equation:

$$N_{\text{RS on S}} = F_{\text{E on S}} + F_{\text{B on S}} - N_{\text{LS on S}}$$
$$\Rightarrow -F_{\text{E on S}}(0.10\text{ m}) - N_{\text{LS on S}}(0.40\text{ m})$$
$$+ (F_{\text{E on S}} + F_{\text{B on S}} - N_{\text{LS on S}})(0.60\text{ m}) = 0$$

Combining the terms with $N_{\text{LS on S}}$ on one side, we get

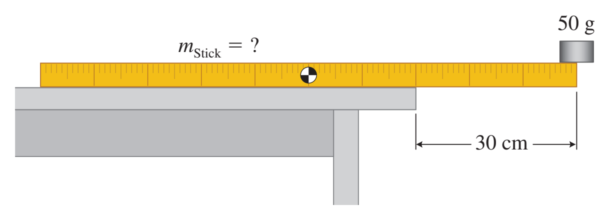
$$N_{\text{LS on S}}(0.40\text{ m} + 0.60\text{ m}) = F_{\text{E on S}}(0.50\text{ m}) + F_{\text{B on S}}(0.60\text{ m})$$
$$= (1.0\text{ N})(0.50\text{ m}) + (50\text{ N})(0.60\text{ m}) = 30.5\text{ N}\cdot\text{m}$$

or $N_{\text{LS on S}} = 30.5$ N. Substituting back into the force condition equation, we find that

$$N_{\text{RS on S}} = 1.0\text{ N} + 50.0\text{ N} - 30.5\text{ N} = 20.5\text{ N}$$

These are the same results we obtained from the original choice of the axis of rotation. The choice of the axis of rotation does not affect the results. This makes sense, in the same way that choosing a coordinate system does not affect the outcome of an experiment. The concepts of axes of rotation and coordinate systems are mental constructs and should not affect the outcome of actual experiments.

Try it yourself A uniform meter stick with a 50-g object on it is positioned as shown below. The stick extends 30 cm over the edge of the table. If you push the stick so that it extends slightly farther over the edge, it tips over. Use this result to determine the mass of the meter stick.



Answer 75 g.

REVIEW QUESTION 8.3 You read the following sentence in a book: "In problem solving, put the axis at the place on the rigid body where the force you know the least about is exerted." Explain why this hint is helpful.

8.4 Center of mass

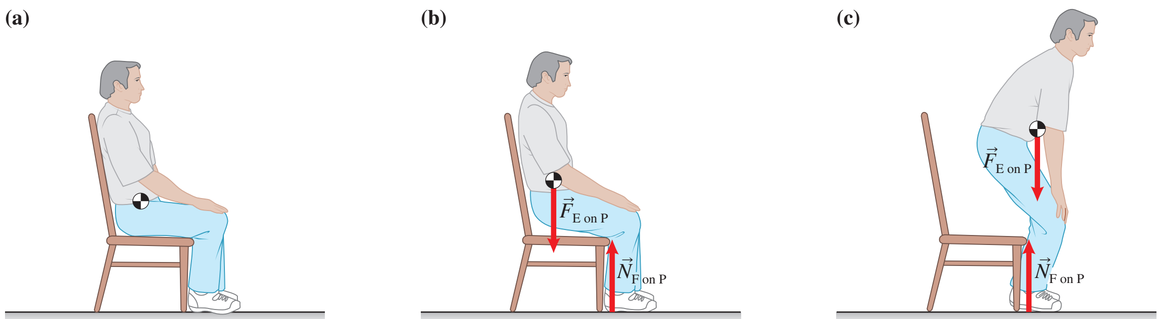
FIGURE 8.13 Where is the jumper's center of mass?

Many extended bodies are not rigid—the human body is a good example. A high jumper crossing the bar is often bent into an inverted U shape (**Figure 8.13**). Why does this shape allow her to jump higher? At the moment shown in the photo, her legs, arms, and head are below the bar as the trunk of the body passes over the top. As each part of the body passes over the bar, the rest of the body is at a lower elevation so that her center of mass is always slightly below the bar. The high jumper does not have to jump as high because she is able to reorganize her body's shape so that her center of mass passes *under* or at least not significantly over the bar.

Without realizing it, we change the position of our center of mass with respect to other parts of the body quite often. Try this experiment. Sit on a chair with your back straight and your feet on the floor in front of the chair (see **Figure 8.14a**). Without using your hands, try to stand up; you cannot. No matter how hard you try, you cannot raise yourself to standing from the chair if your back is vertical.

Why can't you stand? The center of mass of an average person when sitting upright is near the front of the abdomen. Figure 8.14 explains how the location of the center of

FIGURE 8.14 Getting out of a chair without using your hands.



(a) You sit on a chair with back straight and feet on the floor. Your center of mass is over the chair.

(b) With your back straight as you lift yourself from the chair, the normal force exerted by the floor on your feet causes a counterclockwise torque about the center of mass. You fall backward.

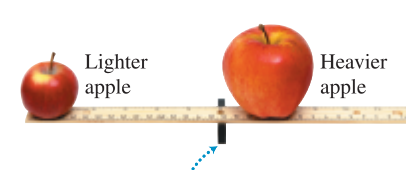
(c) Bending forward so that your center of mass is in front of the floor's normal force causes a clockwise torque so that you can stand.

mass with respect to your feet affects whether the torques exerted on you by the two forces are able to rotate you in the desired direction.

Calculating center of mass

How do we know that the center of mass of a sitting person is near the abdomen? In Section 8.1 we determined the location of an object's center of mass by investigating the directions along which one needs to push the object so it does not turn while being pushed on a flat smooth surface. When pushing in this way at different locations on the object, we found that lines drawn along the directions of these pushing forces all intersected at one point: the center of mass. This is a difficult and rather impractical way to find the center of mass of something like a human. Another method that we investigated consisted of balancing the object on a pointed support. This is also not very practical with respect to humans. Is there a way to predict where an object's center of mass is without pushing or balancing it? Our goal here is to develop a theoretical method that will allow us to determine the location of the center of mass of a complex object, such as the uniform meter stick with two apples shown in **Figure 8.15**. In the next example, we start with an object that consists of three other objects: two people of different masses and a uniform seesaw whose supporting fulcrum (point of support) can be moved. To determine the location of the center of mass of a system involving two people and a uniform beam, we find a place for the fulcrum to support the seesaw and the two people so that the system remains in static equilibrium.

FIGURE 8.15 Where is the center of mass of the meter stick with two apples?

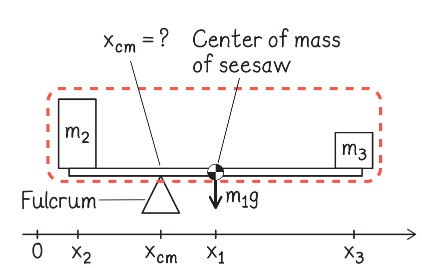


Adjust the position of the fulcrum supporting the seesaw until the system is in static equilibrium.

EXAMPLE 8.4 **Supporting a seesaw with two people**

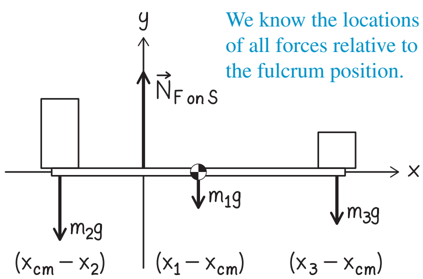
Find an expression for the position of the center of mass of a system that consists of a uniform seesaw of mass m_1 and two people of masses m_2 and m_3 sitting at the ends of the seesaw beam ($m_2 > m_3$).

Sketch and translate The figure at right shows a labeled sketch of the situation. The two people are represented as blocks. We choose the seesaw and two blocks as the system and construct a mathematical equation that lets us calculate the center of mass of that system. We place the x-axis along the seesaw with its origin at some arbitrary position on the left side of the seesaw. The center of mass of the seesaw beam is at x_1 and the two blocks rest at x_2 and x_3. At what *position* x should we place the fulcrum under the seesaw so that the system does not rotate—so that it remains in static equilibrium? At this position, the sum of all torques exerted on the system is zero. This position is the center of mass of the three-object system.



(CONTINUED)

Simplify and diagram We model the seesaw as a rigid body and model each of the people blocks as point-like objects. Assume that the fulcrum does not exert a friction force on the seesaw. As you can see in the force diagram, we know the locations of all forces except the normal force exerted at the fulcrum. We will calculate the unknown position x_{cm} of the fulcrum so the seesaw with two people on it balances—so that it satisfies the second condition of equilibrium.



Represent mathematically Apply the torque condition of equilibrium with the axis of rotation going through the unknown fulcrum position x_{cm}. Then determine the torques around this axis produced by the forces exerted on the system. The gravitational force exerted by Earth on the center of mass of the seesaw has magnitude m_1g, is exerted a distance $x_1 - x_{cm}$ from the axis of rotation, and has clockwise turning ability. The gravitational force exerted by Earth on block 2 has magnitude m_2g, is exerted a distance $x_{cm} - x_2$ from the axis of rotation, and has counterclockwise turning ability. The gravitational force exerted by Earth on block 3 has magnitude m_3g, is exerted a distance $x_3 - x_{cm}$ from the axis of rotation, and has clockwise turning ability. The torque condition of equilibrium for the system becomes

$$m_2g(x_{cm} - x_2) - m_1g(x_1 - x_{cm}) - m_3g(x_3 - x_{cm}) = 0$$

Solve and evaluate Divide all terms of the equation by the gravitational constant g and collect all terms involving x on one side of the equation to get

$$m_2x_{cm} + m_1x_{cm} + m_3x_{cm} = m_2x_2 + m_1x_1 + m_3x_3$$

or

$$x_{cm}(m_2 + m_1 + m_3) = m_2x_2 + m_1x_1 + m_3x_3$$

Divide both sides of the equation by $(m_2 + m_1 + m_3)$ to obtain an expression for the location of the center of mass of the three-object system:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

Let's evaluate this result. The units of x are meters. Next check some limiting cases to see if the result makes sense. Imagine that there are no people sitting on the seesaw $(m_2 = m_3 = 0)$. In this case, $x_{cm} = \frac{m_1x_1}{m_1} = x_1$. The center of mass of the seesaw-only system is at the center of mass of the seesaw x_1, as it should be since we assumed its mass was uniformly distributed. Finally, if we increase the mass of one of the people on the seesaw, the location of the center of mass moves closer to that person.

Try it yourself Where is the center of mass of a 3.0-kg, 2.0-m-long uniform beam with a 0.5-kg object on the right end and a 1.5-kg object on the left?

Answer 0.8 m from the left end of the beam.

In Example 8.4 we arrived at an expression for the location of the center of mass of a three-object system where all objects were located along one straight line. We can apply the same method to a system whose masses are distributed in a two-dimensional x-y plane. For such a two-dimensional system, we get the following:

Center of mass (quantitative definition) If we consider an object as consisting of parts 1, 2, 3, . . . n whose centers of masses are located at the coordinates (x_1, y_1); (x_2, y_2); (x_3, y_3); . . . (x_n, y_n), then the center of mass of this whole object is at the following coordinates:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$
$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n} \quad (8.4)$$

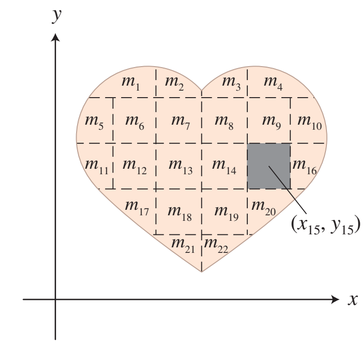
You can think of the position of the center of mass of an object of mass M as a weighted average of the positions of parts 1, 2, 3, etc. $(m_1, m_2, m_3 \ldots m_n$ etc.), or

$$x_{cm} = \frac{m_1}{M}x_1 + \frac{m_2}{M}x_2 + \frac{m_3}{M}x_3 + \cdots + \frac{m_n}{M}x_n$$
$$y_{cm} = \frac{m_1}{M}y_1 + \frac{m_2}{M}y_2 + \frac{m_3}{M}y_3 + \cdots + \frac{m_n}{M}y_n$$

Using Eq. (8.4) for an object with a continuous mass distribution is difficult and involves calculus. For example, in the first section we found the position of the center of mass of a cardboard heart empirically. Now we can calculate its position mathematically (**Figure 8.16**). We subdivide the cardboard into many tiny sections and insert the mass and position of each section into Eq. (8.4), and then add all the terms in the numerator and denominator together to determine the center of mass of the cardboard. For example, section 15 would contribute a term $m_{15}x_{15}$ in the numerator of the x_{cm} equation and a term m_{15} in the denominator. Usually, you will be given the location of the center of mass of such continuous mass distributions.

