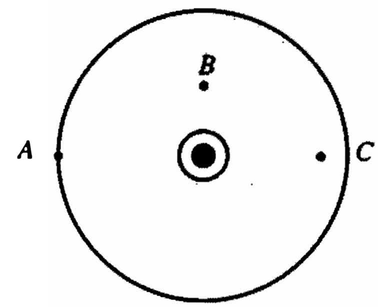
I. Motion with constant angular velocity

A wheel is spinning *counterclockwise* at a constant rate about a fixed axis. The diagram at right represents a snapshot of the wheel at one instant in time.



Top view
Wheel spins *counterclockwise*

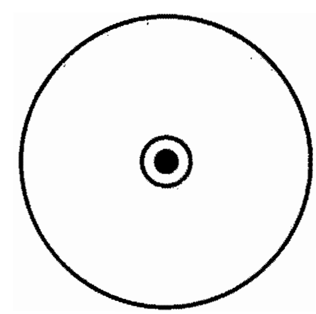
A. Draw arrows on the diagram to represent the direction of the velocity for each of the points *A, B,* and *C* at the instant shown. Explain your reasoning.

Is the time taken by points *B* and *C* to move through one complete circle *greater than, less than,* or *the same as* the time taken by point *A?*

On the basis of your answer above, determine how the speeds of points *A, B,* and *C* compare. Explain.

B. Mark the position of each of the labeled points at a later time when the wheel has completed one half of a turn. Sketch a velocity vector at each point.

For each labeled point, how does the velocity compare to the velocity at the earlier time in part A? Discuss both magnitude and direction.



Top view
Wheel spins *counterclockwise*

Is there one single *linear velocity vector* that applies to every point on the wheel at all times? Explain.

C. Suppose the wheel makes one complete revolution in 2 seconds.

1. For each of the following points, find the change in angle ($\Delta\theta$) of the position vector during one second. (*i.e.*, Find the angle between the initial and final position vectors.)

    - point *A*

    - point *B*

    - point *C*

2. Find the rate of change in the angle for any point on the wheel.

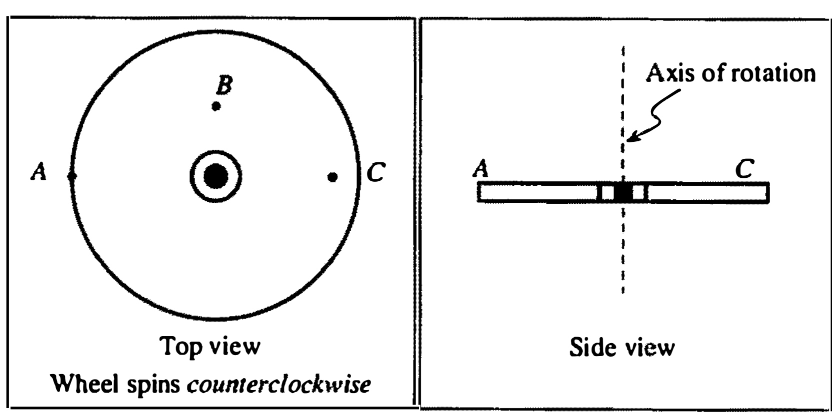
The rate you calculated above is called the *angular speed* of the wheel, or equivalently, the magnitude of the *angular velocity* of the wheel. The angular velocity is defined to be a vector that points along the axis of rotation and is conventionally denoted by the symbol $\vec{\omega}$ (the Greek letter *omega*). To determine the direction of the angular velocity vector, we imagine an observer on the axis of rotation who is looking toward the object. If the observer sees the object rotating counterclockwise, the angular velocity vector is directed toward the observer; if the observer sees it rotating clockwise, the angular velocity vector is directed away from the observer.

D. Would two observers on either side of a rotating object agree on the *direction* of the angular velocity vector? Explain.

Would two observers who use different points on an object to determine the angular velocity agree on the *magnitude* of the angular velocity vector? Explain.