Knowledge of the center of mass helps you answer many questions: Why if you walk with a heavy backpack do you fall more easily? Why does a baseball bat have an elongated, uneven shape? Why do ships carry heavy loads in the bottom rather than near the top of the ship?

FIGURE 8.16 Finding the center of mass of a continuous mass distribution.



$$x_{\text{cm}} = \frac{m_1x_1 + \ldots + m_{15}x_{15} + \ldots + m_{22}x_{22}}{m_1 + \ldots + m_{15} + \ldots + m_{22}}$$

$$y_{\text{cm}} = \frac{m_1y_1 + \ldots + m_{15}y_{15} + \ldots + m_{22}y_{22}}{m_1 + \ldots + m_{15} + \ldots + m_{22}}$$

Mass distribution and center of mass

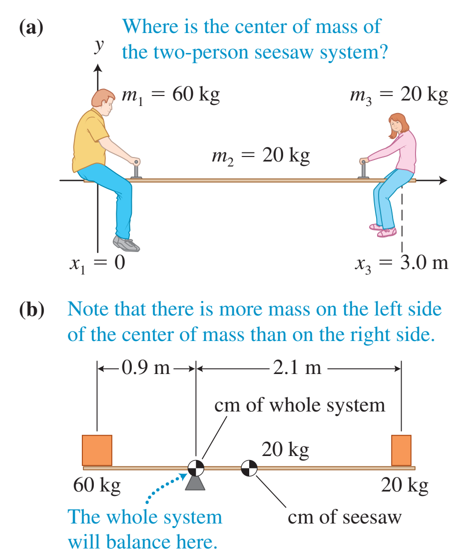
The term "center of mass" is deceiving. It might make you think that the center of mass of an object is located at a place where there is an equal amount of mass on each side. However, this is not necessarily the case. Consider again a uniform seesaw (see **Figure 8.17a**). Suppose that the mass of the seesaw is 20 kg, the length is 3.0 m, the mass of the person on the left end (m_1) is 60 kg, and the mass of the person on the right end (m_3) is 20 kg. Where is the center of mass of this two-person seesaw system, and how much mass is on the left side and the right side of the center of mass?

To find the center of mass, we need a coordinate system with an origin. The origin can be anywhere; we will put it at the location of the more massive person on the left side. The center of mass is then

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$= \frac{(60\text{ kg} \times 0\text{ m}) + (20\text{ kg} \times 1.5\text{ m}) + (20\text{ kg} \times 3.0\text{ m})}{60\text{ kg} + 20\text{ kg} + 20\text{ kg}} = 0.9\text{ m}$$

The seesaw will balance about a fulcrum located 0.9 m from m_1 and 2.1 m from m_2 (see Figure 8.17b). We see that the masses are not equal. The mass on the left side of the center of mass is much greater than on the right side—66 kg versus 34 kg. The larger mass on the left is a shorter distance from the center of mass than the smaller masses on the right, which on average are farther from the center of mass. However, the product of mass and distance on each side balances out, causing torques of equal magnitude. We could rename the center of mass as "the center of torque" to reflect the essence of the concept, but since this is not the term used in physics, we will continue to use the term center of mass.

FIGURE 8.17 The masses on each side of a system's center of mass are unequal.

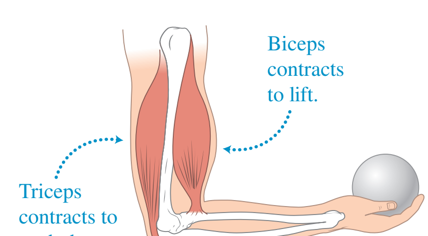


REVIEW QUESTION 8.4 Jade says that the mass of an object is evenly distributed around the location of its center of mass. Do you agree with her? If you disagree, how would you convince her of your opinion?

8.5 Skills for analyzing situations using equilibrium conditions

We often use the equations of equilibrium to determine one or two unknown forces if all other forces exerted on an object of interest are known. Consider the muscles of your arm when you lift a heavy ball or push down on a desktop (**Figure 8.18**). When

FIGURE 8.18 Muscles in the upper arm lift and push down on the forearm.



you hold a ball in your hand, your biceps muscle tenses and pulls up on your forearm in front of the elbow joint. When you push down with your hand on a desk, your triceps muscle tenses and pulls up on a protrusion of the forearm behind the elbow joint. The equations of equilibrium allow you to estimate these muscle tension forces—see the next example, which describes a general method for analyzing static equilibrium problems. The right side of the table applies the general strategies to the specific problem provided.

PROBLEM-SOLVING STRATEGY 8.1 Applying static equilibrium conditions

EXAMPLE 8.5 Using the biceps muscle to lift

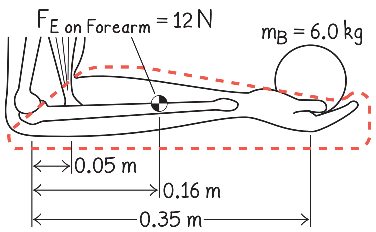
Imagine that you hold a 6.0-kg lead ball in your hand with your arm bent. The ball is 0.35 m from the elbow joint. The biceps muscle attaches to the forearm 0.050 m from the elbow joint and exerts a force on the forearm that allows it to support the ball. The center of mass of the 12-N forearm is 0.16 m from the elbow joint. Estimate the magnitude of (a) the force that the biceps muscle exerts on the forearm and (b) the force that the upper arm exerts on the forearm at the elbow.

Sketch and translate

- Construct a labeled sketch of the situation; mark knowns and unknowns. Choose an axis of rotation.
- Choose a system for analysis.

We choose the axis of rotation to be where the upper arm bone (the humerus) presses on the forearm at the elbow joint. This will eliminate from the torque equilibrium equation the unknown force that the upper arm exerts on the forearm.

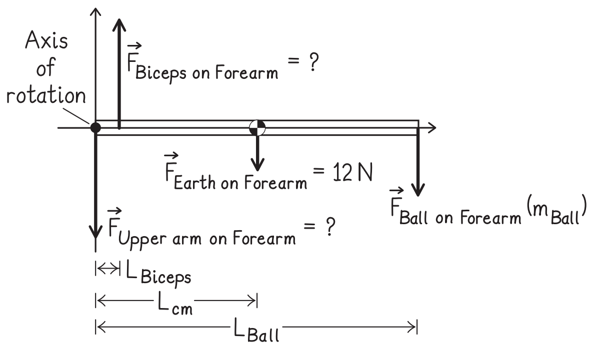
We choose the system of interest to be the forearm and hand.



Simplify and diagram

- Decide whether you will model the system as a rigid body or as a point-like object.
- Construct a force diagram for the system. Include the chosen coordinate system and the axis of rotation (the origin of the coordinate system).

Model the system as a rigid body and draw a force diagram for the forearm and hand.



Represent mathematically

- Use the force diagram to apply the conditions of equilibrium.

$$\Sigma\tau = 0$$

$$(F_{\text{UA on FA}})(0) + (F_{\text{Biceps on FA}})(L_{\text{Biceps}}\sin 90°) + (-F_{\text{E on FA}}(L_{\text{cm}}\sin 90°)) + (-F_{\text{Ball on FA}}(L_{\text{Ball}}\sin 90°)) = 0$$

$$\Sigma F_y = 0$$

$$(-F_{\text{UA on FA}}) + F_{\text{Biceps on FA}} + (-F_{\text{E on FA}}) + (-F_{\text{Ball on FA}}) = 0$$

Solve and evaluate

- Solve the equations for the quantities of interest.
- Evaluate the results. Check to see if their magnitudes are reasonable and if they have the correct signs and units. Also see if they have the expected values in limiting cases.

Substitute $\sin 90° = 1.0$ and rearrange the torque equation to find $F_{\text{Biceps on FA}}$.

$$\begin{aligned} F_{\text{Biceps on FA}} &= \left[(F_{\text{E on FA}})(L_{\text{cm}}) + (F_{\text{Ball on FA}})(L_{\text{Ball}}) \right] / L_{\text{Biceps}} \\ &= \left[(12\ \text{N})(0.16\ \text{m}) + (59\ \text{N})(0.35\ \text{m}) \right] / (0.050\ \text{m}) = 450\ \text{N} \end{aligned}$$

Use the force equation to find $F_{\text{UA on FA}}$:

$$\begin{aligned} F_{\text{UA on FA}} &= F_{\text{Biceps on FA}} - F_{\text{E on FA}} - F_{\text{Ball on FA}} \\ &= 450\ \text{N} - 12\ \text{N} - 59\ \text{N} = 380\ \text{N} \end{aligned}$$

The 450-N force exerted by the biceps on the forearm is much greater than the 59-N force exerted by the ball on the forearm. This difference occurs because the force exerted by the biceps is applied much closer to the axis of rotation than the force exerted by the lead ball.

If the center of mass of the forearm were farther from the elbow, the biceps would have to exert an even larger force.

Try it yourself How would the force exerted by the biceps on the forearm change if the biceps were attached to the forearm farther from the elbow?

Answer

A longer L_{Biceps} in the torque equilibrium equation would mean that the biceps muscle would need to exert a smaller force on the forearm when lifting something.

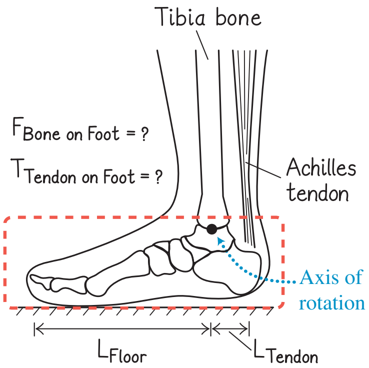
Standing on your toes

Most injuries to the Achilles tendon occur during abrupt movement, such as jumps and lunges. However, we will analyze what happens to your Achilles tendon in a less stressed situation.

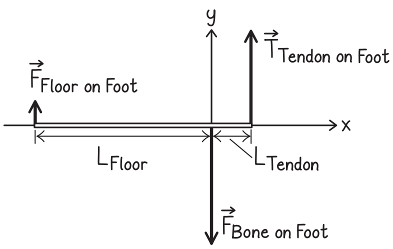
EXAMPLE 8.6 Standing with slightly elevated heel

Suppose you stand on your toes with your heel slightly off the ground. In order to do this, the larger of the two lower leg bones (the tibia) exerts a force on the ankle joint where it contacts the foot. The Achilles tendon simultaneously exerts a force on the heel, pulling up on it in order for the foot to be in static equilibrium. What is the magnitude of the force that the tibia exerts on the ankle joint? What is the magnitude of the force that the Achilles tendon exerts on the heel?

Sketch and translate First we sketch the foot with the Achilles tendon and the tibia. We choose the foot as the system of interest. Three forces are exerted on the foot: the tibia is pushing down on the foot at the ankle joint; the floor is pushing up on the ball of the foot and the toes; and the Achilles tendon is pulling up on the heel. We choose the axis of rotation as the place where the tibia presses against the foot.



Simplify and diagram Model the foot as a very light rigid body. The problem says that the foot is barely off the ground, so we will neglect the angle between the foot and the ground and consider the foot horizontal. The gravitational force exerted on the foot by Earth is quite small compared with the other forces that are being exerted on it, so we will ignore it. A force diagram for the foot is shown at right. When you are standing on the ball and toes of both feet, the floor exerts an upward force on each foot equal to half the magnitude of the gravitational force that Earth exerts on your entire body: $F_{\text{Floor on Foot}} = \dfrac{m_{\text{Body}}g}{2}$. The Achilles tendon pulls up on the heel of the foot, exerting a force $T_{\text{Tendon on Foot}}$. The tibia bone in the lower leg pushes down on the ankle joint exerting a force $F_{\text{Bone on Foot}}$.



Represent mathematically Let's apply the conditions of equilibrium to this system. Note that the distance from the toes to the joint L_{Floor} is somewhat longer than the distance from the joint to the Achilles tendon attachment point L_{Tendon}. The torque condition of equilibrium becomes

$$+\left[T_{\text{Tendon on Foot}}(L_{\text{Tendon}}) \right] + F_{\text{Bone on Foot}}(0) - \frac{m_{\text{Body}}g}{2}(L_{\text{Floor}}) = 0$$

$$\Rightarrow T_{\text{Tendon on Foot}} = \frac{m_{\text{Body}}g}{2}\left(\frac{L_{\text{Floor}}}{L_{\text{Tendon}}} \right)$$

(CONTINUED)

Now, apply the y-scalar component of the force condition of equilibrium:

$$\Sigma F_y = T_{\text{Tendon on Foot}} + (-F_{\text{Bone on Foot}}) + \frac{m_{\text{Body}}g}{2} = 0$$

$$\Rightarrow F_{\text{Bone on Foot}} = T_{\text{Tendon on Foot}} + \frac{m_{\text{Body}}g}{2}$$

Solve and evaluate The distance from the place where the bone contacts the foot to where the floor contacts the foot is about 5 times longer than the distance from the bone to where the tendon contacts the foot. Consequently, the force that the Achilles tendon exerts on the foot is about

$$T_{\text{Tendon on Foot}} = \frac{m_{\text{Body}}g}{2}\left(\frac{L_{\text{Floor}}}{L_{\text{Tendon}}}\right) = \frac{m_{\text{Body}}g}{2}(5) = \frac{5}{2}m_{\text{Body}}g$$

or two and a half times the gravitational force that Earth exerts on the body. Using $g = 10\ \text{N/kg}$ for a 70-kg person, this force will be about 1750 N. That's a very large force for something as simple as standing with your heel slightly elevated! The force exerted on the joint by the leg bone would be

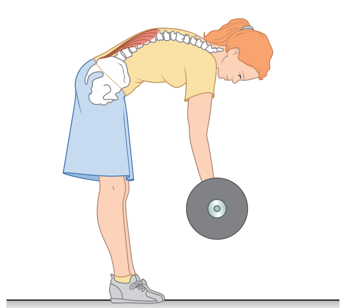
$$\begin{aligned} F_{\text{Bone on Foot}} &= T_{\text{Tendon on Foot}} + \left(\frac{m_{\text{Body}}g}{2}\right) \\ &= 1750\ \text{N} + 350\ \text{N} = 2100\ \text{N} \end{aligned}$$

This force is three times the weight of the person! The forces are much greater when moving. Thus, every time you lift your foot to walk, run, or jump, the tendon tension and joint compression are several times greater than the gravitational force that Earth exerts on your entire body.

Try it yourself Estimate the increase in the magnitude of the force exerted by the Achilles tendon on the foot of the person in this example if his mass were 90 kg instead of 70 kg.

Answer An increase of at least 500 N (110 lb).

FIGURE 8.19 A bad way to lift.



Lifting from a bent position

Back problems often originate with improper lifting techniques—a person bends over at the waist and reaches to the ground to pick up a box or a barbell. In **Figure 8.19**, the barbell pulls down on the woman's arms far from the axis of rotation of her upper body about her hip area. This downward pull causes a large clockwise torque on her upper body. To prevent her from tipping over, her back muscles must exert a huge force on the backbone, thus producing an opposing counterclockwise torque. This force exerted by the back muscles compresses the disks that separate vertebrae and can lead to damage of the disks, especially in the lower back. We can use the equilibrium equations to estimate the forces and torques involved in such lifting.

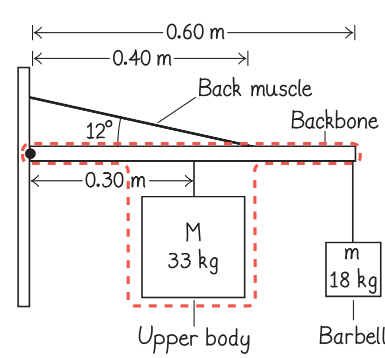
EXAMPLE 8.7 Lifting incorrectly from a bent position

Estimate the magnitude of the force that the back muscle in the woman's back in Figure 8.19 exerts on her backbone and the force that her backbone exerts on the disks in her lower back when she lifts an 18-kg barbell. The woman's mass is 55 kg. Model the woman's upper body as a rigid body.

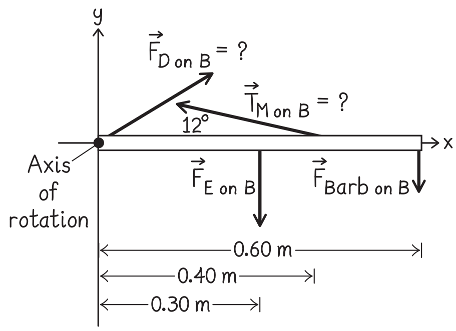
- The back muscle attaches two-thirds of the way from the bottom of her $l = 0.60$ m backbone and makes a 12° angle relative to the horizontal backbone.
- The mass of her upper body is $M = 33$ kg centered at the middle of the backbone and has uniform mass distribution. The axis of rotation is at the left end of the backbone and represents one of the disks in the lower back.

Sketch and translate

The figure at right is our mechanical model of a person lifting a barbell. We want to estimate the magnitudes of the force $T_{\text{M on B}}$ that the back muscle exerts on the backbone and the force $F_{\text{D on B}}$ that the disk in the lower back exerts on the backbone. The force that the disk exerts on the bone is equal in magnitude to the force exerted by the backbone on the disk. The upper body (including the backbone) is the system of interest, but we consider the back muscle to be external to the system since we want to focus on the force it exerts on the backbone. The hinge where the upper body meets the lower body is the axis of rotation.



Simplify and diagram We next draw a force diagram for the upper body. The gravitational force that Earth exerts on the upper body $F_{\text{E on B}}$ at its center of mass is $Mg = (33\ \text{kg})(9.8\ \text{N/kg}) = 323\ \text{N}(73\ \text{lb})$. The barbell exerts a force on the upper body equal to $mg = (18\ \text{kg}) \times (9.8\ \text{N/kg}) = 176\ \text{N}(40\ \text{lb})$. Because of our choice of axis of rotation, the force exerted by the disk on the upper body $F_{\text{D on B}}$ will not produce a torque. The gravitational force exerted by Earth on the upper body and the force that the barbell exerts on the upper body have clockwise turning ability, while the tension force exerted by the back muscles on the upper body has counterclockwise turning ability.



Represent mathematically The torque condition of equilibrium for the upper body is

$$\Sigma\tau = +(F_{\text{D on B}})(0) + [-(Mg)(l/2)\sin 90°] + (T_{\text{M on B}})(2l/3)\sin 12° + [-(F_{\text{Barb on B}})(l)\sin 90°] = 0$$

The x- and y-component forms of the force condition of equilibrium for the backbone are

$$\Sigma F_x = F_{\text{D on B}\,x} + (-T_{\text{M on B}}\cos 12°) = 0$$
$$\Sigma F_y = F_{\text{D on B}\,y} + T_{\text{M on B}}\sin 12° + (-mg) + (-Mg) = 0$$

where $F_{\text{D on B}\,x}$ and $F_{\text{D on B}\,y}$ are the scalar components of the force that the disk exerts on the upper body.

Solve and evaluate We can solve the torque equation immediately to determine the magnitude of the force that the back muscle exerts on the backbone:

$$T_{\text{M on B}} = \frac{(Mg)(l/2)(\sin 90°) + (mg)(l)\sin 90°}{(2l/3)(\sin 12°)}$$

Note that the backbone length l in the numerator and denominator of all of the terms in this equation cancels out. Thus,

$$T_{\text{M on B}} = \frac{(Mg)(1/2)(\sin 90°) + (mg)(1)\sin 90°}{(2/3)(\sin 12°)} = \frac{(323\text{ N})(0.50)(1.0) + (176\text{ N})(1)(1.0)}{(0.667)(0.208)} = 2432\text{ N }(547\text{ lb})$$

We then find $F_{\text{D on B}\,x}$ from the x-component force equation:

$$F_{\text{D on B}\,x} = +T_{\text{M on B}}\cos 12° = +(2432\text{ N})\cos 12° = +2380\text{ N}$$

and $F_{\text{D on B}\,y}$ from the y-component force equation:

$$F_{\text{D on B}\,y} = +Mg + mg - T_{\text{M on B}}\sin 12° = +323\text{ N} + 176\text{ N} - (2432\text{ N})(\sin 12°) = -7\text{ N}$$

Thus, the magnitude of $F_{\text{D on B}}$ is

$$F_{\text{D on B}} = \sqrt{(2380\text{ N})^2 + (-7\text{ N})^2} = 2380\text{ N }(540\text{ lb})$$

The direction of $\vec{F}_{\text{D on B}}$ can be determined using trigonometry:

$$\tan\theta = \frac{F_{\text{D on B}\,y}}{F_{\text{D on B}\,x}} = \frac{-7\text{ N}}{2380\text{ N}} = -0.0029$$

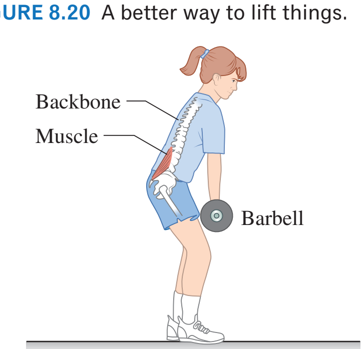
or $\theta = 0.17°$ below the horizontal. We've found that the back muscles exert a force more than four times the gravitational force that Earth exerts on the person and that the disks of the lower back are compressed by a comparable force.

Try it yourself Suppose that a college football lineman stands on top of a 1-inch-diameter circular disk. How many 275-lb linemen, one on top of the other, would exert the same compression force on the disk as that exerted on the woman's disk when she lifts the 40-lb barbell?

Answer The magnitude of the force exerted on the vertebral disk is equivalent to two linemen (2×275 lb $= 550$ lb) standing on the 1-inch-diameter disk.

To lift correctly, keep your back more vertical with the barbell close to your body, as in **Figure 8.20**. Bend your knees and lift with your legs. With this orientation, the back muscle exerts one-third of the force that is exerted when lifting incorrectly. The disks in the lower back undergo one-half the compression they would experience from lifting incorrectly.

FIGURE 8.20 A better way to lift things.



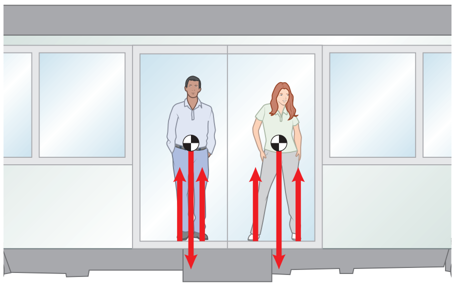
REVIEW QUESTION 8.5 You are trying to hold a heavy dumbbell in one hand so that your arm is perpendicular to your body. Why is it easier to hold it with a bent arm than with a straight arm?

8.6 Stability of equilibrium

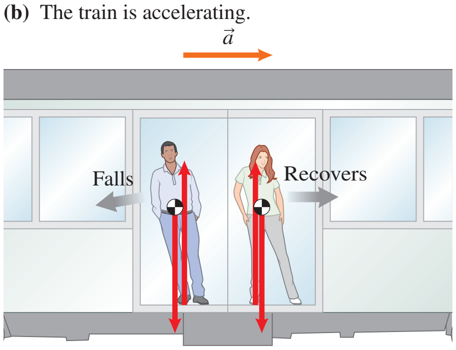
Often, objects can remain in equilibrium for a long time interval—you can sit comfortably for a long time on a living room couch without tipping. But sometimes equilibrium is achieved for only a short time interval—think of sitting on a chair and tilting it backward too far onto its rear legs.

FIGURE 8.21 Balancing on the subway.

(a) The train is at rest.



(b) The train is accelerating.



Equilibrium and tipping objects

You have probably observed that it is easier to balance and avoid falling while standing in a moving bus or subway train if you spread your feet apart in the direction of motion. By doing this you are increasing the **area of support**, the area of contact between an object and the surface it is supported by. To understand area of support, consider two people riding the subway (see **Figure 8.21**).

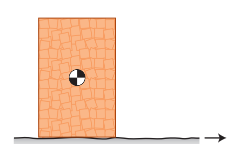
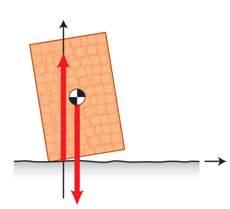
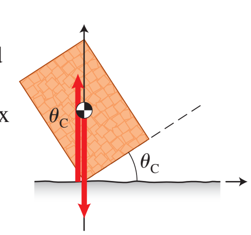
In Figure 8.21a, the man's feet are close together, whereas the woman's feet are farther apart. When the train accelerates toward the right, as shown in Figure 8.21b, the two people tilt to the left. The man falls over because the gravitational force exerted by Earth on his body is outside the area of support provided by his feet. The woman recovers because the gravitational force still points between her feet. These patterns lead us to a tentative rule about tipping:

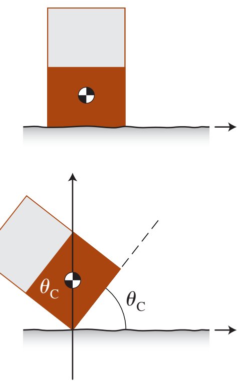
> For an object in static equilibrium, if a vertical line passing through the object's center of mass is within the object's area of support, the object does not tip. If the line is not within the area of support, the object tips.