E. The diagrams at right show top and side views of the spinning wheel in part A.

On each diagram, draw a vector to represent the angular velocity of the wheel. (Use the convention that ⊙ indicates a vector pointing *out of* the page and ⊗ indicates a vector pointing *into* the page.)



Top view
Wheel spins *counterclockwise*
Side view

F. In the space at right sketch the position vectors for point C at the beginning and at the end of a small time interval Δt.

Sketch of position vectors at t_o and $t_o + \Delta t$

1. Label the change in angle ($\Delta\theta$) and the distance between the center of the wheel and point C (r_C). Sketch the path taken by point C during this time interval.

   What is the distance that point C travels during Δt? Express your answer in terms of r_C and $\Delta\theta$.

2. Use your answer above and the definition of linear speed to derive an algebraic expression for the linear speed of point C in terms of the angular speed ω of the wheel.

   What does your equation imply about the relative linear speeds for points farther and farther out on the wheel? Is this consistent with your answer to part A?

II. Motion with changing angular velocity

A. Let $\vec{\omega}_o$ represent the initial angular velocity of a wheel. In each case described below, determine the magnitude of the *change in angular velocity* $|\Delta\vec{\omega}|$ in terms of $|\vec{\omega}_o|$.

1. The wheel is made to spin faster, so that eventually, a fixed point on the wheel is going around twice as many times each second. (The axis of rotation is fixed.)

2. The wheel is made to spin at the same rate but in the opposite direction.

B. Suppose the wheel slows down uniformly, so that $|\vec{\omega}|$ decreases by 8π rad/s every 4 s. (The wheel continues spinning in the same direction and has the same orientation.)

   Specify the angular acceleration $\vec{\alpha}$ of the wheel by giving its magnitude and, relative to $\vec{\omega}$, its direction.

   In linear kinematics we find the acceleration vector by first constructing a *change in velocity vector* $\Delta\vec{v}$ and then dividing that by Δt. Describe the analogous steps that you used above to find the angular acceleration $\vec{\alpha}$.

➪ Discuss your answers above with a tutorial instructor before continuing.

III. Torque and angular acceleration

The rigid bar shown at right is free to rotate about a fixed pivot through its center. The axis of rotation of the bar is perpendicular to the plane of the paper.

A. A force of magnitude F_o is applied to point M as shown. The force is *always* at a right angle to the bar.

For each of the following cases, determine whether the angular acceleration would be in a *clockwise* or *counterclockwise* sense.

- The bar was initially at rest. *(Hint:* Consider $\Delta\vec{\omega}$.*)*

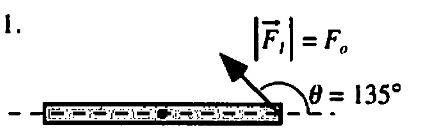
- The bar was spinning at a constant rate before the force was applied.

  Does your answer for the angular acceleration depend on whether the bar was originally spinning clockwise or counterclockwise? Explain.

The application point and direction of a force can affect the rotational motion of the object to which the force is applied. The tendency of a particular force to cause an angular acceleration of an object is quantified as the *torque* produced by the force. The torque $\vec{\tau}$ is defined to be the vector cross product $\vec{r} \times \vec{F}$, where $\vec{r}$ is the vector from the axis of rotation to the point where the force is applied. The magnitude of the torque is simply $|\vec{\tau}| = |\vec{r}|\,|\vec{F}|\,\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$.

B. Compare the magnitude of the *net torque* about the pivot in part A to that in each case below.

1.



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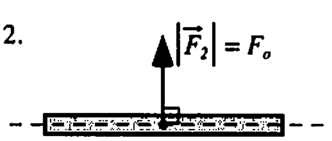

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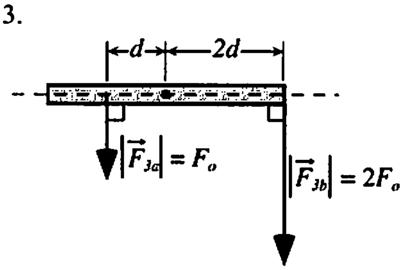

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