If this is a general rule, then we can use it to predict the angle at which an object with a known center of mass will tip over (see Testing Experiment **Table 8.4**).

TESTING EXPERIMENT TABLE 8.4 Testing our tentative rule about tipping

VIDEO TET 8.4

Testing experiment	Prediction	Outcome
Experiment 1. Place a full box of crackers on a flat but rough surface. Its center of mass is at its geometric center. The box's height is 20 cm and the bottom surface is 13 cm × 6.5 cm.	1. The center of mass of a full box of crackers is at its geometric center. 	
Tilt the box along the 13-cm side a little and release it.	2. When you release the slightly tilted box, it returns to the vertical position because of the torque due to the gravitational force exerted by Earth. 	The box returns to the vertical position.
Tilt the box at larger and larger angles. Predict the angle at which the box will tip.	3. When you tilt the box more and more, eventually the line defined by the gravitational force passes over the support point for the box at its bottom edge. Tilting the box more than this critical angle causes it to tip over. 	
	4. For a box with a side of 13 cm and a height of 20 cm, this angle will be $\theta_C = \tan^{-1}(13\text{ cm}/20\text{ cm}) = 33°$.	The outcome matches the prediction.

Testing experiment	Prediction	Outcome
Experiment 2. Repeat the experiment, but this time replace the crackers with a wooden block of the same mass. The block takes up about half the space that the crackers of the same mass did. Predict the angle at which the box will tip.	The center of mass should now be about one-fourth of the way from the bottom (if we neglect the mass of the box). The critical tilt angle should now be $\theta_C = \tan^{-1}(13\text{ cm}/10\text{ cm}) = 52°$. 	The outcome matches the prediction within experimental uncertainty (about 1°).

Conclusion

The outcomes are consistent with the predictions. We've increased our confidence in the tipping rule: for an object to be in static equilibrium, the line defined by the gravitational force exerted by Earth must pass within the object's area of support. If it is not within the area of support, the object tips over.

We now know that an object will tip if it is tilted so that the gravitational force passes beyond its area of support. If the area of support is large or if the center of mass is closer to the ground, more tipping is possible without the object falling over—it is more stable. This idea is regularly used in building construction. Tall towers (like the Eiffel Tower) have a wide bottom and a narrower top. The Leaning Tower of Pisa does not tip because a vertical line through its center of mass passes within the area of support (**Figure 8.22**).

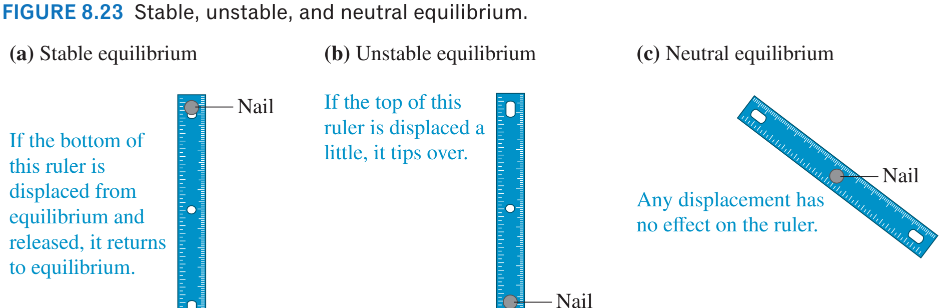
FIGURE 8.22 Because $F_{\text{E on Tower}}$ passes through the base of the tower, the Leaning Tower of Pisa does not tip over.

The equilibrium of a system is stable against tipping if the vertical line through its center of mass passes through the system's area of support.

Equilibrium and rotating objects

Objects that can rotate around a fixed axis can also have either stable or unstable equilibria. Consider a ruler with several holes in it. If you hang the ruler on a nail using a hole near one end, it hangs as shown in **Figure 8.23a**. If you pull the bottom of the ruler to the side and release it, the ruler swings back and forth with decreasing maximum displacement from the equilibrium position, but eventually hangs straight down. This equilibrium position is called *stable* because the ruler always tries to return to that position if free to rotate. However, if you turn the ruler 180° so that the axis of rotation is at the bottom of the ruler (see Figure 8.23b), it can stay in this position only if very carefully balanced. If disturbed, the ruler swings down and never returns. In this case the ruler is in *unstable* equilibrium.

FIGURE 8.23 Stable, unstable, and neutral equilibrium.



The stability or lack of stability can be understood by considering the torque around the axis of rotation due to the force exerted by Earth on the object. In both positions a and b, the sum of the forces exerted on the ruler by Earth and by the nail is zero. The difference is that in the first case the center of mass is *below the axis of rotation*; in the second case it is *above the axis of rotation*. Torques produced by gravitational forces tend to lower the center of mass of objects. In other words, if it is possible for the object to rotate so that its center of mass becomes lower, it will tend to do so.

If we hang the ruler using the center hole (Figure 8.23c), it remains in whatever position we leave it. Both the force exerted by the nail and the gravitational force exerted by Earth produce zero torques. This is called *neutral* equilibrium.

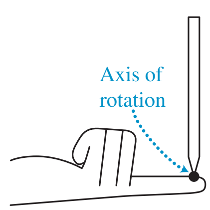
The equilibrium of a system is stable against rotation if the center of mass of the rotating object is below the axis of rotation.

CONCEPTUAL EXERCISE 8.8 Balancing a pencil

Is it possible to balance with stable equilibrium the pointed tip of a pencil on your finger?

Sketch and translate The figure below shows a sketch of the situation.

Simplify and diagram The forces exerted on the pencil are shown at right. The tip of the pencil is the axis of rotation. When the pencil is tilted by only a small angle, the line defined by the gravitational force exerted by Earth on the pencil is not within the area of support of the pencil (which is just the pencil tip). This equilibrium is unstable. Having the center of mass *above* the axis of rotation leads to this instability. To make it stable, we need to lower the center of mass to below the axis of rotation. We can do it by attaching a pocketknife to the pencil, as shown below. Notice that the massive part of the knife is below the tip of the pencil and so is the center of mass of the system. Then when the pencil tilts, the torque due to the gravitational force brings it back to the equilibrium position.



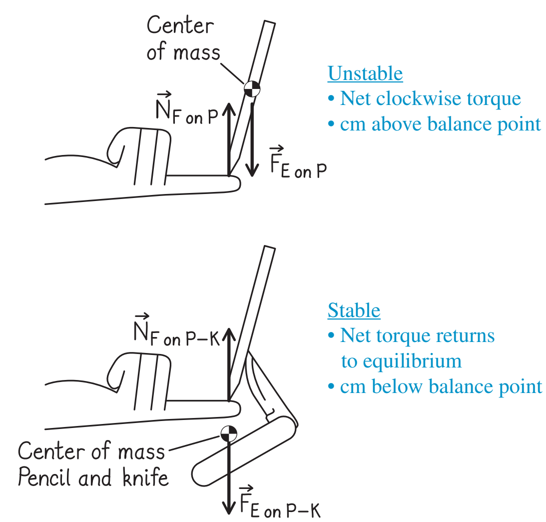
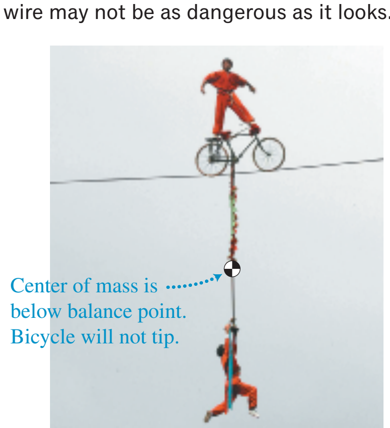


FIGURE 8.24 Balancing a bicycle on a high wire may not be as dangerous as it looks.



TIP Always try to understand new situations in terms of ideas we have already discussed. The trick of balancing the pencil by adding a pocketknife can be understood with the rules we expressed above: (1) the equilibrium is most stable when the center of mass of the system is in the lowest possible position or, equivalently, (2) when the gravitational potential energy of the system has the smallest value.

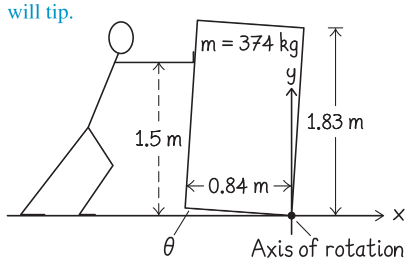
The rules we have learned about equilibrium and stability have many applications, including circus tricks. Think about where the center of mass is located for the bicycle and the two people shown in **Figure 8.24**. Another application involves vending machines.

EXAMPLE 8.9 Tipping a vending machine

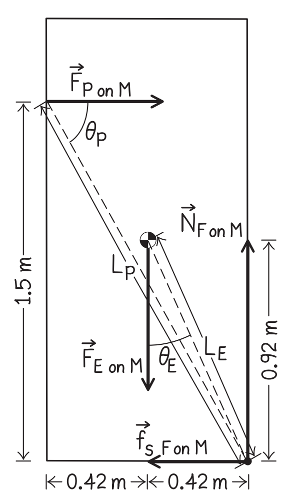
According to the U.S. Consumer Product Safety Commission, tipped vending machines caused 37 fatalities between 1978 and 1995 (2.2 deaths per year). Why is tipping vending machines so dangerous? A typical vending machine is 1.83 m high, 0.84 m deep, and 0.94 m wide and has a mass of 374 kg. It is supported by a leg on each of the four corners of its base. (a) Determine the horizontal pushing force you need to exert on its front surface 1.5 m above the floor in order to just lift its front feet off the surface (so that it will be supported completely by its back two feet). (b) At what critical angle would it fall forward?

Sketch and translate (a) See the sketch of the situation at right. The axis of rotation is through the back support legs of the vending machine.

Find the angle θ at which the vending machine will tip.



Simplify and diagram Model the vending machine as a rigid body. Three objects exert forces on the vending machine (shown in the side view force diagram at right; the tilt is not shown since the vending machine is barely off the floor): the person exerting force $\vec{F}_{\text{P on M}}$ on the machine, Earth exerting gravitational force $\vec{F}_{\text{E on M}}$, and the floor exerting a force on the machine that we represent with two vector components: normal force $\vec{N}_{\text{F on M}}$ and static friction force $\vec{f}_{\text{s F on M}}$ on the machine.



Represent mathematically (a) We use the torque condition of equilibrium to analyze the force needed to tilt the vending machine until the front legs are just barely off the floor. The force exerted by the person has a clockwise turning ability, while the gravitational force has counterclockwise turning ability. The force exerted by the floor on the back legs does not produce a torque, since it is exerted at the axis of rotation.

$$-F_{\text{P on M}}L_{\text{P}} \sin\theta_{\text{P}} + F_{\text{E on M}}L_{\text{E}} \sin\theta_{\text{E}} + N_{\text{F on M}}(0) + f_{\text{s F on M}}(0) = 0$$

(b) We can apply the analysis in Table 8.4 for this situation:

$$\theta_{\text{C}} = \tan^{-1}\left(\frac{\text{depth}}{\text{height}}\right)$$

Solve and evaluate (a) Using the torque equation, we can find the normal force that the person needs to exert on the vending machine to just barely lift its front off the floor:

$$-F_{\text{P on M}}L_{\text{P}}\left(\frac{1.5\text{ m}}{L_{\text{P}}}\right) + (3700\text{ N})L_{\text{E}}\left(\frac{0.42\text{ m}}{L_{\text{E}}}\right) + N_{\text{F on M}}(0) + f_{\text{s F on M}}(0) = 0$$

or $F_{\text{P on M}} = 1000$ N or 220 lb.

(b) We find that the critical tipping angle is

$$\theta_{\text{tipping}} = \tan^{-1}\left(\frac{\text{depth}}{\text{height}}\right) = \tan^{-1}\left(\frac{0.84\text{ m}}{1.83\text{ m}}\right) \approx 24°$$

Both answers seem reasonable.

Try it yourself Determine how hard you need to push against the vending machine to keep it tilted at a 25° angle above the horizontal.

Answer 0 N. A vertical line through the center of mass passes through the axis of rotation, so the net torque about that axis is zero. However, this equilibrium is unstable and represents a dangerous situation.

The chance of being injured by a tipped vending machine is small since a large force must be exerted on it to tilt it up, and it must be tilted at a fairly large angle before it reaches an unstable equilibrium. A more common danger is falling bookcases in regions subject to earthquakes. The base of a typical 2.5-m-tall bookcase is less than 0.3 m deep. The shelves above the base that are filled with books are the same size as the base. If tilted by just

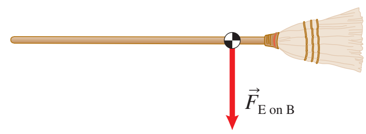
$$\theta_{\text{C}} = \tan^{-1}\left(\frac{\text{depth}}{\text{height}}\right) \approx \tan^{-1}\left(\frac{0.15\text{ m}}{1.25\text{ m}}\right) \approx 7°$$

the bookcase can tip over. In earthquake-prone regions, people often attach a bracket to the top back of the bookcase and then anchor it to the wall.

REVIEW QUESTION 8.6 Why is a ball hanging by a thread in stable equilibrium, while a pencil balanced on its tip is in unstable equilibrium?

Summary

Center of mass The gravitational force that Earth exerts on an object can be considered to be exerted entirely on the object's center of mass.

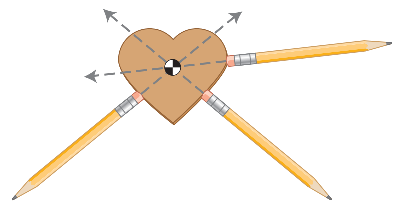


$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

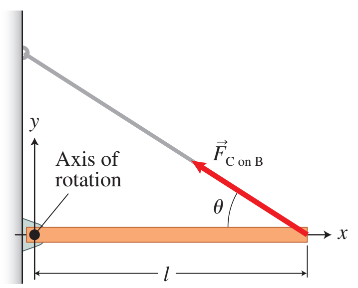
$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots + m_ny_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

Eq. (8.4)

An external force pointing directly toward or away from the center of mass of a free object will not cause the object to turn or rotate. (Sections 8.1 and 8.4)

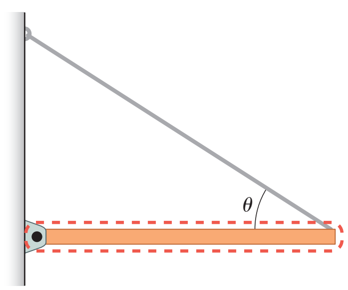


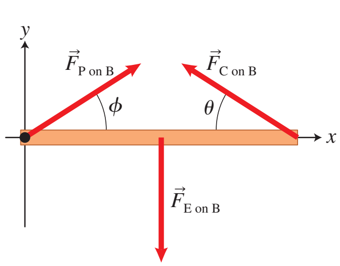
A **torque** τ around an *axis of rotation* is a physical quantity characterizing the turning ability of a force with respect to a particular axis of rotation. The torque is positive if the force tends to turn the object counterclockwise and negative if it tends to turn the object clockwise about the axis of rotation. (Section 8.2)



$$\tau = \pm Fl \sin\theta \qquad \text{Eq. (8.1)}$$

Static equilibrium is a state in which a rigid body is at rest and remains at rest both translationally and rotationally. (Section 8.3)





Translational (force) condition:

$$\Sigma F_{\text{on O } x} = F_{1 \text{ on O } x} + \cdots + F_{n \text{ on O } x} = 0 \qquad \text{Eq. (8.2}x\text{)}$$

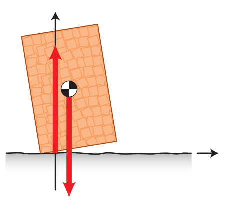
$$\Sigma F_{\text{on O } y} = F_{1 \text{ on O } y} + \cdots + F_{n \text{ on O } y} = 0 \qquad \text{Eq. (8.2}y\text{)}$$

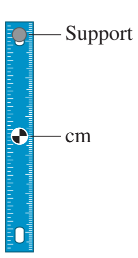
Rotational (torque) condition:

$$\Sigma\tau = \tau_1 + \tau_2 + \cdots + \tau_n = 0 \qquad \text{Eq. (8.3)}$$

The **equilibrium** of a system is stable against tipping as long as the line through its center of mass passes through the system's area of support.

The **equilibrium** of a system is stable against rotating as long as the center of mass of the rotating object is below the axis of rotation. (Section 8.6)



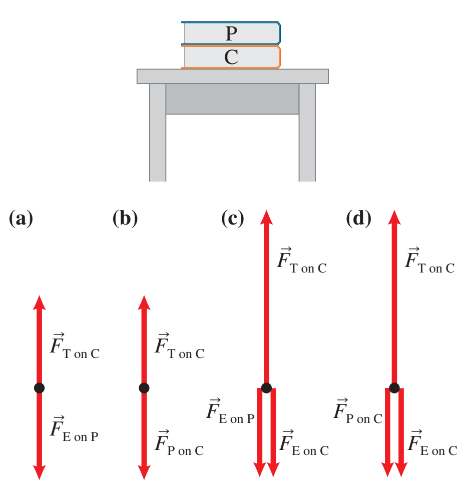


Questions

Multiple Choice Questions

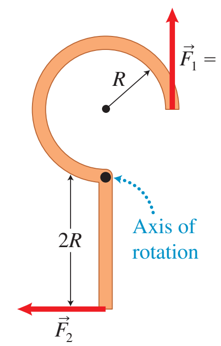
1. A falling leaf usually flutters while falling. However, we have learned that the force that Earth exerts on an object is exerted at its center of mass and thus should not cause rotational motion. How can you resolve this contradiction?
   (a) A leaf is not a rigid body and the rule does not apply.
   (b) There are other forces exerted on the leaf as it falls besides the force exerted by Earth.
   (c) Some forces were not taken into account when we defined the center of mass.
2. You have an irregularly shaped flat object. To find its center of mass you can do which of the following?
   (a) Find a point where you can put a fulcrum to balance it.
   (b) Push the object in different directions and find the point of intersection of the lines of action of the forces that do not rotate the object.
   (c) Separate the object into several regularly shaped objects whose center of masses you know, and use the mathematical definition of the center of mass to find it.
   (d) All of the above
   (e) a and b only
3. A hammock is tied with ropes between two trees. A person lies in it. Under what circumstances are its ropes more likely to break?
   (a) If stretched tightly between the trees
   (b) If stretched loosely between the trees
   (c) The ropes always have equal likelihood to break.
4. Where is the center of mass of a donut?
   (a) In the center of the hole
   (b) Uniformly distributed throughout the donut
   (c) Cannot be found
5. A physics textbook lies on top of a chemistry book, which rests on a table. Which force diagram below best describes the forces exerted by other objects on the chemistry book (**Figure Q8.5**)?

FIGURE Q8.5



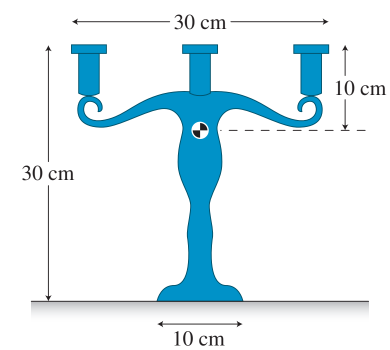
6. What does it mean if the torque of a force is positive?
   (a) The object exerting the force is on the right side of the axis of rotation.
   (b) The object exerting the force is on the left side of the axis of rotation.
   (c) The force points up.
   (d) The force points down.
   (e) None of these choices is necessarily correct.
7. Constant force $\vec{F}_2$ is exerted on the lower arm of the object in **Figure Q8.7**. What is the magnitude of the force $\vec{F}_1$ such that the object is in equilibrium?

FIGURE Q8.7



   (a) $F_1 = F_2$
   (b) $F_1 = \sqrt{2}F_2$
   (c) $F_1 = \frac{\pi}{2}F_2$
   (d) $F_1 = 2F_2$
   (e) $F_1 = 0$
   (f) The object cannot be in equilibrium no matter how large the force F_1 is.
8. Why do you tilt your body forward when hiking with a heavy backpack?
   (a) The backpack pushes you down.
   (b) Bending forward makes the backpack press less on your back.
   (c) Bending forward moves the center of mass of you and the backpack above your feet.
9. What does it mean if the torque of a 10-N force is zero?
   (a) The force is exerted at the axis of rotation.
   (b) A line parallel to and passing through the place where the force is exerted passes through the axis of rotation.
   (c) Both a and b are correct.
10. What is the maximum angle to the horizontal you can tilt the candleholder in **Figure Q8.10** before it tips over? The center of mass of the candleholder is marked on the figure.

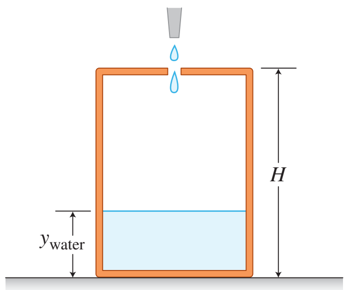
FIGURE Q8.10

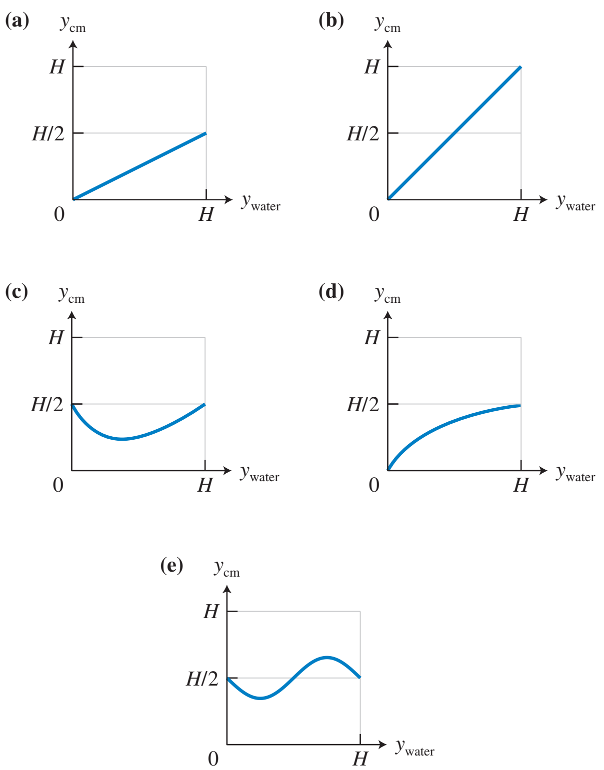


   (a) 26.6°
   (b) 18.4°
   (c) 14.0°
   (d) 9.4°
   (e) None of the above

11. Water starts dripping at a constant rate into an empty plastic can through a small opening at the top, as shown in **Figure Q8.11**. Which graph shows the correct qualitative dependence of the vertical position of the center of mass of the water-can system (y_{cm}) on the height of the water level in the can (y_{water})?

FIGURE Q8.11

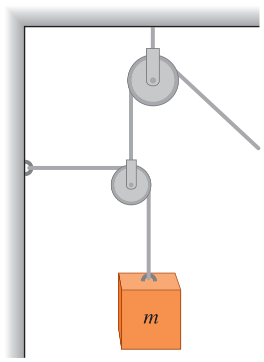




Conceptual Questions

12. Is it possible for an object not to be in equilibrium when the net force exerted on it by other objects is zero? Give an example.
13. Explain the meaning of torque so that a friend not taking physics can understand.
14. Something is wrong with the orientation of the ropes shown in **Figure Q8.14**. Use the first condition of equilibrium for the hanging pulley to help explain this error, and then redraw the sketch as you would expect to see it.

FIGURE Q8.14



15. What are the two conditions of equilibrium? What happens if one or the other condition is not satisfied?
16. Give three examples of situations in which an object is starting to rotate even though the sum of the forces exerted on the object is zero.
17. The force that the body muscles exert on bones that are used to lift various objects is usually five to ten times greater than the gravitational force that Earth exerts on the object being lifted. Explain and give an example.
18. A ladder leans against a wall. Construct a force diagram showing the direction of all forces exerted on the ladder. Identify two interacting objects for each force.
19. Using a crowbar, a person can remove a nail by exerting little force, whereas pulling directly on the nail requires a large force to remove it (you probably can't). Why? Draw a sketch to support your answer.
20. Is it more difficult to do a sit-up with your hands stretched in front of you or with them behind your head? Explain.
21. Sit on a chair with your feet straight down at the front of the chair. Keeping your back perpendicular to the floor, try to stand up without leaning forward. Explain why it is impossible to do it.
22. Can you balance the tip of a wooden ruler vertically on a fingertip? Why is it so difficult? Design a method to balance the ruler on your fingertip. Describe any extra material(s) you will use.
23. Try to balance a sharp wooden pencil on your fingertip, point down. (Hint: A small pocketknife might help by lowering the center of mass of the system.)
24. Design a device that you can use to successfully walk on a tightrope.
25. Explain why it is easier to keep your balance while jumping on two feet than while hopping on one.
26. A carpenter's trick to keep nails from bending when they are pounded into a hard material is to grip the center of the nail with pliers. Why does this help?

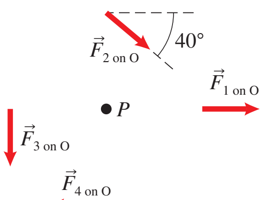
Problems

Below, **BIO** indicates a problem with a biological or medical focus. Problems labeled **EST** ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

8.2 Torque: a new physical quantity

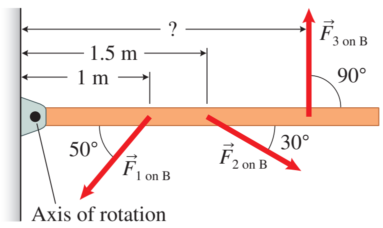
1. Determine the torques about the axis of rotation P produced by each of the four forces shown in **Figure P8.1**. All forces have magnitudes of 120 N and are exerted a distance of 2.0 m from P on some unshown object O.

FIGURE P8.1



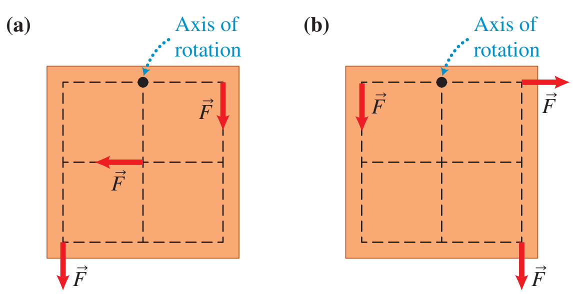
2. Three 200-N forces are exerted on the beam shown in **Figure P8.2**. (a) Determine the torques about the axis of rotation on the left produced by forces $\vec{F}_{1\text{ on B}}$ and $\vec{F}_{2\text{ on B}}$. (b) At what distance from the axis of rotation must $\vec{F}_{3\text{ on B}}$ be exerted to cause a torque that balances those produced by $\vec{F}_{1\text{ on B}}$ and $\vec{F}_{2\text{ on B}}$?

FIGURE P8.2



3. * A 2.0-m-long, 15-kg ladder is resting against a house wall, making a 30° angle with the vertical wall. The coefficient of static friction between the ladder feet and the ground is 0.40, and between the top of the ladder and the wall the coefficient is 0. Make a list of the physical quantities you can determine or estimate using this information and calculate them.
4. **Figure P8.4** shows two different situations where three forces of equal magnitude are exerted on a square board hanging on a wall, supported by a nail. For each case, determine the sign of the total torque that the three forces exert on the board.

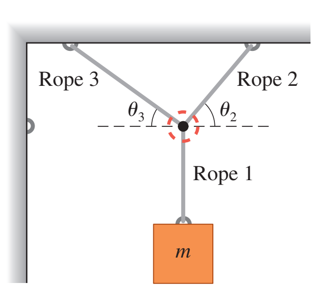
FIGURE P8.4



8.3 Conditions of equilibrium

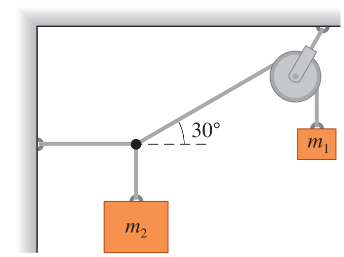
5. Three friends tie three ropes in a knot and pull on the ropes in different directions. Adrienne (rope 1) exerts a 20-N force in the positive x-direction, and Jim (rope 2) exerts a 40-N force at an angle 53° above the negative x-axis. Luis (rope 3) exerts a force that balances the first two so that the knot does not move. (a) Construct a force diagram for the knot. (b) Use equilibrium conditions to write equations that can be used to determine $F_{\text{L on K}\,x}$ and $F_{\text{L on K}\,y}$. (c) Use equilibrium conditions to write equations that can be used to determine the magnitude and direction of $\vec{F}_{\text{L on K}}$.
6. Adrienne from Problem 8.5 now exerts a 100-N force $\vec{F}_{\text{A on K}}$ that points 30° below the positive x-axis, and Jim exerts a 150-N force in the negative y-direction. How hard and in what direction does Luis now have to pull the knot so that it remains in equilibrium?
7. * Kate joins Jim, Luis, and Adrienne in the rope-pulling exercise described in the previous two problems. This time, they tie four ropes to a ring. The three friends each pull on one rope, exerting the following forces: $\vec{T}_{1\text{ on R}}$ (50 N in the positive y-direction), $\vec{T}_{2\text{ on R}}$ (20 N, 25° above the negative x-axis), and $\vec{T}_{3\text{ on R}}$ (70 N, 70° below the negative x-axis). Kate pulls rope 4, exerting a force $\vec{T}_{4\text{ on R}}$ so that the ring remains in equilibrium. (a) Construct a force diagram for the ring. (b) Use the first condition of equilibrium to write two equations that can be used to determine $T_{4\text{ on R}\,x}$ and $T_{4\text{ on R}\,y}$. (c) Solve these equations and determine the magnitude and direction of $\vec{T}_{4\text{ on R}}$.
8. You hang a light in front of your house using an elaborate system to keep the 1.2-kg light in static equilibrium (see **Figure P8.8**). What are the magnitudes of the forces that the ropes must exert on the knot connecting the three ropes if $\theta_2 = 37°$ and $\theta_3 = 0°$? Rope 3 can be tied to the hook on the wall.

FIGURE P8.8



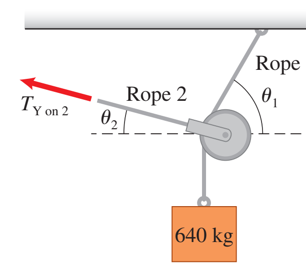
9. * Find the values of the forces the ropes exert on the knot if you replace the light in Problem 8.8 with a heavier 12-kg object and the ropes make angles of $\theta_2 = 63°$ and $\theta_3 = 45°$ (see Figure P8.8).
10. Redraw Figure P8.8 with $\theta_2 = 50°$ and $\theta_3 = 0°$. Rope 2 is found to exert a 100-N force on the knot. Determine m and the magnitudes of the forces that the other two ropes exert on the knot.
11. Determine the masses m_1 and m_2 of the two objects shown in **Figure P8.11** if the force exerted by the horizontal cable on the knot is 64 N.

FIGURE P8.11



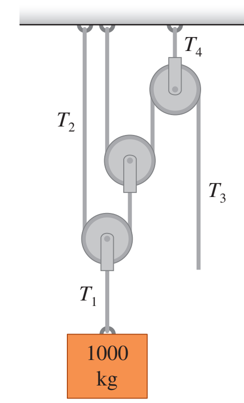
12. * **Lifting an engine** You work in a machine shop and need to move a huge 640-kg engine up and to the left in order to slide a cart under it. You use the system shown in **Figure P8.12**. How hard and in what direction do you need to pull on rope 2 if the angle between rope 1 and the horizontal is $\theta_1 = 60°$?

FIGURE P8.12



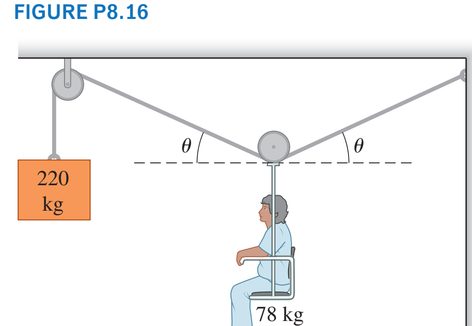
13. * **More lifting** You exert a 630-N force on rope 2 in the previous problem (Figure P8.12). Write the two equations (x and y) for the first condition of equilibrium using the pulley as the object of interest for a force diagram. Calculate θ_1 and θ_2. You may need to use the identity $(\sin\theta)^2 + (\cos\theta)^2 = 1$.
14. **Even more lifting** A pulley system shown in **Figure P8.14** will allow you to lift heavy objects in the machine shop by exerting a relatively small force. (a) Construct a force diagram for each pulley. (b) Use the equations of equilibrium and the force diagrams to determine T_1, T_2, T_3, and T_4.

FIGURE P8.14



15. * **Tightrope walking** A tightrope walker wonders if her rope is safe. Her mass is 60 kg and the length of the rope is about 20 m. The rope will break if its tension exceeds 6700 N. What is the smallest angle at which the rope can bend up from the horizontal on either side of her to avoid breaking?
16. * **Lifting patients** An apparatus to lift hospital patients sitting at the sides of their beds is shown in **Figure P8.16**. At what angle above the horizontal does the rope going under the pulley bend while supporting the 78-kg person hanging from the pulley?

FIGURE P8.16

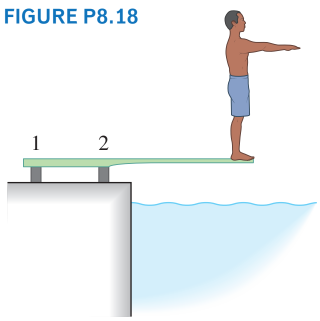


17. A father (80 kg), mother (56 kg), daughter (16 kg), and son (24 kg) try to occupy seats on the seesaw shown in **Figure P8.17** so that the seesaw is in equilibrium. Can they succeed? Explain.

FIGURE P8.17

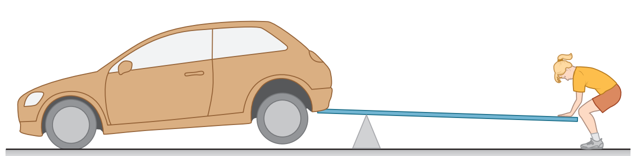
18. * You stand at the end of a uniform diving board a distance d from support 2 (similar to that shown in **Figure P8.18**). Your mass is m, and the distance between the two supports is a. What can you determine from this information? Make a list of physical quantities and show how you will determine them.

FIGURE P8.18



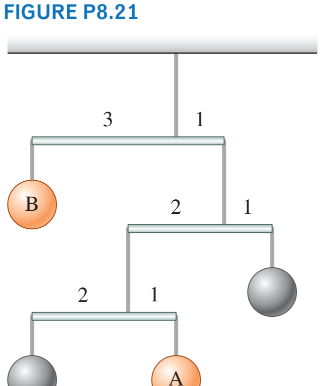
19. * You place a 3.0-m-long board symmetrically across a 0.5-m-wide chair to seat three physics students at a party at your house. If 70-kg Dan sits on the left end of the board and 50-kg Tahreen sits on the right end of the board, where should 54-kg Komila sit to keep the board stable? What assumptions did you make?
20. **Car jack** You've got a flat tire. To lift your car, you make a homemade lever (see **Figure P8.20**). A very light 1.6-m-long handle part is pushed down on the right side of the fulcrum and a 0.10-m-long part on the left side supports the back of the car. How hard must you push down on the handle so that the lever exerts an 8000-N force to lift the back of the car?

FIGURE P8.20



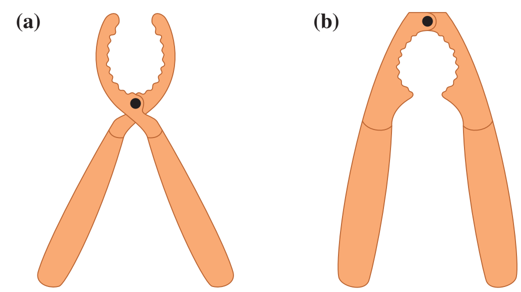
21. * **Mobile** You are building a toy mobile, copying the design shown in **Figure P8.21**. Object A has a 1.0-kg mass. What should be the mass of object B? The numbers in Figure P8.21 indicate the relative lengths of the rods on each side of their supporting cords.

FIGURE P8.21



22. **Another mobile** You are building a toy mobile similar to that shown in Figure P8.21 but with different dimensions and replacing the objects with cups. The bottom rod is 20 cm long, the middle rod is 15 cm long, and the top rod is 8 cm long. You put one penny in the bottom left cup, three pennies in the bottom right cup, eleven pennies in the middle right cup, and five pennies in the top left cup. (a) Draw a force diagram for each rod. (b) Determine the cord attachment points and lengths on each side for each rod. (c) What assumptions did you make in order to solve the problem?
23. **EST** Compare the two different designs of nutcracker shown in **Figure P8.23** and decide which one is more efficient in cracking a nut. Estimate the forces exerted by each cracker on the nut when a 30-N force is exerted on each handle. Indicate any assumptions that you made. You will need a ruler to solve this problem.

FIGURE P8.23

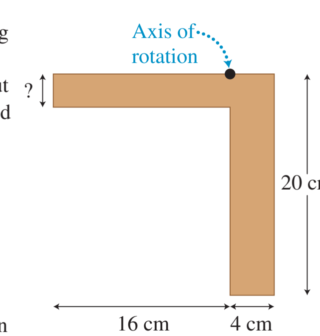


24. Ray decides to paint the outside of his uncle's house. He uses a 4.0-m-long board supported by vertical cables at each end to paint the second floor. The board has a mass of 21 kg. Ray (70 kg) stands 1.0 m from the left cable. What are the forces that each cable exerts on the board?
25. * A 2.0-m-long uniform beam of mass 8.0 kg supports a 12.0-kg bag of vegetables at one end and a 6.0-kg bag of fruit at the other end. At what distance from the vegetables should the beam rest on your shoulder to balance? What assumptions did you make?
26. * A uniform beam of length l and mass m supports a bag of mass m_1 at the left end, another bag of mass m_2 at the right end, and a third bag m_3 at a distance l_3 from the left end ($l_3 < 0.5l$). At what distance from the left end should you support the beam so that it balances?

8.4 Center of mass

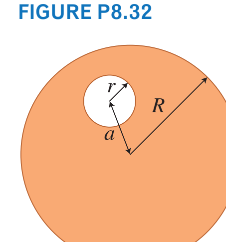
27. A person whose height is 1.88 m is lying on a light board placed on two scales so that scale 1 is under the person's head and scale 2 is under the person's feet. Scale 1 reads 48.3 kg and scale 2 reads 39.3 kg. Where is the center of mass of the person?
28. * A seesaw has a mass of 30 kg, a length of 3.0 m, and a fulcrum beneath its midpoint. It is balanced when a 60-kg person sits on one end and a 75-kg person sits on the other end. Locate the center of mass of the seesaw. Where is the center of mass of a uniform seesaw that is 3.0 m long and has a mass of 30 kg if two people of masses 60 kg and 75 kg sit on its ends?
29. * You decide to cut an L-shaped object out of cardboard so that the object is in static equilibrium if hung as shown in **Figure P8.29**. Find the missing dimension of the object. Cut the object out of some cardboard and check if your result is correct.

FIGURE P8.29



30. * You have a 1-m-wide, 2-m-long 14-kg wooden board. If you place a 5.0-kg pot of soup in one corner of the board, where is the center of mass of the board-pot system?
31. * An 80-kg clown sits on a 20-kg bike on a tightrope attached between two trees. The center of mass of the clown is 1.6 m above the rope, and the center of mass of the bike is 0.7 m above the rope. A load of what mass should be fixed onto the bike and hang 1.5 m below the rope so that the center of mass of the clown-bike-load system is 0.5 m below the rope? What is the force that the rope exerts on each tree if the angle between the rope and the horizontal is 10°?
32. ** **Figure P8.32** shows a disk of radius R with a circular hole of radius r cut a distance a from the center of the disk. Where is the disk's center of mass? (Hint: You can think of cutting the hole as adding material of negative mass to the original object.)

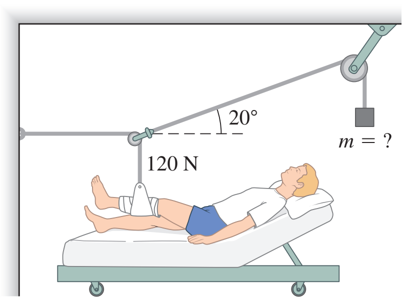
FIGURE P8.32



8.5 Skills for analyzing situations using equilibrium conditions

33. **Leg support** A person's broken leg is kept in place by the apparatus shown in **Figure P8.33**. If the rope pulling on the leg exerts a 120-N force on it, how massive should be the block hanging from the rope that passes over the pulley? First, derive a general expression for m in terms of relevant parameters and then determine the mass of the block.

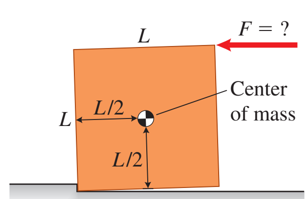
FIGURE P8.33



34. * **Diving board** The diving board shown in Figure P8.18 has a mass of 28 kg and its center of mass is at the board's geometrical center. Determine the forces that support posts 1 and 2 (separated by 1.4 m) exert on the board when a 60-kg person stands on the end of the board 2.8 m from support post 2.

35. ** A uniform cubical box of mass m and side L sits on the floor with its bottom left edge pressing against a ridge. Derive the expression for the least force you need to exert horizontally at the top right edge of the box that will cause its bottom right edge to be slightly off the floor, as shown in **Figure P8.35**. (Note: With the right edge slightly off the floor, the ground and ridge exert their forces on the bottom left edge of the box.)

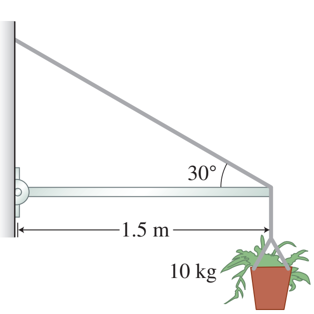
FIGURE P8.35



36. * If the force F shown in **Figure P8.35** is 840 N and the bottom right edge of the box is slightly off the ground, what is the mass of the cubical box of side 1.2 m?

37. * You decide to hang a new 10-kg flowerpot using the arrangement shown in **Figure P8.37**. Can you use a slanted rope attached from the wall to the end of the beam if that rope breaks when the tension exceeds 170 N? The mass of the beam is not known, but it looks light.

FIGURE P8.37



38. * You decide to hang another plant from a 1.5-m-long 2.0-kg horizontal beam that is attached by a hinge to the wall on the left. A cable attached to the right end goes 37° above the beam to a connecting point above the hinge on the wall. You hang a 100-N pot from the beam 1.4 m away from the wall. What is the force that the cable exerts on the beam?

39. * Now you decide to change the way you hang the pot described in Problems 8.37 and 8.38. You orient the beam at a 37° angle above the horizontal and orient the cable horizontally from the wall to the end of the beam. The beam still holds the 2.0-kg pot and plant hanging 0.1 m from its end. Now determine the force that the cable exerts on the beam and the force that the wall hinge exerts on the beam (its x- and y-components and the magnitude and direction of that force).

40. * What mechanical work must you do to lift a log that is 3.0 m long and has a mass of 100 kg from the horizontal to a vertical position? (Hint: Use the work-energy principle.) You are lifting one end of the log while the other end is on the ground all the time.

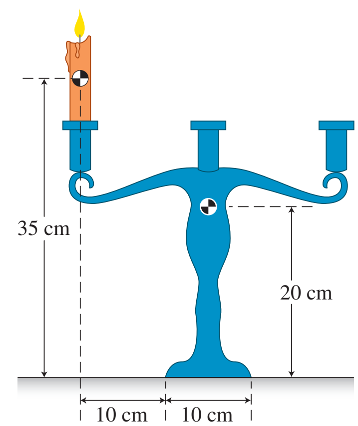
8.6 Stability of equilibrium

41. * A 70-g meter stick has a 30-g piece of modeling clay attached to the end. Where should you drill a hole in the meter stick so that you can hang the stick horizontally in equilibrium on a nail in the wall? Draw a picture to help explain your decision.

42. * You are trying to tilt a very tall refrigerator (2.0 m high, 1.0 m deep, 1.4 m wide, and 100 kg) so that your friend can put a blanket underneath to slide it out of the kitchen. Determine the force that you need to exert on the front of the refrigerator at the start of its tipping. You push horizontally 1.4 m above the floor.

43. Can you put a 0.2-kg candle on a 0.6-kg candleholder, as shown in **Figure P8.43**, without tipping the candleholder over? Explain. The centers of mass of the candle and candleholder are marked on the figure.

FIGURE P8.43



44. * You have an Atwood machine (see Figure 4.9) with two blocks each of mass m attached to the ends of a string of length l. The string passes over a frictionless pulley down to the blocks hanging on each side. While pulling down on one block, you release it. Both blocks continue to move at constant speed, one up and the other down. Is the system still in equilibrium? Find the vertical component of the center of mass of the two-block system. Indicate all of your assumptions and the coordinate system used.

45. * **EST** You stand sideways in a moving train. Estimate how far apart you should keep your feet so that when the train accelerates at $2.0\ m/s^2$ you can still stand without holding anything. List all your assumptions.

General Problems

46. **EST** Your hand holds a liter of milk (mass about 1 kg) while your arm is bent at the elbow in a 90° angle. Estimate the torque caused by the milk on your arm about the elbow joint. Indicate all numbers used in your calculations. This is an estimate, and your answer may differ by 10 to 50% from the answers of others.

47. **EST** **Body torque** You hold a 4.0-kg computer. Estimate the torques exerted on your forearm about the elbow joint caused by the downward force exerted by the computer on the forearm and the upward 340-N force exerted by the biceps muscle on the forearm. Ignore the mass of the arm. Indicate any assumptions you make.

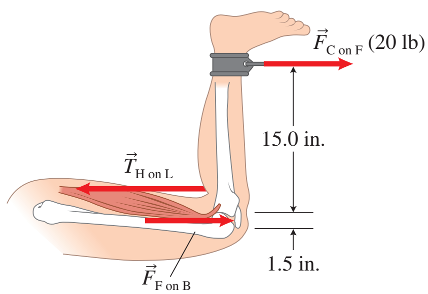
48. * **Using biceps to hold a child** A man is holding a 16-kg child using both hands with his elbows bent in a 90° angle. The biceps muscle provides the positive torque he needs to support the child. Determine the force that each of his biceps muscles must exert on the forearm in order to hold the child safely in this position. Ignore the triceps muscle and the mass of the arm.

49. **BIO** **Using triceps to push a table** A man pushes on a table exerting a 20-N downward force with his hand. Determine the force that his triceps muscle must exert on his forearm in order to balance the upward force that the table exerts on his hand. Ignore the biceps muscle and the mass of the arm. If you did not ignore the mass of the arm, would the force you determined be smaller or larger? Explain.

50. * **BIO** **Using biceps to hold a dumbbell** Find the force that the biceps muscle shown in Example 8.5 exerts on the forearm when you lift a 16-kg dumbbell with your hand. Also determine the force that the bone in the upper arm (the humerus) exerts on the bone in the forearm at the elbow joint. The mass of the forearm is about 5.0 kg and its center of mass is 16 cm from the elbow joint. Ignore the triceps muscle.

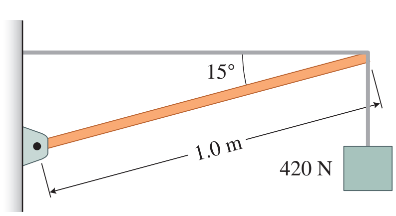
51. * **BIO** **Hamstring** You are exercising your hamstring muscle (the large muscle in the back of the thigh). You use an elastic cord attached to a hook on the wall while keeping your leg in a bent position (**Figure P8.51**). Determine the magnitude of the tension force $\vec{T}_{\text{H on L}}$ exerted by the hamstring muscles on the leg and the magnitude of compression force $\vec{F}_{\text{F on B}}$ at the knee joint that the femur exerts on the calf bone. The cord exerts a 20-lb force $\vec{F}_{\text{C on F}}$ on the foot.

FIGURE P8.51



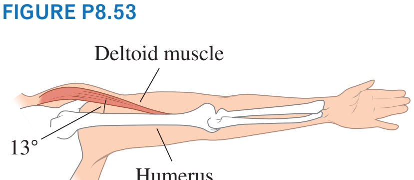
52. * **BIO** **Lift with bent legs** You injure your back at work lifting a 420-N radiator. To understand how it happened, you model your back as a weightless beam (**Figure P8.52**), analogous to the backbone of a person in a bent position when lifting an object. (a) Determine the tension force that the horizontal cable exerts on the beam (which is analogous to the force the back muscle exerts on the backbone) and the force that the wall exerts on the beam at the hinge (which is analogous to the force that a disk in the lower back exerts on the backbone). (b) Why do doctors recommend lifting objects with the legs bent?

FIGURE P8.52



53. * **BIO** **Dumbbell lift I** A woman lifts a 3.6-kg dumbbell in each hand with her arm in a horizontal position at the side of her body and holds it there for 3 s (see **Figure P8.53**). What force does the deltoid muscle in her shoulder exert on the humerus bone while holding the dumbbell? The deltoid attaches 13 cm from the shoulder joint and makes a 13° angle with the humerus. The dumbbell in her hand is 0.55 m from the shoulder joint, and the center of mass of her 4.0-kg arm is 0.24 m from the joint.

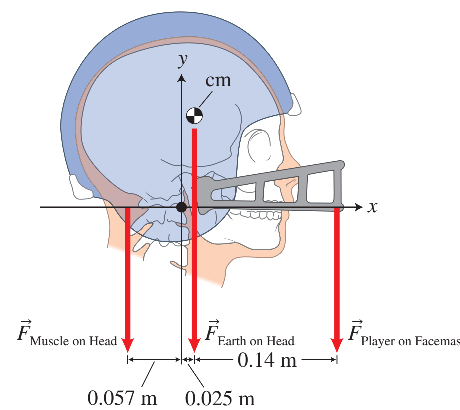
FIGURE P8.53



54. ** **BIO** **Dumbbell lift II** Repeat the previous problem with a 7.2-kg dumbbell. Determine both the force that the deltoid exerts on the humerus and the force that the lifter's shoulder joint exerts on her humerus.

55. * **BIO** **Facemask penalty** The head of a football running back (see **Figure P8.55**) can be considered as a lever with the vertebra at the bottom of the skull as a fulcrum (the axis of rotation). The center of mass is about 0.025 m in front of the axis of rotation. The torque caused by the force that Earth exerts on the 8.0-kg head/helmet is balanced by the torque caused by the downward forces exerted by a complex muscle system in the neck. That muscle system includes the trapezius and levator scapulae muscles, among others (effectively 0.057 m from the axis of rotation). (a) Determine the magnitude of the force exerted by the neck muscle system pulling down to balance the torque caused by the force exerted by Earth on the head. (b) If an opposing player exerts a downward 180-N (40-lb) force on the facemask, what muscle force would these neck muscles now need to exert to keep the head in equilibrium?

FIGURE P8.55



56. * Eiichi has purchased an adjustable hand grip to use for strengthening hands, fingers, and wrists. The hand grip consists of two handles that can rotate around a common axis and a spring that connects the handles (see **Figure P8.56**). The force needed to squeeze the hand grip can be adjusted from small (Figure P8.56a) to large (Figure P8.56b) by changing the position at which the lower end of the spring is hooked to the left handle. Eiichi, Yuko, and Lars have different explanations for why it is harder to squeeze the hand grip in case (b) compared to (a).

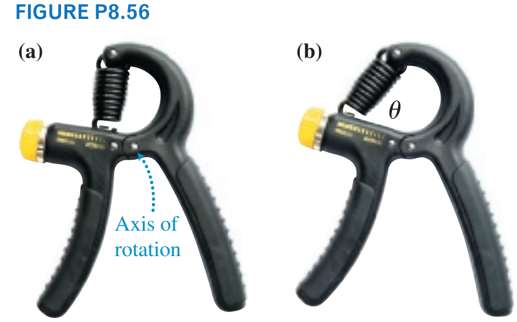
Eiichi: It is harder to squeeze the hand grip because the distance between the axis of rotation and the point where the force is exerted on the left handle by the spring is larger in case (b) than in case (a).

Yuko: It is harder to squeeze the hand grip because the extension of the spring in case (b) is larger than in case (a).

Lars: It is harder to squeeze the hand grip because the angle θ in case (b) is smaller than in case (a).

First, comment on the students' explanations and decide whose ideas are correct and whose aren't. Then construct a complete correct explanation. Indicate any assumptions that you made.

FIGURE P8.56

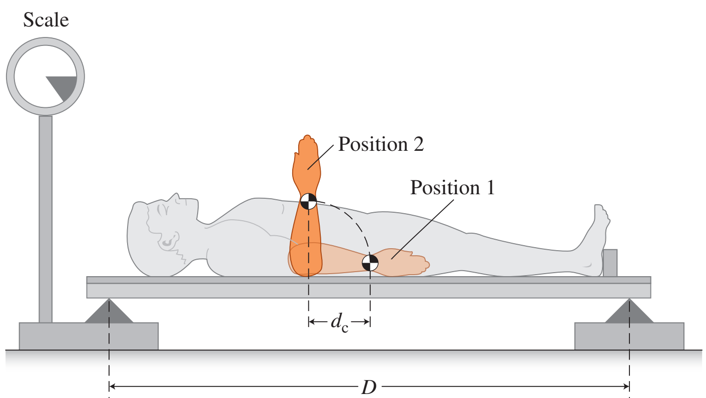


57. * BIO While browsing books on neurophysiology, you come across a book published in 1967 by Soviet neurophysiologist Nikolai Aleksandrovich Bernstein, *The Co-ordination and Regulation of Movements*, in which he describes a technique to determine the mass of a part of the human body if the position of the center of mass of that body part is known. His technique for determining the mass of the forearm is described as follows: The person lies on a support board, as shown in **Figure P8.57**. Two readings of the scale are taken: first with the forearm held in position 1 (m_1) and second with it in position 2 (m_2). Knowing the distance from the elbow to the center of mass of the forearm (d_c) and the distance between the knife edges supporting the board (D), the mass of the forearm and hand can be calculated from the following expression:

$$m_{fh} = \frac{D(m_2 - m_1)}{d_c}$$

(a) First, without deriving the expression, evaluate it to see if it is reasonable. Are the units correct? Is the sign of the expression positive? Are qualitative dependences reasonable? (b) Now derive the expression. (c) Why do you think it is important to place the support board on knife edges instead of rigid blocks?

FIGURE P8.57



58. ** **Touch detector** You have two force sensors connected to a computer and a meter stick of known mass. The sensors are used to keep the stick horizontal; there are no other supports. You push on the stick with your finger in an arbitrary location. (a) Design an experimental setup that will allow you to determine the magnitude of your pushing force F and the location of your finger x based on the readings of the two force sensors. (b) Derive an expression that can be used as a computer algorithm to calculate x and F using the readings of the force sensors and the parameters of your setup. (c) Evaluate the expression, analyzing the limiting cases.

59. ** Design two experiments to determine the mass of a ruler, using different methods. Your available materials are the ruler, a spring, and a set of objects of standard mass: 50 g, 100 g, and 200 g. One of the methods should involve your knowledge of static equilibrium. After you design and perform the experiment, decide whether the two methods give you the same or different results.

60. * An 80-kg person stands at one end of a 130-kg boat. He then walks to the other end of the boat so that the boat moves 80 cm with respect to the bottom of the lake. (a) What is the length of the boat? (b) How much did the center of mass of the person-boat system move when the person walked from one end to the other? (Hint: Note that the total momentum of the person-boat system remains constant.)

61. EST Two people (50 kg and 75 kg) holding hands stand on Rollerblades 1.0 m apart. (a) Estimate the location of their center of mass. (b) The two people push each other and roll apart. Estimate the new location of the center of mass when they are 4.0 m apart. What assumptions did you make? (Hint: Note that the total momentum of the two people remains constant.)

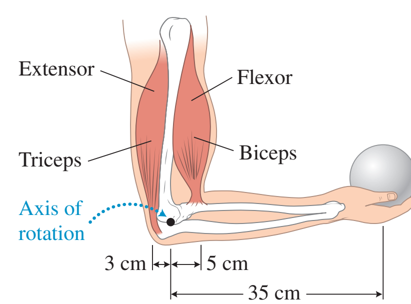
62. ** Find the center of mass of an L-shaped object. The vertical leg has a mass of m_a of length a and the horizontal leg has a mass of m_b of length b. Both legs have the same width w, which is much smaller than a or b.

Reading Passage Problems

BIO **Muscles work in pairs** Skeletal muscles produce movements by pulling on tendons, which in turn pull on bones. Usually, a muscle is attached to two bones via a tendon on each end of the muscle. When the muscle contracts, it moves one bone toward the other. The other bone remains in nearly the original position. The point where a muscle tendon is attached to the stationary bone is called the *origin*. The point where the other muscle tendon is attached to the movable bone is called the *insertion*. The origin is like the part of a door spring that is attached to the doorframe. The insertion is similar to the part of the spring that is attached to the movable door.

During movement, bones act as levers and joints act as axes of rotation for these levers. Most movements require several skeletal muscles working in groups, because a muscle can only exert a pull and not a push. In addition, most skeletal muscles are arranged in opposing pairs at joints. Muscles that bring two limbs together are called flexor muscles (such as the biceps muscle in the upper arm in **Figure 8.25**). Those that cause the limb to extend outward are called extensor muscles (such as the triceps muscle in the upper arm). The flexor muscle is used when you hold a heavy object in your hand; the extensor muscle can be used, for example, to extend your arm when you throw a ball.

FIGURE 8.25 Muscles often come in flexor-extensor pairs.



63. You hold a 10-lb ball in your hand with your forearm horizontal, forming a 90° angle with the upper arm (Figure 8.25). Which type of muscle produces the torque that allows you to hold the ball?
(a) Flexor muscle in the upper arm
(b) Extensor muscle in the upper arm
(c) Flexor muscle in the forearm
(d) Extensor muscle in the forearm

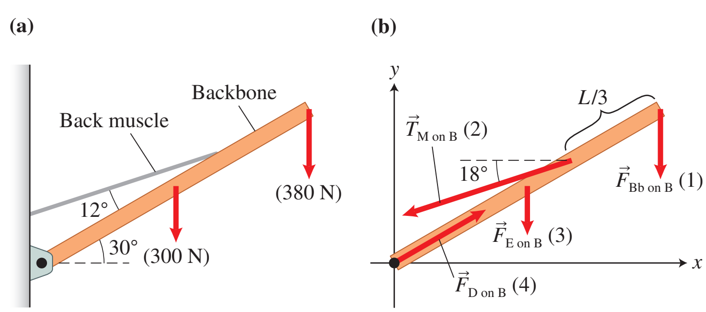
64. In Figure 8.25, how far in centimeters from the axis of rotation are the forces that the ball exerts on the hand, that the biceps exerts on your forearm, and that the upper arm exerts on your forearm at the elbow joint?
(a) 0, 5, 35 (b) 35, 5, 0 (c) 35, 5, 3
(d) 35, 5, −3 (e) 30, 5, 0

65. Why is it easier to hold a heavy object using a bent arm than a straight arm?
(a) More flexor muscles are involved.
(b) The distance from the joint to the place where gravitational force is exerted by Earth on the object is smaller.
(c) The distance from the joint to the place where force is exerted by the object on the hand is smaller.
(d) There are two possible axes of rotation instead of one.

66. Why are muscles arranged in pairs at joints?
(a) Two muscles can produce a bigger torque than one.
(b) One can produce a positive torque and the other a negative torque.
(c) One muscle can pull on the bone and the other can push.
(d) Both a and b are true.

BIO Improper lifting and the back A careful study of human anatomy allows medical researchers to use the conditions of equilibrium to estimate the internal forces that body parts exert on each other while a person lifts in a bent position (see Figure 8.19). Suppose an 800-N (180-lb) person lifts a 220-N (50-lb) barbell in a bent position. The situation can be represented with a mechanical model (**Figure 8.26a**). The cable (the back muscle) exerts a tension force $\vec{T}_{\text{M on B}}$ on the backbone and the support at the bottom of the beam (the disk in the lower back) exerts a compression force $\vec{F}_{\text{D on B}}$ on the backbone. The backbone in turn exerts the same magnitude force on the 2.5-cm-diameter fluid-filled disks in the lower backbone. Such disk compression can cause serious back problems. A force diagram of this situation is shown in Figure 8.26b. The magnitude of the gravitational force $\vec{F}_{\text{E on B}}$ that Earth exerts on the center of mass of the upper stomach-chest region is 300 N. Earth exerts a 380-N force on the head, arms, and 220-N barbell held in the hands. Using the conditions of equilibrium, we estimate that the back muscle exerts a 3400-N (760-lb) force $\vec{T}_{\text{M on B}}$ on the backbone and that the disk in the lower back exerts a 3700-N (830-lb) force $\vec{F}_{\text{D on B}}$ on the backbone. This is like supporting a grand piano on the 2.5-cm-diameter disk.

FIGURE 8.26 Analysis of a person's backbone when lifting from a bent position.



67. Rank in order the magnitudes of the torques caused by the four forces exerted on the backbone (see Figure 8.26b), with the largest torque listed first.
(a) $1 > 2 > 3 > 4$ (b) $2 = 3 > 1 > 4$
(c) $3 > 2 > 1 > 4$ (d) $2 > 1 > 3 > 4$
(e) $1 = 2 = 3 = 4$

68. What are the signs of the torques caused by forces 1, 2, 3, and 4, respectively, about the origin of the coordinate system shown in Figure 8.26b?
(a) $+, +, +, +$ (b) $-, +, -, 0$
(c) $+, -, +, 0$ (d) $-, -, -, 0$
(e) $+, -, +, -$

69. Which expression below best describes the torque caused by force $F_3 = F_{\text{E on B}}$, the force that Earth exerts on the upper body at its center of mass for the backbone of length L?
(a) 0 (b) $F_3(2L/3)\sin 12°$
(c) $F_3(L/2)\cos 30°$ (d) $-F_3(2L/3)\sin 12°$
(e) $-F_3(L/2)\cos 30°$

70. Which expression below best describes the torque caused by force $F_2 = T_{\text{M on B}}$ exerted by the muscle on the backbone?
(a) 0 (b) $F_2(2L/3)\sin 12°$
(c) $F_2(L)\cos 30°$ (d) $-F_2(2L/3)\sin 12°$
(e) $-F_2(L)\cos 30°$